1. (b)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+e^x}dx = \int_{-\pi/2}^{\pi/2} \frac{\sin^2(-x)}{1+e^{-x}}dx$$

$$\Rightarrow I + I = \int_{-\pi/2}^{\pi/2} \sin^2 x \frac{e^x+1}{1+e^x}dx$$

$$\Rightarrow I = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \sin^2 x\, dx$$

$$\text{or} \quad I = \int_0^{\pi/2} \sin^2 x\, dx$$

2. (a)

$$I = \int_{-2n}^{2n+\frac{1}{2}} (\sin \pi x)\left\{\frac{x}{2}\right\}dx$$

a. $$= 2n\int_0^2 (\sin \pi x)\frac{x}{2}dx + \int_0^{1/2} (\sin \pi x)\frac{x}{2}dx$$

$$= \frac{-2n\pi+1}{\pi^2}$$

3. (b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 6 & 2 & 0 \\ 5 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ x^2 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ 2\alpha x + \beta x^2 \\ 5x + \gamma x^2 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ 6x + 2x^2 \\ 5x + 4x^2 + 3 \end{bmatrix} = \begin{bmatrix} x \\ 2\alpha x + \beta x^2 \\ 5x + \gamma x^2 + 3 \end{bmatrix}$$

$\therefore$ $f(y) = f(1) + y^2 - 1 + y - 1 = y^2 + y + 1$

$\{\because f(1) = 3\}$

$\therefore f(x) = x^2 + x + 1$

$f'(x) = 2x + 1$

$$\therefore \int_{-1}^{1} (\alpha f(x) + \beta f'(x) + \gamma)dx = 20$$

4. (d)

Integrating by parts, the given integral is equal to

$$x \tan^{-1}\sqrt{\sqrt{x}-1}\Big|_1^{16} - \int_1^{16} \frac{x}{\sqrt{x}}\frac{1}{4\sqrt{x}\sqrt{\sqrt{x}-1}}dx$$

$$= \frac{16}{3}\pi - \frac{1}{4}\int_1^{16}\frac{dx}{\sqrt{\sqrt{x}-1}}$$

$$= \frac{16}{3}\pi - \frac{1}{4}\int_0^{\sqrt{3}}\frac{4t(1+t^2)}{t}dt \quad (\sqrt{x} = 1 + t^2)$$

$$= \frac{16}{3}\pi - \left(\sqrt{3}+\sqrt{3}\right) = \frac{16}{3}\pi - 2\sqrt{3}$$

5. (b)

$R_2 \to R_2 - R_1 \quad R_3 = R_3 - 3R_1$

$$\Delta(x) = \begin{vmatrix} 1+x+2x^2 & x+3 & 1 \\ -1 & -3 & 2 \\ -3 & 2 & 6 \end{vmatrix}$$

$$R_1 = R_1 - \frac{R_2}{2}$$

$$\Delta(x) = \begin{vmatrix} 1+x+2x^2+1/2 & x+3+3/2 & 0 \\ -1 & -3 & 2 \\ -3 & 2 & 6 \end{vmatrix}$$

$$R_2 = R_2 - \frac{R_3}{3}$$

$$\Delta(x) = \begin{vmatrix} \frac{3}{2}+x+2x^2 & x+\frac{9}{2} & 0 \\ 0 & -3-\frac{2}{3} & 0 \\ -3 & 2 & 6 \end{vmatrix}$$

$$\Delta(x) = \begin{vmatrix} \frac{3}{2}+x+2x^2 & x+\frac{9}{2} & 0 \\ 0 & -\frac{11}{3} & 0 \\ -3 & 2 & 6 \end{vmatrix}$$

$$\Delta(x) = 6\left(-\frac{11}{3}\left(\frac{3}{2}+x+2x^2\right)\right)$$

$$\int_0^1 \Delta(x) = \left[\frac{-66}{3}\left[\frac{3}{2}x + \frac{x^2}{2} + \frac{2x^3}{3}\right]\right]_0^1$$

