**1. (a)**

The equation of the circle will be $x^2+(y\pm r)^2=r^2$. Hence the constant to be eliminated is *r*.

$\therefore$ order of differential equation =1

**2. (c)**

$y=c_1e^x+c_2e^{2x}+c_3e^{3x}+c_4e^x.e^{c_5}$

$=(c_1+c_4.e^{c_5})e^x+c_2e^{2x}+c_3e^{3x}=c_1'e^x+c_2e^{2x}+c_3e^{3x}$

where $c_1'=c_1+c_4.e^{c_5}$. So there are 3 arbitrary constant associated with different terms. Hence the order of the differential equation formed, will be 3.

**3. (a)**

The parametric form of the given equation is x = t, y = $t^2$. The equation of any tangent at t is 2xt = y + $t^2$. Differentiating we get $2t=y_1\left(=\frac{dy}{dx}\right)$ Putting this value in the above equation, we have $2x\frac{y_1}{2}=y+\left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1=4y+y_1^2$.

The order of this equation is 1.

**4. (a)**

Let the curve be y = f (x). The equation of the tangent at any point (x, y) is given by Y – y = f′ (x) (X – x). So the portion of the axis of X which is cut off between the origin and the tangent at any point is obtained by putting Y = 0. Therefore

$x-\frac{y}{f'(x)}=ky$

$\Rightarrow x-y\frac{dx}{dy}=ky$

$\Rightarrow \frac{dx}{dy}-\frac{x}{y}=-k$

which is a linear equation in x, and its integrating factor is $e^{-\int 1/y\,dy}=y^{-1}$.

Therefore, multiplying by $y^{-1}$ we have

$\frac{d}{dy}(xy^{-1})=-ky^{-1}$

$\Rightarrow xy^{-1}=-k\log y+c$

or x = y (c – k log y)

**5. (b)**

$y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$

$\Rightarrow y-ay^2=a\frac{dy}{dx}+x\frac{dy}{dx}$

$\Rightarrow y(1-ay)=(a+x)\frac{dy}{dx}$

$\Rightarrow \frac{dy}{(a+x)}=\frac{dy}{y(1-ay)}$

On integrating both sides, we get

$\int\frac{dx}{(a+x)}=\int\frac{dy}{y(1-ay)}$

$\Rightarrow \log(a+x)=\int\left[\frac{1}{y}+\frac{a}{(1-ay)}\right]dx$

$\Rightarrow \log(a+x)=\log y+\frac{a\log(1-ay)}{-a}+\log c$

$\Rightarrow \log(a+x)=\log y-\log(1-ay)+\log c$

$\Rightarrow \log(x+a)(1-ay)=\log cy$

$\Rightarrow (x+a)(1-ay)=cy$

**6. (a)**

Clearly, highest order derivative involved is $\frac{dy}{dx}$, having order 1.

Expressing the above differential equation as a polynomial in derivative, we have $\left(y-x\frac{dy}{dx}\right)^2=a^2\left(\frac{dy}{dx}\right)^2+b^2$

*i.e.*, $(x^2-a^2)\left(\frac{dy}{dx}\right)^2-2xy\frac{dy}{dx}+y^2-b^2=0$ In this equation, the power of highest order derivative is 2. So its degree is 2.

**7. (a)**

The highest order derivative involved is $\frac{d^2y}{dx^2}$ which is the 2nd order derivative. Hence order of the differential equation is 2. Making the above equation free from radical, as far as the derivatives are concerned, we have

$\left(\frac{d^2y}{dx^2}+x^{\frac{1}{4}}\right)^3=-\frac{dy}{dx}$ *i.e.* $\left(\frac{d^2y}{dx^2}+x^{\frac{1}{4}}\right)^3+\frac{dy}{dx}=0$

The exponent of highest order derivative $\frac{d^2y}{dx^2}$ will be 3.

Hence degree of the differential equation is 3.

