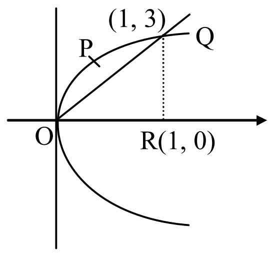
1. **(c)**

Required area



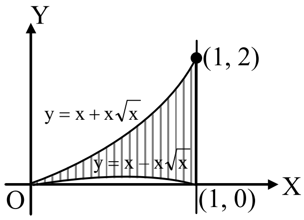
= area OPQRO - area ΔOQR

$$= \int_0^1 \sqrt{9x}\, dx - \frac{1}{2} \times 1 \times 3 = 3\ \frac{2}{3}\ x^{3/2}\Big|_0^1 - \frac{3}{2} = \frac{1}{2}.$$

2. **(b)**

Given curve is $(y - x)^2 = x^3$

$\Rightarrow y - x = \pm x\sqrt{x}$ or $y = x \pm x\sqrt{x}$



$\therefore$ Required area = $\left|\int_0^1 [(x + x\sqrt{x} - (x - x\sqrt{x})]dx\right| =$

$$\left|2\int_0^1 x^{3/2}dx\right| = \left|2.\frac{2}{5}\right| = \frac{4}{5} \text{ sq. units.}$$

3. **(b)**

Given curve is $y = x^4 - 2x^3 + x^2 + 3$ $\frac{dy}{dx} = 4x^3 - 6x^2 + 2x$

$\frac{d^2y}{dx^2} = 12x^2 - 12x + 2$ For maxima and minima $\frac{dy}{dx} = 0$ $4x^3 - 6x^2 + 2x = 0$ then $x = 0, \frac{1}{2}, 1$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{x=0} = 2, \left.\frac{d^2y}{dx^2}\right|_{x=\frac{1}{2}} = -1 \text{ nd } \left.\frac{d^2y}{dx^2}\right|_{x=1} = 2$$

$\therefore$ Points of minimum are x = 0 and x = 1

$$\therefore \text{Required area} = \int_0^1 (x^4 - 2x^3 + x^2 + 3)\,dx$$

$$= \left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} + 3x\right]_0^1$$

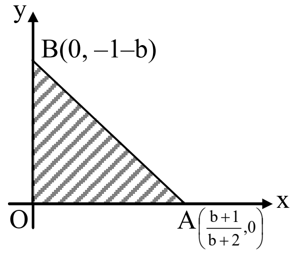
$$= \frac{1}{5} - \frac{2}{4} + \frac{1}{3} + 3 = \frac{1}{5} - \frac{1}{2} + \frac{1}{3} + 3 = \frac{27}{10} \text{ sq. units.}$$

4. **(a)**

Equation of tangent at (1, 1) is

$y - 1 = (2 + b)(x - 1)$ for first quadrant $b < 0$ $\therefore \phi 0$ Αρεα = 2

$\frac{1}{2} \times \frac{b+1}{b+2} \times (-1 - b) = 2 \Rightarrow (b + 1)^2 + 4b + 8 = 0 \Rightarrow b^2 + 6b + 9 = 0$



$\Rightarrow (b + 3)^2 = 0$ $\therefore \phi 0$ $\beta = -3$.

5. **(b)**

Given curve $a^4y^2 = (2a - x)\,x^5$

cutt off x-axis when y = 0 0 = (2a - x) x^5 x = 0, 2aHence the area bounded by the curve

$a^4y^2 = (2a - x)\,x^5$ is

$A_1 = \int_0^{2a} \frac{\sqrt{(2a - x)}}{a^2} x^{5/2}\,dx$ Put $x = 2a\sin^2\theta$ $\therefore dx = 4a\sin\theta\cos\theta\, d\theta$ $\therefore \phi 0$ $A_1 = \int_0^{\pi/2} \frac{\sqrt{2a}.\cos\theta(2a)^{5/2}.\sin^5\theta\ 4a\sin\theta\cos\theta}{a^2} d\theta$

$$= \int_0^{\pi/2} 32a^2 \sin^6\theta \,.\, \cos^2\theta\, d\theta$$

$$= 32a^2 . \frac{(5.3.1)(1)}{8.6.4.2} . \frac{\pi}{2} = \frac{5\pi a^2}{8} \text{ (By walli's formula)}$$