$$= \frac{-66}{3}\left(\frac{3}{2}+\frac{1}{2}+\frac{2}{3}\right)$$

$$= -22\left[\frac{9+3+4}{6}\right] = \frac{-176}{3}$$

6. (a)

$$I = \int_0^{\pi} \log(1+\cos x)\,dx$$

$$= \int_0^{\pi} \log\left(2\cos^2\frac{x}{2}\right)dx$$

$$= \int_0^{\pi} \log 2\,dx + \int_0^{\pi} \log\cos^2\frac{x}{2}\,dx$$

$$= \pi\log 2 + 2\int_0^{\pi} \log\cos\frac{x}{2}\,dx$$

$$= \pi\log 2 + 4\int_0^{\pi/2} \log\cos x\,dx$$

$$= \pi\log 2 + 4\int_0^{\pi/2} \log\cos(\pi/2 - x)\,dx$$

$= \pi \log 2 + 4 \int_0^{\pi/2} \log \sin x \, dx$

$= \pi \log 2 + 4k$

7. (a)

$I = \int_{2n}^{2n+\frac{1}{2}} (\sin x) \left\{\frac{x}{2}\right\} dx$

$= 2n \int_0^2 (\sin \pi x) \frac{x}{2} dx + \int_0^{1/2} (\sin \pi x) \frac{x}{2} dx$

$= \frac{-2n\pi + 1}{\pi^2}$

8. (c)

$f(x) = \frac{\sin x}{x}$ is a decreasing function and

$\frac{\sin x}{x} > 0$ for all x in $(0, \pi)$

Since $\sin x < x < \tan x$

$\Rightarrow \frac{\sin(\sin x)}{\sin x} > \frac{\sin x}{x} > \frac{\sin(\tan x)}{\tan x}$ for $\frac{\pi}{6} < x < \frac{\pi}{3}$

$\therefore I_2 > I_1 > I_3$

9. (c)

Put $1 + x = t^2$, then

$I = \int_{\sqrt{2}}^{\infty} \frac{2t}{(t^2 - 1)} \frac{1}{t} dt$

Integrating by parts, we get

$I = -\frac{1}{(t^2-1)t}\Big]_{\sqrt{2}}^{\infty} - \int_{\sqrt{2}}^{\infty} \frac{dt}{(t^2-1)t^2}$

$= \frac{1}{\sqrt{2}} - \int_{\sqrt{2}}^{\infty} \left[\frac{1}{t^2-1} - \frac{1}{t^2}\right] dt$

$= \frac{1}{\sqrt{2}} - \left(\frac{1}{2}\log\left|\frac{t-1}{t+1}\right| + \frac{1}{t}\right)\Bigg]_{\sqrt{2}}^{\infty}$

$= \frac{1}{\sqrt{2}} - \frac{1}{2}\log\left|\frac{1-1/t}{1+1/t}\right|\Bigg]_{t=\infty} + 0$

$+ \frac{1}{2}\log\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right| + \frac{1}{\sqrt{2}}$

$= \sqrt{2} - \log(\sqrt{2} - 1)$

10. (a)

Put $\sqrt{x} = t$ or $x = t^2$, so that

$I = 2\int_0^{\infty} \frac{t^2}{(1+t^2)(2+t^2)(3+t^2)} dt$

$= \int_0^{\infty} \left(-\frac{1}{1+t^2} + \frac{4}{2+t^2} - \frac{3}{3+t^2}\right) dt$

$= \left(-\tan^{-1} t + \frac{4}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) - \frac{3}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right)\Bigg]_0^{\infty} = -\frac{\pi}{2} + 2$

$\sqrt{2}\left(\frac{\pi}{2}\right) - \sqrt{3}\left(\frac{\pi}{2}\right)$

$= \frac{\pi}{2}(2\sqrt{2} - \sqrt{3} - 1)$

11. (b)

We can write

$I = \int_0^1 \frac{x(1-x)}{\sqrt{1-x^2}} dx$

$= \int_0^1 \left(\frac{x}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} + \frac{1-x^2}{\sqrt{1-x^2}}\right) dx$

$\left(-\sqrt{1-x^2} - \sin^{-1} x\right)\Big]_0^1$

$\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right)\Big]_0^1$

$-\frac{\pi}{2} + 1 + \frac{1}{2}\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{4}$

12. (a)

We can write

$I = I_1 + I_2 + I_3$

where $I_1 = \int_{-3}^{2} |x+1| dx$ etc.

