1. (d)

Let $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$, $A' = \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix}$

Since A is orthogonal, $\therefore \phi 0 \ AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}\begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation the corresponding elements, we have

$$\left.\begin{matrix} 4\beta^2+\gamma^2 = 1 \\ 2\beta^2-\gamma^2 = 0 \end{matrix}\right\} \Rightarrow \beta = \pm\frac{1}{\sqrt{6}}, \gamma = \pm\frac{1}{\sqrt{3}}$$

$$\alpha^2+\beta^2+\gamma^2 = 1 \Rightarrow \alpha^2 + \frac{1}{6} + \frac{1}{3} = 1 \Rightarrow a = \pm\frac{1}{\sqrt{2}}.$$

Hence (D) is correct answer.

2. (c)

$A^T A = I \Rightarrow |A^T A| = |I| \Rightarrow |A|^2 = 1$

$\Rightarrow (a^3 + b^3 + c^3 - 3abc)^2 = 1$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 1$

(since a, b, c are positive real number

$\Rightarrow a^3 + b^3 + c^3 \geq 3abc$ from AM $\geq$ GM)

$\Rightarrow a^3 + b^3 + c^3 = 4$.Hence (C) is correct answer.

3. (c)

For $PP' = 1$,

$$\begin{bmatrix} 2/3 & 3k & a \\ -1/3 & -4k & b \\ 2/3 & -5k & c \end{bmatrix}\begin{bmatrix} 2/3 & -1/3 & 2/3 \\ 3k & -4k & -5k \\ a & b & c \end{bmatrix}$$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.Performing matrix multiplication, we have

$$\frac{4}{9} + 9k^2 + a^2 = 1,\ \frac{1}{9} + 16k^2 + b^2 = 1,\ \frac{4}{9} + 25k^2 + c^2 = 0$$

$$\Rightarrow a^2 = \frac{169}{450}, b^2 = \frac{256}{450}, c^2 = \frac{25}{450}$$

Also $\frac{4}{9} - 15k^2 + ac = 0, -\frac{2}{9} + 20k^2 + bc = 0,$

$-\frac{2}{9} - 12k^2 + ab = 0$

$\Rightarrow ab = \frac{208}{450}, bc = -\frac{80}{450}, ac = \frac{-65}{450}$.Hence $a = \pm\frac{13}{5\sqrt{2}}$, b

$= \pm\frac{16}{5\sqrt{2}}, c \mp \frac{1}{3\sqrt{2}}$.

Hence (C) is correct answer.

4. (a)

As A is a non-singular matrix $A^{-1} = \frac{1}{|A|}$ (adj A)

$\Rightarrow$ adj A = $|A|\ A^{-1}$ = B(say).

Now,

adj (adj A) = adj (B) = $|B|B^{-1} = ||A|A^{-1}|\ (|A|\ A^{-1})^{-1}$

$= |A|^3\ |A^{-1}|\ |A|^{-1}\ (A^{-1})^{-1}$

[using scalar multiple property of determinants]

$= |A|^3\ \frac{1}{|A|} \cdot \frac{1}{|A|} A$

$= |A|$ A.

$= |A|^3\ \frac{1}{|A|} \cdot \frac{1}{|A|} A$

$= |A|$ A.

5. (b)

We have

$[A\ (A + B)^{-1}\ B]^{-1} = B^{-1}\ ((A + B)^{-1})^{-1}\ A^{-1}$

$= B^{-1}(A + B)A^{-1} = (B^{-1}\ A + I)\ A^{-1}$

$= B^{-1}\ I + IA^{-1} = B^{-1} + A^{-1}$.

Hence (B) is correct answer.

6. (c)

det $(B^{-1}\ AB)$ = det (B^{-1}) det A det B= det (B^{-1}).det B. det A = det $(B^{-1}\ B)$.det A

= det (I).det A = 1.det A = det A.

Hence (C) is correct.