Area of circle $A_2 = \pi a^2$

$\therefore \frac{A_1}{A_2} = \frac{5}{8} \Rightarrow A_1 : A_2 = 5 : 8.$

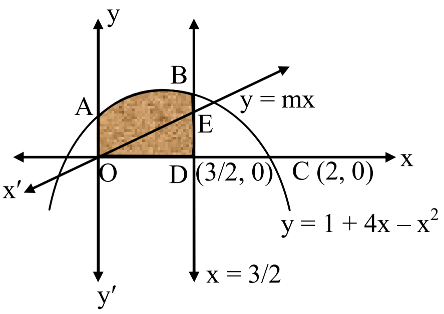
6. **(a)**

We have, $y = 1 + 4x - x^2$

$\Rightarrow x^2 - 4x = -y + 1$

$\Rightarrow (x - 2)^2 = -(y - 5)$

This equation represents a parabola having vertex at (2, 5) and opens downward. The area enclosed by this parabola and the line x = 0, y = 0, x = $\frac{3}{2}$ is shaded figure.



Since y = mx bisects the area of the shaded region. Therefore, Area of the region ODBAO = 2 Area of the region ODEO

$\Rightarrow \int_0^{3/2} (1 + 4x - x^2)\,dx = 2\int_0^{3/2} mx\,dx$

$\Rightarrow \left[x + 2x^2 - \frac{x^3}{3}\right]_0^{3/2} = 2m\left[\frac{x^2}{2}\right]_0^{3/2}$

$\Rightarrow \frac{3}{2} + \frac{9}{2} - \frac{9}{8} = m \times \frac{9}{4} \Rightarrow \frac{39}{8} = \frac{9m}{4} \Rightarrow m = \frac{13}{6}$.

Hence (A) is the correct answer.

7. (a)

It is given that $f(x) > x$, for all $x > 1$. So, Area bounded by $y = f(x)$, $y = x$ and the lines $x = 1$, $x = t$ is given by

$\int_1^t \{f(x) - x\}dx$ But this area is given equal to $(t + \sqrt{1+t^2} - \sqrt{2} - 1)$ sq. units. Therefore,

$\int_1^t \{f(x) - x\}\,dx = t + \sqrt{1+t^2} - \sqrt{2} - 1$, for all $t > 1$ On

differentiating both sides w.r.t. t, we get

$f(t) - t = 1 + \frac{t}{\sqrt{1+t^2}}$ for all $t > 1$

$\Rightarrow f(t) = t + 1 + \frac{t}{\sqrt{1+t^2}}$ for all $t > 1$

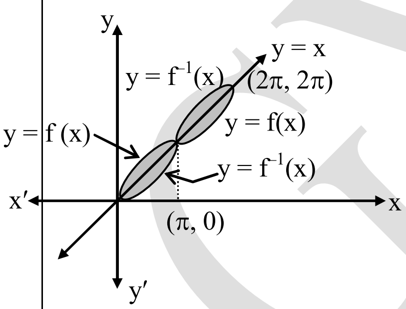
Hence $f(x) = x + 1 + \frac{x}{\sqrt{1+x^2}}$ for all $x > 1$.

Hence (A) is the correct answer.

8. (d)

Clearly, $f : [0, 2\pi] \to [0, 2\pi]$ given by

$f(x) = x + \sin x$ is a bijection. So, its inverse exists. The graph of $f^{-1}(x)$ is the mirror image of the graph of $f(x)$ in the line $y = x$.

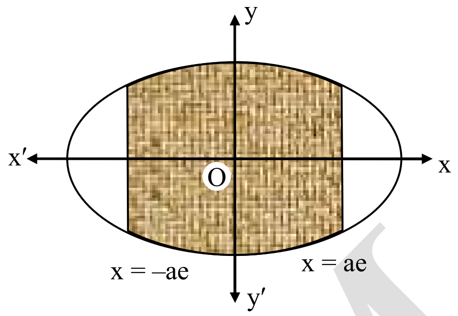


$\therefore$ Required area = Area of the shaded region = 4(Area of the one loop) $= 4\left[\int_0^{\pi}(x + \sin x)dx - \int_0^{\pi} x\,dx\right] = 4\int_0^{\pi}\sin x\,dx = -4[\cos\pi - \cos 0] = 8$ sq. units. Hence (D) is the correct answer.