**8. (d)**

The above equation cannot be written as a polynomial in derivatives due to the term $x^2\log\left(\frac{d^2y}{dx^2}\right)$. Hence degree of the differential equation is 'not defined'.

**9. (c)**

To eliminate the arbitrary constants *g*, *f* and *c*, we need 3 more equations, that by differentiating the equation 3 times. Hence highest order derivative will be $\frac{d^3y}{dx^3}$. Hence order of the differential equation will be 3.

**10. (a)**

To eliminate a the above equation is differentiated once and exponent of $\frac{dy}{dx}$ will be1. Hence degree is 1

**11. (b)**

The given differential equation can be re-written as $y = e^{\frac{dy}{dx}}$

$\Rightarrow \ln y = \frac{dy}{dx}$

This is a polynomial in derivative. Hence order is 1 and degree 1.

**12. (b)**

As the highest order derivative involved is $\frac{d^2y}{dx^2}$. Hence order is 2.

The given differential equation cannot be written as a polynomial in derivatives, the degree is not defined.

**13. (d)**

$y = c_1x + \frac{c_2}{x}$ .....(i)

There are two arbitrary constants. To eliminate these constants, we need to differentiate (i) twice.

Differentiating (i) with respect to $x$,

$\frac{dy}{dx} = c_1 - \frac{c_2}{x^2}$ .....(ii)

Again differentiating with respect to $x$,

$\frac{d^2y}{dx^2} = \frac{2c_2}{x^3}$ ......(iii)

From (iii), $c_2 = \frac{x^3}{2}\frac{d^2y}{dx^2}$ and from (ii), $c_1 = \frac{dy}{dx} + \frac{c_2}{x^2}$;

$\therefore\ c_1 = \frac{dy}{dx} + \frac{x}{2}\frac{d^2y}{dx^2}$

From (i), $y = \left(\frac{dy}{dx} + \frac{x}{2}\cdot\frac{d^2y}{dx^2}\right)x + \frac{x^2}{2}\cdot\frac{d^2y}{dx^2} \Rightarrow$

$y = x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$\therefore\ \frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0$

**14. (b)**

We have $y = \frac{x}{x+1} \Rightarrow \frac{1}{y} = \frac{x+1}{x} = 1 + \frac{1}{x}$

Differentiating $w.r.t.\ x$,

$-\frac{1}{y^2}\frac{dy}{dx} = 0 - \frac{1}{x^2}$

$\therefore\ x^2\frac{dy}{dx} = y^2$

**15. (a)**

The equation of a parabola whose axis is parallel to $y$-axis may be expressed as

$(x-\alpha)^2 = 4a(y-\beta)$ ........(i)

There are three arbitrary constants $\alpha, \beta$ and $a$.

We need to differentiate (i) 3 times

Differentiating (i) $w.r.t.\ x$, $2(x-\alpha) = 4a\frac{dy}{dx}$

Again differentiating $w.r.t.\ x$,

$2 = 4a\frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{2a}$

Differentiating $w.r.t.\ x$,

$\frac{d^3y}{dx^3} = 0$

**16. (b)**

The slope of the tangent to the family of curves is $m_1 = \frac{dy}{dx}$

Equation of the hyperbola is $xy = c^2 \Rightarrow y = \frac{c^2}{x}$

$\therefore\ \frac{dy}{dx} = -\frac{c^2}{x^2}$

$\therefore$ Slope of tangent to $xy = c^2$ is $m_2 = -\frac{c^2}{x^2}$

Now $\tan\frac{\pi}{4} = \frac{m_1 - m_2}{1 + m_1m_2} \Rightarrow 1 = \frac{\frac{dy}{dx} + \frac{c^2}{x^2}}{1 - \frac{c^2}{x^2}\frac{dy}{dx}} \Rightarrow$

$\frac{dy}{dx}\left(1 + \frac{c^2}{x^2}\right) = \left(1 - \frac{c^2}{x^2}\right)$

