1. (a)

$$\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$$

$$=\cos^{-1}\left[\cos\left(2\pi-\frac{\pi}{3}\right)\right]+\sin^{-1}\left[\sin\left(2\pi-\frac{\pi}{3}\right)\right] \quad =\frac{\pi}{3}-\frac{\pi}{3}=0$$

2. (a)

Given equation is $2\cos^{-1}x+\sin^{-1}x=\frac{11\pi}{6}$

$$\Rightarrow \cos^{-1}x+(\cos^{-1}x+\sin^{-1}x)=\frac{11\pi}{6} \Rightarrow \cos^{-1}x=\frac{11\pi}{6}-\frac{\pi}{2}$$

$\Rightarrow \cos^{-1}x=\frac{4\pi}{3}$, which is not possible as $\cos^{-1}x\in[0,\pi]$.

3. (b)

$$\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$$

$$\Rightarrow \frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{2}-\cos^{-1}y=\frac{2\pi}{3} \Rightarrow \cos^{-1}x+\cos^{-1}y=\frac{\pi}{3}$$

4. (b)

We know that $\sin^{-1}y+\cos^{-1}y=\frac{\pi}{2}, |y|\le 1$

$\therefore$ According to question,

$$x-\frac{x^2}{2}+\frac{x^3}{4}-.........\ =x^2-\frac{x^4}{2}+\frac{x^6}{4}-.........\ ...$$

$$\Rightarrow \frac{x}{1+\frac{x}{2}}=\frac{x^2}{1+\frac{x^2}{2}},\ (\because 0<|x|<\sqrt{2})$$

$$\Rightarrow \frac{x}{2+x}=\frac{x^2}{2+x^2} \Rightarrow 2x+x^3=2x^2+x^3 \Rightarrow x=x^2$$

$\therefore x-x^2=0 \Rightarrow x(1-x)=0 \Rightarrow x=0$ and $x=1$, but $x\ne 0$.

So, $x=1$

5. (c)

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ $\tan^{-1}\sqrt{x(x+1)}$ is defined, when $x(x+1)\ge 0$ ....(i)

$\sin^{-1}\sqrt{x^2+x+1}$ is defined, when $0\le x(x+1)+1\le 1$ or $0\le x(x+1)\le 0$ ....(ii)

From (i) and (ii), $x(x+1)=0$ or $x=0$ and -1.

Hence, number of solutions is 2

6. (b)

$$\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=\cot^{-1}\left(\frac{\sqrt{1-\frac{1}{5}}}{\frac{1}{\sqrt{5}}}\right)+\cot^{-1}3=$$

$$\cot^{-1}(2)+\cot^{-1}(3)=\cot^{-1}\left(\frac{2\times 3-1}{3+2}\right)=\cot^{-1}(1)=\frac{\pi}{4}$$

7. (d)

Given, $\sin^{-1}C=\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13}$

$$\therefore \sin^{-1}C=\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}$$

$$=\sin^{-1}\left\{\frac{3}{5}\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{9}{25}}\right\}=\sin^{-1}\left(\frac{56}{65}\right)\Rightarrow C=\frac{56}{65}$$

8. (c)

$$\tan^{-1}\frac{12}{5}+\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{63}{16}=\pi+\tan^{-1}\frac{48+15}{20-36}+\tan^{-1}\frac{63}{16}$$

$$(xy>1)=\pi-\tan^{-1}\frac{63}{16}+\tan^{-1}\frac{63}{16}=\pi$$

9. (b)

$$\sin^{-1}2x=\sin^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}x=\sin^{-1}\left[\frac{\sqrt{3}}{2}.\sqrt{1-x^2}-x\sqrt{1-\frac{3}{4}}\right]$$

$$\therefore\ 2x=\frac{\sqrt{3}}{2}\sqrt{1-x^2}-\frac{x}{2}$$

$$\therefore \left(\frac{5x}{2}\right)^2=\frac{3}{4}(1-x^2) \text{ or } 28x^2=3 \Rightarrow x=\sqrt{\frac{3}{28}}=\frac{1}{2}\sqrt{\frac{3}{7}},$$