9. (a)

The required area

$= 4\int_0^{ae} y\,dx = \frac{4b}{a}$

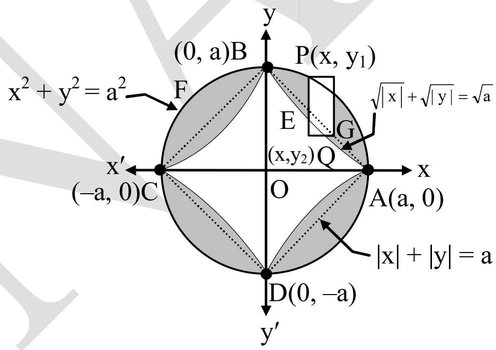


$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$

$= \frac{4b}{2a}[ae\sqrt{a^2 - a^2e^2} + a^2\sin^{-1} e]$

$= 2ab\,(e\sqrt{1-e^2} + \sin^{-1} e)$ sq. units.

10. (a)



The shaded region in figure represents the region enclosed by $x^2 + y^2 = a^2$ and $\sqrt{|x|} + \sqrt{|y|} = \sqrt{a}$. From the symmetry, it is evident that

Required area = 4 [Area of the region bounded by the two curves in first quadrant only]

$= 4\int_0^a (y_1 - y_2)dx = 4\int_0^a \{\sqrt{a^2 - x^2} - (\sqrt{a} - \sqrt{x})^2\}\,dx$

$= 4\int_0^a (\sqrt{a^2 - x^2} - a - x + 2\sqrt{a}\sqrt{x})\,dx$

$= 4\left[\frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\sin^{-1}\frac{x}{a} - ax - \frac{x^2}{2} + \frac{4\sqrt{a}}{3}x^{3/2}\right]_0^a$

$= 4\left[\left\{\frac{1}{2}a^2 \times \frac{\pi}{2} - a^2 - \frac{a^2}{2} + \frac{4}{3}a^2\right\} - 0\right]$

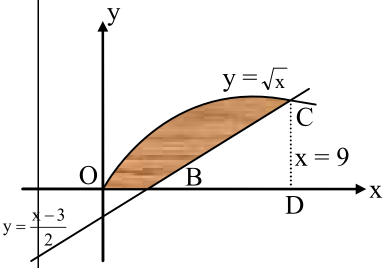
$= \left\{\pi a^2 - \frac{2}{3}a^2\right\}$ sq. units.

11. (c)

The two curves meet where $\sqrt{x} = \frac{x-3}{2}$ ... (i)

$\Rightarrow 4x = x^2 - 6x + 9 \Rightarrow x^2 - 10x + 9 = 0 \Rightarrow x = 9, 1$

But x = 1 does not satisfy (i)



$\therefore$ The two curves meet where x = 9.

$\therefore$ Required area (shown shaded)

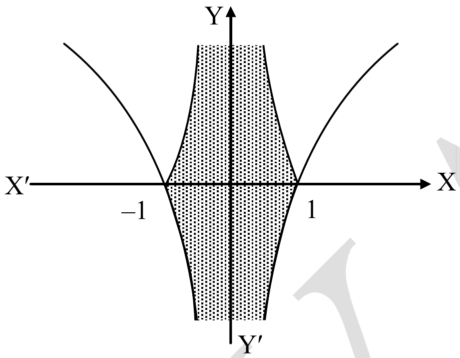
= area OBCO = area OBDCO - area BDC

$$= \int_0^9 \sqrt{x}\, dx - \int_3^9 \left(\frac{x-3}{2}\right) dx$$

$$= \frac{2}{3}\,[x^{3/2}]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9 = 9.$$

12. (a)

Note that ln x is defined for x > 0 and ln | x | is defined for | x | > 0. Also, | ln x | ≥ 0 and also | ln | | x | = | ln | x | | ≥ 0.