$\therefore\ \frac{dy}{dx} = \frac{x^2 - c^2}{x^2 + c^2}$

**17. (a)**

Separating the variables, we can re-write the given differential equation as

$\frac{xdx}{1+x^2} = \frac{dy}{1+y^2} \Rightarrow \int\frac{2x\,dx}{1+x^2} = 2\int\frac{dy}{1+y^2} \Rightarrow$

$2\tan^{-1}y = \log_e(1+x^2) + c$

**18. (b)**

We have $dy = (x^2 + \sin 3x)dx \Rightarrow \int dy = \int (x^2 + \sin 3x)dx \Rightarrow$

$y = \frac{x^3}{3} - \frac{\cos 3x}{3} + c$

**19. (a)**

Given equation may be re-written as $dx = (y^2 + \sin y)dy$

Integrating, $\int dx = \int (y^2 + \sin y)\,dy$

$\therefore\ x = \frac{y^3}{3} - \cos y + c$

**20. (c)**

Let $4x + y + 1 = z \Rightarrow 4 + \frac{dy}{dx} = \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 4$

$\therefore\ \frac{dy}{dx} = (4x + y + 1)^2$

$\Rightarrow \frac{dz}{dx} - 4 = z^2 \Rightarrow \frac{dz}{dx} = z^2 + 4 \Rightarrow \frac{dz}{z^2+4} = dx$

$\Rightarrow \frac{1}{2}\tan^{-1}\frac{z}{2} = x + c \Rightarrow \tan^{-1}\left(\frac{4x+y+1}{2}\right) = 2x + 2c$

$\therefore 4x + y + 1 = 2\tan(2x + 2c)$

**21. (a)**

Given equation may be expressed as $\frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right) + 1\right]$ …….(i)

Let $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ From (i), $v + x\frac{dv}{dx} = v(\log v + 1) \Rightarrow x\frac{dv}{dx} = v\log v$

$\Rightarrow \frac{dv}{v\log v} = \frac{dx}{x} \Rightarrow \int \frac{1}{\log v} d(\log v) = \int \frac{dx}{x}$

$\therefore \log(\log v) = \log x + \log c \Rightarrow \log(\log v) = \log(cx) \Rightarrow \log v = cx \Rightarrow v = e^{cx} \Rightarrow \frac{y}{x} = e^{cx}$, $\therefore y = xe^{cx}$

**22. (a)**

Given equation may be re-written as

$\frac{y}{x}\cdot\frac{dy}{dx} = \left(\frac{y}{x}\right)^2 + \frac{\phi((y/x)^2)}{\phi'((y/x)^2)}$ .....(i)

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ and $\frac{y}{x} = v$

$\therefore$From (i), $v\left(v + x\frac{dv}{dx}\right) = v^2 + \frac{\phi(v^2)}{\phi'(v^2)} \Rightarrow vx\frac{dv}{dx} = \frac{\phi(v^2)}{\phi'(v^2)}$

$\Rightarrow \frac{\phi'(v^2)(2v\,dv)}{\phi(v^2)} = 2\frac{dx}{x}$

Integrating, $\ln(\phi(v^2)) = 2\ln x + \ln c \Rightarrow \phi(v^2) = cx^2$

$\therefore \phi(y^2/x^2) = cx^2$

**23. (a)**

Given equation is homogeneous. Let $y = vx$ $\therefore$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{y^3 + 2x^2y}{x^3 + 2xy^2} = v + x\frac{dv}{dx} \Rightarrow \frac{(y/x)^3 + 2(y/x)}{1 + 2(y/x)^2} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{v^3 + 2v}{1 + 2v^2} = v + x\frac{dv}{dx} \Rightarrow x\frac{dv}{dx} = v\left\{\frac{v^2+2}{1+2v^2} - 1\right\} = v\left\{\frac{1-v^2}{1+2v^2}\right\}$

