1. (b)

According to the given conditions, $5 = \frac{x}{1-r}$, r being the common ratio $\Rightarrow r = 1 - \frac{x}{5}$

Now, $|r| < 1$ *i.e.* $-1 < r < 1 \Rightarrow -1 < 1 - \frac{x}{5} < 1 \Rightarrow -2 < -\frac{x}{5} < 0$

$\Rightarrow 2 > \frac{x}{5} > 0$ *i.e.* $0 < \frac{x}{5} < 2$, $\quad 0 < x < 10$

2. (b)

Given a^{-1}, b^{-1}, c^{-1} are in A.P. $\Rightarrow$ a, b, c are in H.P.

$\Rightarrow \frac{a^{101} + c^{101}}{2} > (\sqrt{ac})^{101} > b^{101} \; (\because \sqrt{ac} > b)$

$\Rightarrow 2b^{101} - a^{101} - c^{101} < 0$

Let $f(x) = x^2 - kx + 2b^{101} - a^{101} - c^{101}$

$f(-\infty) = (\infty) > 0$

$f(0) < 0$; $f(\infty) > 0$

Hence equation $f(x) = 0$ has one root in $(-\infty, 0)$ and other in $(0, \infty)$

3. (b)

$(2 + b)^2 = (3 + a)(3 + c)$

$4 + b^2 + 4b = 9 + 3(a + c) + ac$

$4 + b^2 + 4b = 9 + 6b + ac$

$ac = b^2 - 2b - 5$

$= (b - 1)^2 - 6$

$ac \geq -6$

4. (b)

Let $S = 1 + 2.2 + 3.2^2 + 4.2^3 + ... + 100.2^{99}$ .....(i)

$2S = 1.2 + 2.2^2 + 3.2^3 + ... + 99.2^{99} + 100.2^{100}$ .....(ii)

Equation (i) - Equation (ii) gives,

$-S = 1 + (1.2 + 1.2^2 + 1.2^3 + ... \text{upto 99 terms}) - 100.2^{100} = 1 + \frac{2(2^{99} - 1)}{2 - 1} - 100.2^{100}$

$\Rightarrow S = -1 - 2^{100} + 2 + 100.2^{100} = 1 + 99.2^{100}$

5. (b)

Here $x = \frac{1}{1 - \cos^2\theta} = \operatorname{cosec}^2\theta$, $y = \sec^2\theta$,

$z = \frac{1}{1 - \sin^2\theta\cos^2\theta}$

$z = \frac{1}{1 - \frac{1}{x} \cdot \frac{1}{y}} = \frac{xy}{xy - 1} \Rightarrow xyz = xy + z$

6. (c)

Let d be common difference of A.P. $\Rightarrow d = a_i - a_{i-1}$

Now $\frac{1}{a_{i+1}^{2/3} + a_{i+1}^{1/3}.a_i^{1/3} + a_i^{2/3}} = \frac{a_{i+1}^{1/3} - a_i^{1/3}}{a_{i+1} - a_i} = \frac{1}{d}\left[a_{i+1}^{1/3} - a_i^{1/3}\right]$

Thus $\sum_{i=1}^{n} \frac{n}{a_{i+1}^{2/3} + a_{i+1}^{1/3}.a_i^{1/3} + a_i^{2/3}}$

$= \frac{1}{d}\sum_{i=1}^{n-1}\left(a_{i+1}^{1/3} - a_i^{1/3}\right)$

$= \frac{1}{d}\left(a_n^{1/3} - a_1^{1/3}\right) = \frac{1}{d}\;\frac{(a_n - a_1)}{a_n^{2/3} + a_n^{1/3}.a_1^{1/3} + a_1^{2/3}}$

$= \frac{(n-1)}{a_n^{2/3} + a_n^{1/3}.a_1^{1/3} + a_1^{2/3}}$

[C] is correct.

7. (a)

$a_1, a_2, a_3 \ldots\ldots a_n = c \Rightarrow a_1\, a_2\, a_3 \ldots a_{n-1}(2a_n) = 2c$

$\because$ AM $\geq$ GM

$\frac{a_1 + a_2 + a_3 + ...a_{n-1} + 2a_n}{n} \geq (a_1\, a_2\, a_3 \ldots 2a_n)^{1/n}$

$a_1 + a_2 + a_3 + \ldots a_{n-1} + 2a_n \geq n(2c)^{1/n}$

8. (c)

Let common difference be k. $(k \in I)$

$\Rightarrow d = a + 3k = a^2 + (a + k)^2 + (a + 2k)^2$

$\Rightarrow 5k^2 + 3(2a - 1)k + 3a^2 - a = 0$

$k \in R \Rightarrow D \geq 0$

$\Rightarrow 9(2a - 1)^2 - 4.5(3a^2 - a) \geq 0$

$\Rightarrow 24a^2 + 16a - 9 \leq 0 \Rightarrow a = -1, 0$

If $a = 0 \Rightarrow k = 0, 3/5 \Rightarrow k = 0$

which is not possible.