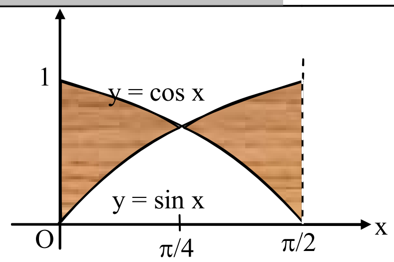
Required area is symmetrical in all the four quadrants and is equal to $4\int_0^1 |\ln x|\, dx = -4\int_0^1 \ln x\, dx$ [In (0, 1), ln x < 0]

$= -4\,[x \ln x - x]_0^1 = -4\,(-1) = 4.$

$(\because \lim_{x \to 0} x \ln x = 0)$

13. (d)

Required area = $\int_0^{\pi/2} (\sin x - \cos x)dx = \int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx$



$$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (0 + 1) - \left\{1 - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right\}$$

$$= \frac{4}{\sqrt{2}} - 2 = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1).$$

14. (a)

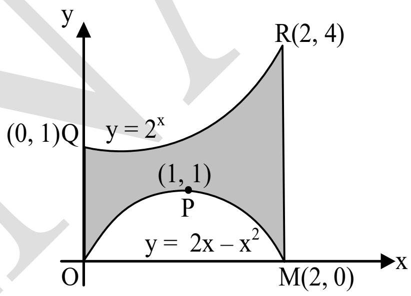
Figure is self explanatory. Given curves are

$y = 2^x$ ...(i)

$(x - 1)^2 = -(y - 1)$ ...(ii)

The required area

$$= \int_0^2 (y_1 - y_2)\, dx$$



Where $y_1 = 2x$ and $y_2 = 2x - x^2$

$$= \int_0^2 (2^x - 2x + x^2)dx = \left[\frac{2}{\log 2} - x^2 + \frac{1}{3}x^3\right]_0^2$$

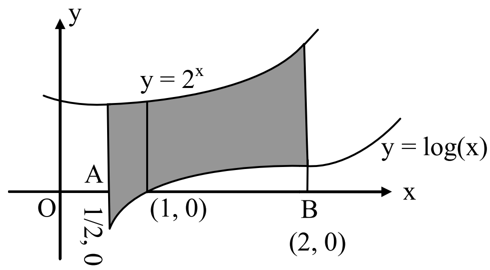
$$= \left(\frac{4}{\log 2} - 4 + \frac{8}{3}\right) - \frac{1}{\log 2} = \frac{3}{\log 2} - \frac{4}{3}$$

15. (b)

Given curves are

y = log x... (i)

$y = 2^x$ ... (ii)



So, the required area

$$= \left|\int_{1/2}^{2}(y_1 - y_2)\,dx\right| = \left|\int_{1/2}^{2}(2^x - \log x)\,dx\right|$$

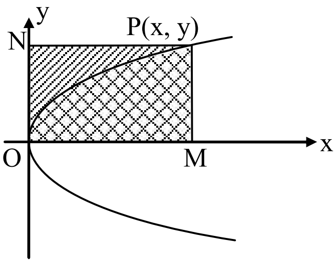
$$= \left|\left[\frac{2^x}{\log 2} - (x\log x - x)\right]_{1/2}^{2}\right|$$

$$= \left|\frac{4-\sqrt{2}}{\log 2} - (2\log 2 - 2) + \left(\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\right)\right|$$

$$= \frac{4-\sqrt{2}}{\log 2} - \frac{5}{2}\log 2 + \frac{3}{2}.$$

16. (b)

Let P(x, y) be the point on the curve passing through the origin *O*(0, 0), and let PN and PM be the liens parallel to the x- and y-axes, respectively (Fig.). If the equation of the curve is y = y(x), the area



POM equals $\int_0^x y\,dx$ and the area PON equals

$xy - \int_0^x y\,dx$ Assuming that 2(POM) = PON, we therefore have

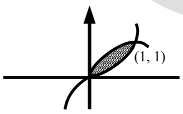
$2\int_0^x y\,dx = xy - \int_0^x y\,dx \Rightarrow 3\int_0^x y\,dx = xy$. Differentiating both sides of this gives

$$3y = x\frac{dy}{dx} + y \Rightarrow 2y = x\frac{dy}{dx} \Rightarrow \frac{dy}{y} = 2\frac{dx}{x}$$

$\Rightarrow \log|y| = 2\log|x| + C \Rightarrow y = Cx^2$, with C being a constant. This solution represents a parabola. We will get a similar result if we had started instead with 2 (PON) = POM.