$\Rightarrow \frac{1+2v^2}{v(1-v^2)}dv = \frac{dx}{x} \Rightarrow \frac{1+2v^2}{v(1-v)(1+v)}dv = \frac{dx}{x}$

$\Rightarrow \left(\frac{A}{v} + \frac{B}{1-v} + \frac{D}{1+v}\right)dv = \frac{dx}{x}$

where $A(1-v)(1+v) + Bv(1+v) + Dv(1-v) = 1 + 2v^2$ Putting

$v = 0$, $A = 1$

$v = 1$, $B = \frac{3}{2}$

$v = -1$, $D = -\frac{3}{2}$

$\therefore \left(\frac{1}{v} + \frac{3}{2}\frac{1}{1-v} - \frac{3}{2}\frac{1}{1+v}\right)dv = \frac{dx}{x}$

Integrating both side, we get

$\ln v + \frac{3}{2}\frac{\ln(1-v)}{-1} - \frac{3}{2}\ln(1+v) = \ln x + \ln c$

$\Rightarrow \ln v - \frac{3}{2}\ln(1-v) - \frac{3}{2}\ln(1+v) = \ln cx \Rightarrow v/\{(1-v)(1+v)\}^{3/2} = cx$

$\Rightarrow \left(\frac{v}{cx}\right)^2 = (1-v^2)^3 \Rightarrow \left(\frac{y}{cx^2}\right)^2 = \left(1 - \frac{y^2}{x^2}\right)^3$

$\Rightarrow (x^2 - y^2)^3 = \frac{x^2y^2}{c^2}$

$\therefore (x^2 - y^2)^3 = Bx^2y^2, \left(\because \frac{1}{c^2} = B\right)$

**24. (b)**

Given equation is non-homogeneous

Let $x = X + h$, $y = Y + k$

$\Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$

$\therefore \frac{dY}{dX} = \frac{(X+h) - 3(Y+k) + 2}{3(X+h) - (Y+k) + 6} = \frac{X - 3Y + (h - 3k + 2)}{3X - Y + (3h - k + 6)}$ Let us select $h$ and $k$ so that $h - 3k + 2 = 0$ and $3h - k + 6 = 0$

Solving, $k = 0$, $h = -2$ $\therefore X = x - h = x + 2$, $Y = y - k = y$

$\therefore \frac{dY}{dX} = \frac{X - 3Y}{3X - Y}$, which is homogeneous

$\Rightarrow \frac{dY}{dX} = v + X\frac{dv}{dX} \Rightarrow \frac{X - 3Y}{3X - Y} = v + X\frac{dv}{dX}$

$\Rightarrow \frac{1 - 3(Y/X)}{3 - (Y/X)} = v + X\frac{dv}{dX} \Rightarrow \frac{1 - 3v}{3 - v} = v + X\frac{dv}{dX}$

$\Rightarrow X\frac{dv}{dX} = \frac{1-3v}{3-v} - v = \frac{v^2 - 6v + 1}{3 - v} \Rightarrow \frac{(3-v)dv}{v^2 - 6v + 1} = \frac{dX}{X}$

$\Rightarrow \frac{2v - 6}{v^2 - 6v + 1}dv = -2\frac{dX}{X}$

Integrating, $\ln(v^2 - 6v + 1) = -2\ln X + \ln c$

$\Rightarrow \ln(v^2 - 6v + 1) + \ln X^2 = \ln c \Rightarrow X^2(v^2 - 6v + 1) = c$

$\Rightarrow Y^2 - 6XY + X^2 = c$

$\therefore y^2 - 6(x+2)y + (x+2)^2 = c$

**25. (c)**

We have $xdx + (ydx + xdy) = 0 \Rightarrow xdx + d(xy) = 0$

Integrating, $\frac{x^2}{2} + xy = \frac{c}{2}$

$\therefore x^2 + 2xy = c$