1. (c)

We have $F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$,

$F(\alpha')=\begin{bmatrix}\cos\alpha' & -\sin\alpha' & 0\\ \sin\alpha' & \cos\alpha' & 0\\ 0 & 0 & 1\end{bmatrix}$

$F(\alpha).F(\alpha')=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha' & -\sin\alpha' & 0\\ \sin\alpha' & \cos\alpha' & 0\\ 0 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}\cos(\alpha+\alpha') & -\sin(\alpha+\alpha') & 0\\ \sin(\alpha+\alpha') & \cos(\alpha+\alpha') & 0\\ 0 & 0 & 1\end{bmatrix}=F(\alpha+\alpha')$

2. (b)

$A^2=A.A=\begin{bmatrix}1&1&0\\1&2&1\\2&1&0\end{bmatrix}\begin{bmatrix}1&1&0\\1&2&1\\2&1&0\end{bmatrix}=\begin{bmatrix}2&3&1\\5&6&2\\3&4&1\end{bmatrix}$,

$A^3=A.^2A=\begin{bmatrix}2&3&1\\5&6&2\\3&4&1\end{bmatrix}\begin{bmatrix}1&1&0\\1&2&1\\2&1&0\end{bmatrix}=\begin{bmatrix}7&9&3\\15&19&6\\9&12&4\end{bmatrix}$

$A^3-3.A^2=\begin{bmatrix}7&9&3\\15&19&6\\9&12&4\end{bmatrix}-\begin{bmatrix}6&9&3\\15&18&6\\9&12&3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$

$\Rightarrow A^3-3A^2-I=0$

3. (b)

$adj\ A=\begin{bmatrix}4&-2\\-3&4\end{bmatrix}$

$|adj\,A|=\begin{vmatrix}4&-2\\-3&4\end{vmatrix}=16-6=10$

4. (b)

Here $|A|=\begin{vmatrix}a&b\\c&d\end{vmatrix}=ad-bc$, $adj(A)=\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$.

Hence $A^{-1}=\dfrac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$

5. (a)

We have, $A=\begin{bmatrix}1&-1&1\\-2&1&-3\\1&1&1\end{bmatrix}$, $\therefore\ |A|=1(4)+1(5)+1(1)=10$

and $adj(A)=\begin{bmatrix}4&2&2\\-5&0&5\\1&-2&3\end{bmatrix}$

Then $A^{-1}=\dfrac{1}{10}\begin{bmatrix}4&2&2\\-5&0&5\\1&-2&3\end{bmatrix}$

According to question, B is the inverse of matrix A. Hence $\alpha=5$

6. (d)

For invertible, $|A|\neq 0$ i.e., $\begin{vmatrix}1&0&-K\\2&1&3\\K&0&1\end{vmatrix}\neq 0$

$\Rightarrow 1(1)-K(-K)\neq 0 \Rightarrow |A|=K^2+1\neq 0$, which is true for all real K .

7. (a)

$|f(\alpha)|=\begin{vmatrix}\cos\alpha&-\sin\alpha&0\\ \sin\alpha&\cos\alpha&0\\0&0&1\end{vmatrix}=1$, adj of

$f(\alpha)=\begin{vmatrix}\cos\alpha&\sin\alpha&0\\-\sin\alpha&\cos\alpha&0\\0&0&1\end{vmatrix}$

$[f(\alpha)]^{-1}=\begin{vmatrix}\cos\alpha&\sin\alpha&0\\-\sin\alpha&\cos\alpha&0\\0&0&1\end{vmatrix}$ ......(i) and

$f(-\alpha)=\begin{vmatrix}\cos\alpha&\sin\alpha&0\\-\sin\alpha&\cos\alpha&0\\0&0&1\end{vmatrix}$ ......(ii)

From (i) and (ii), $[f(\alpha)]^{-1}=f[-\alpha]$

8. (b)

Determinant of unit matrix of any order =1.

9. (a)

$125=|A^3|=|A|^3\Rightarrow|A|=5$ and $|A|=\alpha^2-4=5\Rightarrow\ \alpha^2=9$

$\Rightarrow\ \alpha=\pm3$

10. (a)

We know that, det. $(-A)=(-1)^n\det A$, where n is order of square matrix

If A is square matrix of order 3, Then $n=3$. Hence $|-2A|=(-2)^3|A|=-8|A|$.