Put $x + 1 = t$, so that

$I_1 = \int_{-2}^{3} |t| dt = \int_{-2}^{0} (-t) dt + \int_0^3 t \, dt$

$= -\frac{1}{2}t^2\Big]_{-2}^{0} + \frac{1}{2}t^2\Big]_0^3 = \frac{13}{2}$

Similarly, $I_2 = I_3 = \frac{9}{2}$

Thus, $I = \frac{31}{2}$

13. (c)

We can write

$I = \int_{-2}^{0} [(x+1)^3 - 1 + (x+1)\cos(x+1)] dx$

Put $x + 1 = t$, so that

$I = \int_{-1}^{1} [t^3 - 1 + t \cos t] dt$

$= \int_{-1}^{1} (-1) dt = -t\Big]_{-1}^{1} = -2$

as $t^3 + t \cos t$ is an odd function

14. (d)

Put $\frac{1}{2}\cos x = t$, so that -sin x dx = 2dt and

$$I = \int_{1/2}^{-1/2} e^{|t|}\,(2\sin t + 3\cos t)\,(-2)\,dt$$

As $e^{|t|}$ sin t is an odd function, and $e^{|t|}$ cos t is an even function,

$$I = 6\int_0^{1/2} e^t \cos t\,dt = 6e^t \cos t]_0^{1/2} + 6\int_0^{1/2} e^t \sin t\,dt$$

$$I = 6\left[\sqrt{e}\cos\left(\frac{1}{2}\right) - 1\right] + 6e^t \sin t]_0^{1/2} - 6\int_0^{1/2} e^t \cos t\,dt$$

$$\Rightarrow 7I = 6\sqrt{e}\left(\cos\left(\frac{1}{2}\right) + \sin\left(\frac{1}{2}\right) - 1\right)$$

15. (c)

Put $\frac{1}{x} = t$, so that

$$I = \int_e^{1/e} \left|\log\frac{1}{t}\right| (-1)\,dt$$

$$= \int_{1/e}^{e} |-\log t|\,dt$$

$$= \int_{1/e}^{1} (-\log t)\,dt + \int_1^e (\log t)\,dt$$

$$= (-t\log t + t)]]_{1/e}^{1} + (t + \log t - t)\,]_1^e$$

$$= 1 - \frac{1}{e} - \frac{1}{e} + e - e + 1$$

$= 2(1 - 1/e)$

16. (c)

Writing $I = \int_0^{\pi/2} \frac{\sin 2kx}{\sin x}\cos x\,dx$

and using the given identity, we can write

$$I = \int_0^{\pi/2} 2[\cos x + \cos 3x + \ldots\ldots + \cos(2k-1)x]\cos x\,dx$$

$$= \int_0^{\pi/2} [(1 + \cos 2x) + (\cos 4x + \cos 2x) + \ldots.. +$$

$(\cos 2kx + \cos (2k-2)x)]\,dx$

$$= \left(x + \sin 2x + \frac{1}{2}\sin 4x + \ldots.. + \frac{1}{2k}(\sin 2kx)\right)\Big]_0^{\pi/2} = \frac{\pi}{2}$$

17. (a)

Put $b - x = t^2$, so that

$$I = \int_{\sqrt{b-a}}^{0} \sqrt{\frac{b - t^2 - a}{t^2}}\,(-2t)\,dt$$

$$= 2\int_0^c \sqrt{c^2 - t^2}\,dt \text{ where } c = \sqrt{b-a}$$

$$= 2\left[\frac{1}{2}t\sqrt{c^2 + t^2} + \frac{c^2}{2}\sin^{-1}\left(\frac{t}{c}\right)\right]_0^c$$