17. (c)

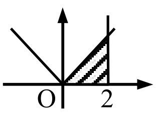
$$A = \int_0^1(\sqrt{x} - x^3)\,dx$$



$$2/3(x^{3/2})_0^1 - (x^4/4)_0^1 = 5/12$$

18. (c)

Area = $1/2 \times 2 \times 2 = 2$

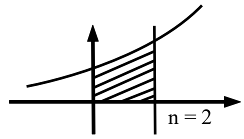


19. (b)

$$\text{Area} = \int_0^2 2^{kx}\,dx$$

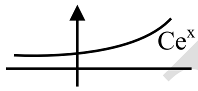
$\Rightarrow (2^{kx}/(\ell n2)k)_0^2 = 3/\log 2$

$\Rightarrow 2^{2k} = 3k + 1 \Rightarrow k = 1$



20. (b)

$$\text{area} = \int_p^q C\,e^x dx$$



$= C(e^x)_p^q \Rightarrow C\,(e^q - e^p) \Rightarrow |F(q) - F(p)|$

21. (b)

$\text{area} = 2\int_0^a x\sqrt{\frac{a-x}{a}} \Rightarrow$ Put $x = a\sin^2\theta \quad dx$

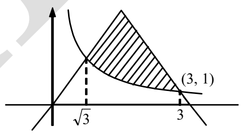
$= 2a\sin\theta\cos\theta\,d\theta$

$$= 2\int_0^{\pi/2}(a\sin^2\theta)\cdot\cos\theta(2a\sin\theta\cos\theta)$$

$$= 4a^2\int_0^{\pi/2}\sin^3\theta\cos^2\theta\,d\theta \Rightarrow 4a^2\frac{(2)}{5.3} = 8/15\,a^2$$

22. (b)

$$ax = \int_{\sqrt{3}}^{3}\left(2-|2-x|\right) - \left(\frac{3}{x}\right)dx$$

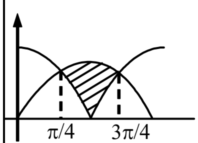


$$= \int_{\sqrt{3}}^{2}x\,dx + \int_2^3 4 - x - \int_{\sqrt{3}}^{3}\frac{3}{x}dx$$

$$(x^2/2)_{\sqrt{3}}^2 + (4x - x^2/2)_2^3 - (3\cos n)_{\sqrt{3}}^3 \Rightarrow 4 - 3\log 3/2$$

23. (c)

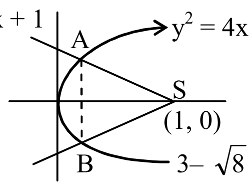
$$\text{area} = \int_{\pi/4}^{3\pi/4}(\sin^2 x - \cos^2 x)\,dx$$



$$= -\int_{\pi/4}^{3\pi/4}\cos 2x\,dx = 1$$

24. (d)

$|y| = -x + 1$

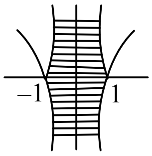


Required area = $2\int_0^{3-\sqrt{8}} \sqrt{4x}\,dx + \Delta ABS$

$= 2\left[\frac{4}{3}\left(x^{3/2}\right)_0^{3-\sqrt{8}} + \frac{1}{2}\left(2\sqrt{2}-2\right)^2\right]$

$= 2\left[\frac{4}{3}(5\sqrt{2}-7)+6-4\sqrt{2}\right]$ Sq. units

25. (a)



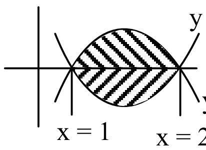
log x is defined for x > 0
log |x| is defined for $x \in R - \{0\}$
Also $|\log x| \ge 0$ & $|\log |x|| \ge 0$
Required area is symmetrical in all four quadrants. So,