11. (c)

$X = A^{-1}B$

$X=\dfrac{(adjA)B}{|A|}$

Clearly if the system has infinite solutions |A| = 0 and (adj A)B = O

12. (a)

$AB=\begin{bmatrix}-2&3&-1\\-1&2&-1\\-6&9&-4\end{bmatrix}\begin{bmatrix}1&3&-1\\2&2&-1\\3&0&-1\end{bmatrix}$

$=\begin{bmatrix}-2+6-3&-1+4-3&-6+18-12\\-6+6+0&-3+4+0&-18+18+0\\2-3+1&1-2+1&6-9+4\end{bmatrix}$

$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $BA=\begin{bmatrix}1&3&-1\\2&2&-1\\3&0&-1\end{bmatrix}\begin{bmatrix}-2&3&-1\\-1&2&-1\\-6&9&-4\end{bmatrix}$

$$= \begin{bmatrix} -2+6-3 & -4-2+6 & -6+0+6 \\ 3+6-9 & 6+4-9 & 9+0-9 \\ 6-6+0 & -2-2+4 & -3+0+4 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow AB = BA$$

13. (d)

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

$$|A^T A^{-1}| = 1$$

14. (d)

[sum of leading diagonal elements is called Trace of matrix]

$$2+(-3)+16=15$$

15. (b)

$$AA' = I$$

$$\therefore |AA'|\,|I|$$

$$\Rightarrow \quad |A|\,|A| = 1$$

$$[\,|A'| = |A| \text{ for any square matrix}]$$

$$\therefore \ |A| = \pm 1$$

16. (c)

Here $A^2 = A$

$$\Rightarrow \begin{bmatrix} 2 & -2 & -16-4x \\ -1 & 3 & 16+4x \\ 4+x & -8-2x & -12+x^2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$$

On comparing $16+4x=4 \Rightarrow x=-3$

17. (c)

We evaluate A^2 and A^3 and write the given equation as AA^{-1}

$= I = \frac{1}{6}[A^3 + cA^2 + dA]$.Comparing the corresponding elements on both the sides, we get c = - 6, d = 11.

18. (a)

$AA^T = I$

$$\begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\alpha^2 & 0 & 0 \\ 0 & 6\beta^2 & 0 \\ 0 & 0 & 3\gamma^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2\alpha^2 = 1,\ 6\beta^2 = 1,\ 3\gamma^2 = 1$$

$$\therefore \alpha = \pm\frac{1}{\sqrt{2}},\ \beta = \pm\frac{1}{\sqrt{6}},\ \gamma = \pm\frac{1}{\sqrt{3}}$$

19. (d)

$$A = \pm I,\ \pm\begin{bmatrix} 1 & 0 \\ c & -1 \end{bmatrix}, \begin{bmatrix} 1 & b \\ \frac{1}{b} & 0 \end{bmatrix}, \begin{bmatrix} a & b \\ \frac{1-a^2}{b} & -a \end{bmatrix}$$

where a, b, c are arbitrary and h $\neq$ 0.

20. (c)

$$M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$$

$|M_r| = r^2 - (r-1)^2 = 2r - 1$

$$\Rightarrow \sum_{r=1}^{n} |M_r| = \sum_{r=1}^{n} (2r-1) = n^2$$

21. (b)

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \sqrt{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

22. (c)

$$A^T = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix},\ 2A^T = \begin{bmatrix} 0 & 2 & -4 \\ -2 & 0 & -6 \\ 4 & 6 & 0 \end{bmatrix} 2A^T + A$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} = A^T$$

<u>Alternate</u>

$A^T = -A$ ($\because$ A is skew symmetric)

So $2A^T + A = A^T + A - A = A^T$.

23. (d)

$|A|\,|\text{adj}(A)| = |A|^3$

$|\text{adj } A| = |A|^2 = 2^2$

24. (c)

We have

$$[1\ \ 2]\left(\begin{bmatrix} -2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$$

$$= [1\ \ 2]\begin{bmatrix} -2+10 \\ 3+4 \end{bmatrix} = [1\ \ 2]\begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

$= [8 + 14] = [2\ \ 2]$

25. (a)

We have

$$|A| = \begin{vmatrix} 1 & w^2 & w \\ w^2 & w & 1 \\ w & 1 & w^2 \end{vmatrix} = \begin{vmatrix} 1+w^2+w & w^2 & w \\ w^2+w+1 & w & 1 \\ w+1+w^2 & 1 & w^2 \end{vmatrix}$$

[Using $C_1 \to C_1 + C_2 + C_3$]

$$= \begin{vmatrix} 0 & w^2 & w \\ 0 & w & 1 \\ 0 & 1 & w^2 \end{vmatrix} = 0$$

A is a singular matrix.