Hence $a = -1 \Rightarrow k = 1$

So $a + b + c + d = 2$

9. (b)

Product $= \frac{a^c b^a c^b}{a^b b^c c^a}$

$a = mn^{p-1}$, $b = mn^{q-1}$, $c = mn^{r-1}$

Hence product

$= (mn^{p-1})^{(r-q)d}\;(mn^{q-1})^{(p-r)d}\;(mn^{r-1})^{(q-p)d}$

$= 1$

10. (d)

Since a, b, c are in A.P.

$b = a + d$, $c = a + 2d$,

where d is a common difference, $d > 0$

Again since a^2, b^2, c^2 are in G.P.

$a^2, (a + d)^2, (a + 2d)^2$ are in G.P.

Þ $(a + d)^4 = a^2(a + 2d)^2$

Or $(a + d)^2 = \pm a(a + 2d)$

$\Rightarrow a^2 + d^2 + 2ad = \pm(a^2 + 2ad)$

Taking (+) sign, d = 0 (not possible as $a < b < c$)

Taking (-) sign,

$2a^2 + 4ad + d^2 = 0$

$\Rightarrow 2a^2 + 4a\left(\frac{1}{2} - a\right) + \left(\frac{1}{2} - a\right)^2 = 0$

$\left[\therefore a + b + c = \frac{3}{2} \Rightarrow a + d = \frac{1}{2}\right]$

$\Rightarrow 4a^2 - 4a - 1 = 0$

$a = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$.

Here $d = \frac{1}{2} - a > 0$. So, $a < \frac{1}{2}$

Hence, $a = \frac{1}{2} - \frac{1}{\sqrt{2}}$.

11. (b)

Now, $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + ... + \frac{1}{a_{4000} a_{4001}} =$

$\left(\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 a_3} + ... + \frac{a_{4001} - a_{4000}}{a_{4000} a_{40001}}\right)$

$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + ... + \frac{1}{a_{4000}} - \frac{1}{a_{4001}}\right)$

$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{4001}}\right) = \frac{4000}{a_1 a_{4001}} = 10$ (given)

$\Rightarrow a_1 a_{4001} = 400$ …(i)

$a_1 + a_{4001} = a_2 + a_{4000} = 50$ …(ii)

$(a_1 - a_{4001})^2 = (a_1 + a_{4001})^2 - 4a_1a_{4001}$

$= (50)^2 - 1600$

Þ $|a_1 - a_{4001}| = 30$.

12. (a)

Let $f(x) = ax^2 + bx + c$

$f(1) = a + b + c$

$f(-1) = a - b + c$

Since, $f(1) = f(-1)$

$\Rightarrow a + b + c = a - b + c$

$\Rightarrow 2b = 0 \Rightarrow b = 0$

So $f(x) = ax^2 + c$

$f'(x) = 2ax$

'¢(a) $= 2a^2$, $f'(b) = 2ab$, $f'(c) = 2ac$

Now, we take $2f'(b) = f'(a) + f'(c)$

$\Rightarrow 2.2ab = 2a^2 + 2ac$

$\Rightarrow 2b = a + c$

$\Rightarrow$ a, b, c are in AP

$\Rightarrow f'(a), f'(b), f'(c)$ are in AP.

13. (a)

$\frac{\frac{n}{2}[2.3+(n-1).2]}{\frac{10}{2}[2.5+(10-1).3]} = 7 \Rightarrow \frac{n}{10}\left(\frac{4+2n}{37}\right) = 7$

$\Rightarrow n(2+n) = 37 \times 35 \quad n = 35$

14. (c)

$\because a + b + c = 3b = 3/2 \quad b = 1/2$

$a = \frac{1}{2} - d$ & $c = \frac{1}{2} + d$

Now $(\frac{1}{2} - d)^2, \frac{1}{4}, (\frac{1}{2} + d)^2$ are in GP

(using $b^2 = ac$) Þ $\frac{1}{4} = d^2 - \frac{1}{4} \Rightarrow d = 1/\sqrt{2}$

$a = \frac{1}{2} - d = \frac{1}{2} - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-2}{2\sqrt{2}}$

15. (b)

$2\alpha\beta / \alpha+\beta = 4$

16. (a)

$x = \frac{1}{1-a}$ & $y = \frac{1}{1-b}$

$s = \frac{1}{1-ab}$ put $a = 1 - \frac{1}{x}$ & $b = 1 - \frac{1}{y}$

17. (c)

$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

The line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ passes through the pt

(1, -2)

18. (c)