7. (d)

Here the rank of A is 3

Therefore, the minor of order 3 of A $\neq 0$.

$$\Rightarrow \begin{vmatrix} y+a & b & c \\ a & y+b & c \\ a & b & y+c \end{vmatrix} \neq 0$$

$$\Rightarrow (y+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & y+b & c \\ 1 & b & y+c \end{vmatrix} \neq 0$$

[Applying $C_1 \to C_1 + C_2 + C_3$ and taking $(y + a + b + c)$ common from C_1]

$\Rightarrow (y+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & y & 0 \\ 0 & 0 & y \end{vmatrix} \neq 0$[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$\Rightarrow (y + a + b + c)\ (y^2) \neq 0$ [Expanding along C_1]$\Rightarrow y \neq 0$ and $y \neq -(a + b + c)$

Hence (D) is correct answer.

8. (a)

Let $B = I + A + A^2 + \ldots + A^{k-1}$

Post multiply both sides by (I - A), so that

$B(I - A) = (I + A + A^2 + \ldots + A^{k-1})(I - A)$

$= I - A + A - A^2 + A^2 - A^3 + \ldots - A^{k-1} + A^{k-1} - A^k$

$= I - A^k = I$, since $A^k = 0$

$\Rightarrow B = (I - A)^{-1}$

Hence $(I - A)^{-1} = I + A + A^2 + \ldots + A^{k-1}$.

Thus, p = -1.

Hence (A) is correct answer.

9. (b)

By using Cramer's rule, the solution of the system is

$x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$, where $\Delta = \begin{vmatrix} 3 & m \\ 2 & -5 \end{vmatrix} = -(15 + 2m)$

$\Delta_x = \begin{vmatrix} m & m \\ 20 & -5 \end{vmatrix} = -25m$, $\Delta_y = \begin{vmatrix} 3 & m \\ 2 & 20 \end{vmatrix} = 60 - 2m$

$\Rightarrow x = \frac{-25m}{-(15+2m)} = \frac{25m(15+2m)}{(15+2m)^2} > 0,$

for $m > 0$, or $m < -\frac{15}{2}$.

Also $y = \frac{60-2m}{-(15+2m)} = \frac{2(m-30)(15+2m)}{(15+2m)^2} > 0$

for $m > 30$, or $m < -\frac{15}{2}$.

$\Rightarrow x > 0, y > 0$ for $m > 30$ or $m < -\frac{15}{2}$

For $m = -\frac{15}{2}$, the system has no solution.

Hence (B) is correct answer.

10. (b)

$A = BX \Rightarrow B^{-1}A = (B^{-1}B)X \Rightarrow B^{-1}A = X$

$\because B^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow X = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$

11. (b)

$$\begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix} \Big/ 1+\tan^2\theta/2$$

$$= \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix} \Big/ 1+\tan^2\theta/2$$

$$= \begin{bmatrix} \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} & \frac{-2\tan\theta/2}{1+\tan^2\theta/2} \\ \frac{2\tan\theta/2}{1+\tan^2\theta/2} & \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

12. (c)

Let $A = k\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \therefore \phi 0\ A^T = \kappa \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

Since A is orthogonal $AA^T = I$

$\Rightarrow k^2 \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = I$

$\Rightarrow k^2 \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = I \Rightarrow 9k^2 I = I \Rightarrow k^2 = 1/9 \Rightarrow k = \pm 1/3$

13. (c)

$AB = \begin{bmatrix} \cos^2\theta\cos^2\phi + \sin\theta\cos\theta\cos\phi\sin\phi & \cos^2\theta\sin\phi\cos\phi + \sin\theta\cos\theta\sin^2\phi \\ \cos\theta\sin\theta\cos^2\phi + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\sin\phi\cos\phi + \sin^2\theta\sin^2\phi \end{bmatrix}$

$AB = \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \sin\phi\cos\theta\cos(\theta-\phi) \\ \sin\theta\cos\phi\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$

$\because AB = 0 \quad \Rightarrow \cos(\theta - \phi) = 0$

$\Rightarrow \cos(\theta - \phi) = \cos(2n+1)\frac{\pi}{2}$ where n = 0, 1, 2, …

$\theta = (2n+1)\frac{\pi}{2} + \phi$ where n = 0, 1, 2, ……

14. (d)

${}^3C_1 + {}^3C_1 \cdot {}^3C_2 = 3 + 3.3 = 3 + 9 = 12$

15. (b)

$A^2 + B^2 = AA + BB = A(BA) + B(AB)$

$= (AB)A + (BA)B = BA + AB = A + B$

16. (a)