Required area = $4\int_0^1 |\log x|\,dx = -4\int_0^1 \log x\,dx$ [in (0,1), log x < 0]

= - 4 [- 1] = 4 sq. unit

26. (d)

Required area



$= 2\int_1^2 \left(-x^2+3x-2\right)dx = 2\left[-\frac{x^3}{3}+3.\frac{x^2}{2}-2x\right]_1^2$

$= 2\left[-\frac{8}{3}+2+\frac{5}{6}\right] = \frac{1}{3}$ sq unit

27. (a)

Required area

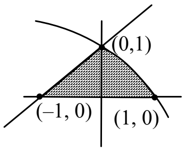
$A = \int_{-1}^{1}(-x^2+2)dx + \int_1^2 (2x-1)dx$

$= \left[-\frac{x^3}{3}+2x\right]_{-1}^{1} + \left[x^2-x\right]_1^2$

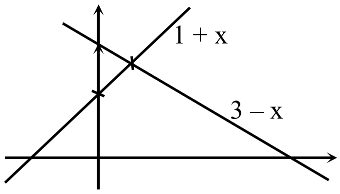
$= \frac{10}{3} + 2 = \frac{16}{3}$ sq. units

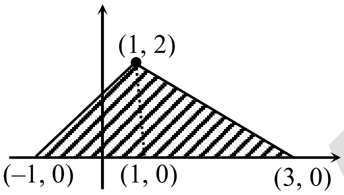
28. (a)

Area = $\int_0^1 (x_2 - x_1)\,dx = \int_0^1 [(1-y^2)-(y-1)]dy = \frac{7}{6}$ sq. units



29. (d)



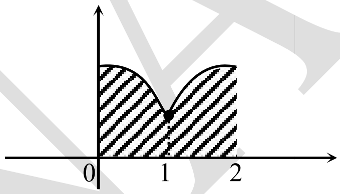


$= \frac{1}{2}\times 2\times 2 + \frac{1}{2}\times 2\times 2 \Rightarrow \frac{8}{2} = 4$

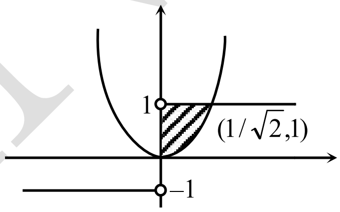
30. (a)

Area = $\int_0^1 \sqrt{4-x^2}\,dx + \int_1^3 \sqrt{3x}\,dx$

$= \frac{2\pi-\sqrt{3}+36}{6}$



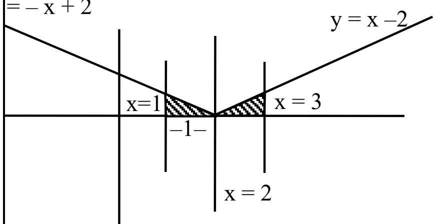
31. (b)



Area = $\int_0^1 \sqrt{y/2}\,dy$

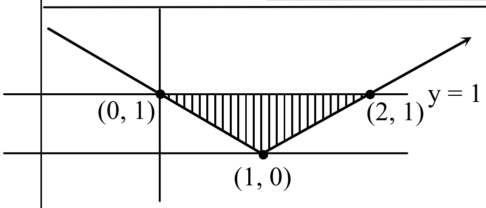
$= \frac{1}{\sqrt{2}}.\frac{2}{3}\left(y^{\frac{3}{2}}\right)_0^2 = \frac{2\sqrt{2}}{6}$

32. (a)



Area = $2\times\frac{1}{2}\times|x| = 1$

33. (a)



Area = $\frac{1}{2}$ base × height = $\frac{1}{2} \times 2 \times 1 = 1$

34. (a)

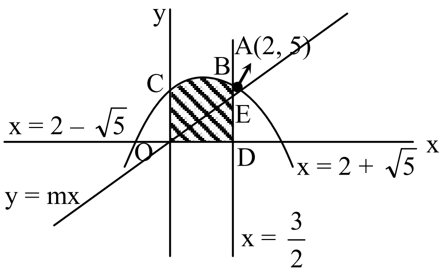
$y = 1 + 4x - x^2$ .... (1) gives $(x - 2)^2 = -(y - 5)$, which is parabola with vertex at (2, 5). Also it cuts x-axis, where $(x - 2)^2 = -(0 - 5)$

$\Rightarrow \; x = 2 \pm \sqrt{5}$

$\Rightarrow \; x = 2 + \sqrt{5},\ x = 2 - \sqrt{5} < 0$

The line y = mx divides the shaded area ODEBCO bounded by x = 0, y = 0, x = 3/2 and parabola into two equal parts ODEO and OEBCO.