$= 0 + c^2 \sin^{-1}(1) - 0$

$$= \frac{\pi}{2}(b - a)$$

18. (b)

Case -1

If $a < b < 0$, $\int_a^b \frac{|x|}{x}\,dx = -\int_a^b dx$

$= -(b - a) = |b| - |a|$

$(\because a < b < 0;\ |b| = -b$ and $|a| = -a)$

Case- 2

If $a < 0 < b$, $\int_a^b \frac{|x|}{x}dx = -\int_a^0 dx + \int_0^b dx$

$= a + b = |b| - |a|$ $\quad(\because a < 0,\ |a| = -a)$

Case- 3

If $0 < a < b$, $\int_a^b \frac{|x|}{x}dx$

$$= \int_a^b dx = b - a = |b| - |a|$$

19. (b)

$$I = \int_{-\pi/2}^{\pi/2} |\sin x|\,dx = 2\int_0^{\pi/2} |\sin x|\,dx \quad \text{(Even function)}$$

$$= 2\int_0^{\pi/2} \sin x = -2[\cos x]_0^{\pi/2} = 2$$

20. (b)

$$\int_0^1 \frac{dx}{\sqrt{1 - (x\sin\alpha)^2}} = \left[\frac{\sin^{-1}(x\sin\alpha)}{\sin\alpha}\right]_0^1$$

$$= \frac{\sin^{-1}(\sin\alpha) - 0}{\sin\alpha} = \frac{\pi - \alpha}{\sin\alpha}$$

$(\because$ for $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ $\sin^{-1}\sin\alpha = \pi - \alpha)$

$= \left|\frac{\pi - \alpha}{\sin\alpha}\right|$ (For given interval $\frac{\pi - \alpha}{\sin\alpha}$ $\left|\frac{\pi - \alpha}{\sin\alpha}\right|$)

21. (a)

$|z - 1|$ is a positive real number

arg $|z - 1| = 0$

$$\int_0^{50} [\arg|z - 1|]\,dx = \int_0^{50} 0\,dx = 0$$

22. (b)

$$\int_{-1}^{1} \max\{x, x^3\}\,dx = \int_{-1}^{0} x^3\,dx + \int_0^1 x\,dx$$

$= \frac{-1}{4} + \frac{1}{2} = \frac{1}{4}$

23. (a)

Clearly min. {|x|, [x]} = [x]

$\therefore \phi 2 \int_{-2}^{2}[x]dx = \int_{-2}^{-1}[x]dx + \int_{-1}^{0}[x]dx + \int_{0}^{1}[x]dx +$

$\int_{1}^{2}[x]dx$

$= -2\int_{-2}^{-1}dx - \int_{-1}^{0}dx + 0 + \int_{1}^{2}dx$

= -2 -1 + 1 = -2

24. (a)

$g(x+\pi) = \int_{0}^{x+\pi}\cos^4(t)\,dt$

$= \int_{0}^{x}\cos^4(t)\,dt + \int_{x}^{x+\pi}\cos^4 t\,dt$ ... (3)

$= \int_{0}^{x}\cos^4 t\,dt + \int_{0}^{\pi}\cos^4 t\,dt$ ($\because \cos^4 t$ is periodic function)

$= g(x) + g(\pi)$

25. (d)

Let $I = \int_{1}^{4}\frac{2e^{\sin x^2}}{x}dx = \int_{1}^{4}\frac{2xe^{\sin x^2}}{x^2}$

Put $x^2 = y$, then $2x\,dx = dy$

x	1	4
y	1	16

$\therefore I = \int_{1}^{16}\frac{e^{\sin y}}{y}dy = |F(y)|\int_{1}^{16} = F(16) - F(1)$

thus one of the possible value of k = 16