$a + e = b + d = c + c \Rightarrow a - 4b + 6c - 4d + e$

$= (2c) - 4(2c) + 6c = 2c - 8c + 6c = 0$

19. (a)

We have $b^2 = ac$ …..(1)

For $ax^2 + 2bx + c = 0 \Rightarrow D = 4b^2 - 4ac = 0$ by (1)

Roots of $ax^2 + 2bx + c = 0$ are equal.

hence if root is a then

$\alpha + \alpha = -\frac{2b}{a} \Rightarrow \alpha = -\frac{b}{a}$ …(2)

this α is also a root of $dx^2 + 2ex + f = 0$

$d.\frac{b^2}{a^2} + 2e\left(-\frac{b}{a}\right) + f = 0$

$\Rightarrow \frac{d}{a^2} - \frac{2e}{ab} + \frac{f}{b^2} = 0 \Rightarrow \frac{\alpha}{a} - \frac{2e}{b} + \frac{fa}{ac} = 0 \quad \because b^2 = ac$

$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0 \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$ hence $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

20. (b)

$T_n = \frac{\frac{n(n+1)}{2\cdot 2}}{\sum n^3} = \frac{\frac{n(n+1)}{4}}{\frac{n^2(n+1)^2}{4}} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

$S_n = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} ..... - \frac{1}{n+1} = 1 - \frac{1}{n+1}$

$= \frac{n}{n+1}$

21. Ans.

(i) [A]

(ii) [A]

(iii) [B]

(iv) [D]

Sol. (i) 6

(ii) 6

(iii) 9

(iv) 1

22. Ans.

(i) [C]

(ii) [A]

(iii) [D]

(iv) [B]

Sol. (i) 52

(ii) 42

(iii) 2045

(iv) 1620

23. Ans.

(i) [C]

(ii) [A]

(iii) [B]

(iv) [D]

Sol. (i) $\because$ S = 1 + 5 + 13 + 29 ……….+ t_{10}

S = 1 + 5 + 13 ……..+ t_9 + t_{10}

Subtracting

0 = 1 + 4 + 8 + 16 …….- t_{10}

t_{10} = 1 + (4 + 8 + 16 ……..+ 9_{terms})

t_{10} = 2045

(ii) $\because$ Sum of all two digit numbers = $\frac{90}{2}[10 + 99]$

$= 45 \times 109$

sum of all two digit numbers which are divisible by 2

$= \frac{45}{2}[10 + 98] = 45 \times 54$

sum of all two digit numbers which are divisible by 3

$= \frac{30}{2}[12 + 99] = 15 \times 111$

& sum of all two digit numbers which are divisible by 6

$= \frac{15}{2}[12 + 96] = 15 \times 54$

required sum = $45 \times 109 + 15 \times 54 - 15 \times 111 - 45 \times 54$

$= 1620$

(iii) $\because S = \frac{5}{1^2.4^2} + \frac{11}{4^2.7^2} + \frac{17}{7^2.10^2} + \ldots\ldots..$

$3S = \frac{15}{1^2.4^2} + \frac{33}{4^2.7^2} + \frac{51}{7^2.10^2} + \ldots\ldots.$

$= \left(\frac{1}{1^2} - \frac{1}{4^2}\right) + \left(\frac{1}{4^2} - \frac{1}{7^2}\right) + \left(\frac{1}{7^2} - \frac{1}{10^2}\right)\ldots\ldots$

$3S = 1 \Rightarrow S = \frac{1}{3}$

(iv) $\because$ H.M. of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \& \frac{1}{5}$ is

$$\frac{4}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}} = \frac{240}{77}$$

24. Ans.

(i) [C]

(ii) [C]

(iii) [B]

(iv) [A]

Sol. (i) $\because$ x, y, z are in G.P. $\Rightarrow$ ex, ey, ez are in G.P.

$\log_e ex, \log_e ey, \log_e ez$ are in A.P.

$\log_{ex} e, \log_{ey} e, \log_{ez} e$ are in H.P.

(ii) $\frac{a-b}{b-c} = \frac{a}{c} \Rightarrow ac - bc = ab - ac \Rightarrow 2ac = ab + bc$

$\Rightarrow \frac{2}{b} = \frac{1}{c} + \frac{1}{a}$; a, b, c are in H.P.

(iii) If $2^{ax+1}, 2^{bx+1}, 2^{cx+1}$ are in G.P.$\Rightarrow$ a, b, c are in A.P.

(iv) $x = \frac{a}{1-a} + 1 \Rightarrow x = \frac{1}{1-a} \Rightarrow 1 - a = \frac{1}{x}$

$a = 1 - \frac{1}{x}$ similarly, $b = 1 - \frac{1}{y}$, $c = 1 - \frac{1}{z}$

given 2b = a + c Þ $\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.