$\therefore$ Area ODEBCO = 2. Area ODE.

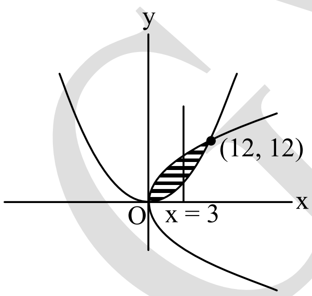


$$\int_0^{3/2} (1 + 4x - x^2)\,dx = 2\int_0^{3/2} mx\,dx$$

$$\Rightarrow \frac{3}{2} + 2\left(\frac{3}{2}\right)^2 - \frac{1}{3}\left(\frac{3}{2}\right)^3 = m\left(\frac{3}{2}\right)^2$$

$$\Rightarrow m = \frac{13}{6}$$

35. (a)



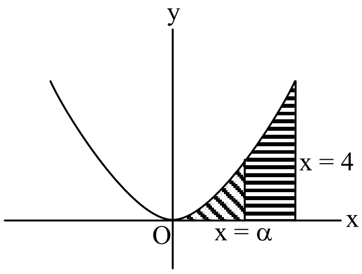
Total area = $\frac{16 \times 3 \times 3}{3}$ A = 48

Area between x = 0 and x = 3

$$A_1 = \int_0^3 (y_2 - y_1)\,dx = \int_0^3 \left(\sqrt{12x} - \frac{x^2}{12}\right)dx = \frac{45}{4}$$

$$A_2 = A - A_1 = 48 - \frac{45}{4} = \frac{147}{4} \quad \frac{A_1}{A_2} = \frac{\frac{45}{4}}{\frac{147}{4}} = 15 : 49$$

36. (a)



$$2\int_0^{\alpha} \frac{x^2}{4}dx = \int_0^4 \frac{x^2}{4}dx$$

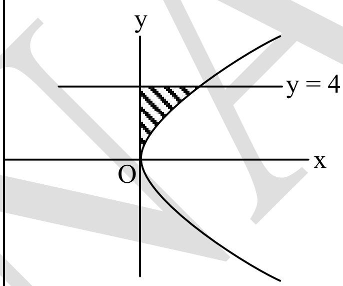
$$\Rightarrow 2.\frac{\alpha^3}{12} = \frac{1}{4}\frac{(4)^3}{3}$$

$$\Rightarrow \frac{\alpha^3}{12} = \frac{1}{2} \times \frac{16}{3}$$

$$\Rightarrow \alpha^3 = 32$$

$$\Rightarrow \alpha = (32)^{1/3}$$

37. (c)

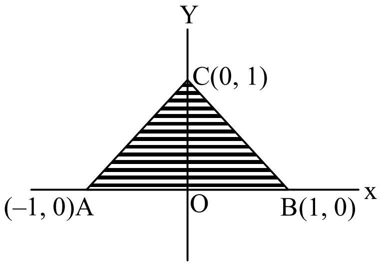


$$\text{Area} = \int_0^4 x\,dy = \int_0^4 y^2\,dy$$

38. (b)

$y + |x| = 1 \;\Rightarrow\; y + x = 1$ if $x > 0$

$y - x = 1$ if $x < 0$



Area = $\frac{1}{2} \times 2 \times 1 = 1$

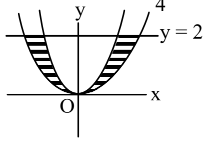
39. (b)

$$\text{Area} = \int_{-1}^{1} \cos x\,dx = 2\int_0^1 \cos x\,dx$$

$= 2 \sin 1$

40. (a)

$x^2 = \frac{y}{4}$, $x^2 = 9y$



$$\text{Area} = 2\int_0^2 \left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right)dy$$