Given $1 + P + P^2 + \ldots\ldots + P^n = O$

$P^{-1} + (P^{-1}P) + (P^{-1}P)P + \ldots.. + (P^{-1}P)P^{n-1} = O$

$\Rightarrow P^{-1} + I + IP + \ldots\ldots.. + IP^{n-1} = O$

$\Rightarrow P^{-1} = -(-P^n) \Rightarrow P^{-1} = P^n$

17. (a)

$P^T Q^{2005} P = P^T (PAP^T)^{2005} P$

$= P^T \frac{\{(PAP^T)(PAP^T)\ldots\ldots..(PAP^T)\}}{2005 \text{ times}} P$

$= \frac{(P^TP)A(P^TP)A(P^TP)\ldots\ldots..(P^TP)A(P^TP)}{2005 \text{ times}} = A^{2005}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $A^3 = A^2A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ …. and so on.

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix} \Rightarrow P^T Q^{2005} P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

18. (a)

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow A^{4n} = (I)^n = I$`

19. (a)

$\therefore X^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix} \Rightarrow X^n = 2^{n-1} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

20. (b)

$A^3 - 3A^2 - I = \begin{bmatrix} 7 & 9 & 3 \\ 15 & 19 & 6 \\ 9 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 9 & 3 \\ 15 & 18 & 6 \\ 9 & 12 & 3 \end{bmatrix}$

$a - \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = 0 \Rightarrow A^3 - 3A^2 - I = 0$

21. (b)

$|3AB| = 3 \times 3 \times 3\ |A|\ |B| = 27\ (-1)\ (3) = -81$

22. (c)

$\left(\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix} - 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)\left(\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix} - 3\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)$

$= \begin{bmatrix}2 & 2\\-1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\-1 & -2\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = 0$

23. (d)

$\begin{bmatrix}\alpha & \beta\\\gamma & -\alpha\end{bmatrix}^2 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}\alpha & \beta\\\gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\\gamma & -\alpha\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}\alpha^2 + \beta\gamma & 0\\0 & \alpha^2 + \beta\gamma\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \Rightarrow \alpha^2 + \beta\gamma = 1$

24. (d)

Inverse is $\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$

25. (c)

Use $A.A^{-1} = I$

26. (b)

we have A = iB

$\Rightarrow\ A^2 = (iB)^2 = i^2B^2 = -B^2$

$= \begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix} = -2B$

$\Rightarrow\ A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow\ A^8 = (A^4)^2 = (8B)^2 = 64B^2 = 128B$

27. (b)

As AB = I, we get $B = A^{-1}$

$B = \dfrac{1}{1+\tan^2\theta}\begin{bmatrix}1 & -\tan\theta\\\tan\theta & 1\end{bmatrix}$

$\Rightarrow (\sec^2\theta)\ B = \begin{bmatrix}1 & -\tan\theta\\\tan\theta & 1\end{bmatrix} = A(-\theta)$

28. (a)

(B' A B)' = B'. A' (B')' = B' A' B

if A is symmetric

then (B'. A . B)' = B'. A . B

hence symmetric

29. (d)

$\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}\alpha^2 & 0\\\alpha+1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$

$\Rightarrow \alpha^2 = 1\alpha + 1 = 5$

$\Rightarrow \alpha = +1, -1$

satisfying $\alpha = 4$ (AB surd)

30. (b)

$\begin{bmatrix}1, & 0\\-1, & 7\end{bmatrix}\begin{bmatrix}1, & 0\\-1, & 7\end{bmatrix} = 8\begin{bmatrix}1, & 0\\-1, & 7\end{bmatrix} + k\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

or $\begin{bmatrix}1+0 & 0+0\\-1-7 & 0+49\end{bmatrix} = z\begin{bmatrix}8 & 0\\-8 & 56\end{bmatrix} + \begin{bmatrix}k & 0\\0 & k\end{bmatrix}$

or $\begin{bmatrix}1 & 0\\-8 & 49\end{bmatrix} = \begin{bmatrix}k+8 & 0\\-8 & k+56\end{bmatrix}$

$\Rightarrow 1 = k + 8$

$49 = k + 56$

$\Rightarrow k = -7$