(not $-\frac{1}{2}\sqrt{\frac{3}{7}}$)

10. (d)

If we denote $\cos^{-1}x$ by y, then

Since $0\le\cos^{-1}x\le\pi \Rightarrow 0\le 2y\le 2\pi$ ....(1)

Also since $-\frac{\pi}{2}\le\sin^{-1}\left(2x\sqrt{1-x^2}\right)\le\frac{\pi}{2}$

$$\Rightarrow -\frac{\pi}{2}\le\sin^{-1}\sin(2y)\le\frac{\pi}{2}$$

$\Rightarrow -\frac{\pi}{2}\le 2y\le\frac{\pi}{2}$ ....(2)

From (1) and (2) we find $0\le 2y\le\frac{\pi}{2}$

$$\Rightarrow 0\le y\le\frac{\pi}{4}$$

$$\Rightarrow 0\le\cos^{-1}x\le\frac{\pi}{4}$$

which holds if $\frac{1}{\sqrt{2}}\le x\le 1$

11. (a)

Given equation can be written as

$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$$\Rightarrow \tan^{-1}\frac{x-1+x+1}{1-(x-1)(x+1)}=\tan^{-1}\frac{3x-x}{1+3x^2}$$

$$\Rightarrow \frac{2x}{2-x^2}=\frac{2x}{1+3x^2}$$

$\Rightarrow x+3x^3=2x-x^3$

$\Rightarrow 4x^3-x=0$

$\Rightarrow x(4x^2-1)=0$

$\Rightarrow x=0, x=\pm\frac{1}{2}$

none of which satisfies $1<x<\sqrt{2}$

12. (b)

$(\cot^{-1} x - 1)(\cot^{-1} x - 2) > 0 \Rightarrow x < \cot 2$ or $x > \cot 1$

($\cot^{-1} x$ is a decreasing function)

13. (d)

$\tan \cos^{-1} \frac{4}{5} = \frac{3}{4}$, $\tan \tan^{-1} \frac{2}{3} = \frac{2}{3} \Rightarrow$ expression in the question $= \frac{3/4 + 2/3}{1 - 3/4 \times 2/3} = \frac{17}{6}$

$\Rightarrow$ Choice (d) is correct.

14. (a)

$\tan^2(\sin^{-1} x) > 1$

$\Rightarrow \frac{\pi}{4} < \sin^{-1} x < \frac{\pi}{2}$ or $-\frac{\pi}{2} < \sin^{-1} x < -\frac{\pi}{4}$

$x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ or $x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$

$\Rightarrow x \in \left(-1, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$

Hence (A) is the correct answer.

15. (b)

$\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x = \frac{\pi}{2} - \tan^{-1} x$ We know

$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} > -\tan^{-1} x > -\frac{\pi}{2}$ $\quad \therefore 0 < \frac{\pi}{2} - \tan^{-1} x < \frac{\pi}{4}$

16. (a)

We have $\cos^{-1}\left[\frac{x}{a}\cdot\frac{y}{b} - \sqrt{\left(1 - \frac{x^2}{a^2}\right)}\sqrt{\left(1 - \frac{y^2}{b^2}\right)}\right] = \alpha$

$\Rightarrow \frac{xy}{ab} - \sqrt{\left(1 - \frac{x^2}{a^2}\right)}\sqrt{\left(1 - \frac{y^2}{b^2}\right)} = \cos\alpha$

$\therefore \left(\frac{xy}{ab} - \cos\alpha\right)^2 = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2}$

$\frac{x^2y^2}{a^2b^2} + \cos^2\alpha - \frac{2xy}{ab}\cos\alpha = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2}$

$\Rightarrow \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = 1 - \cos^2\alpha = \sin^2\alpha$

17. (a)

$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

Let $s^2 = \frac{a+b+c}{abc}$

$\therefore \theta = \tan^{-1}\sqrt{a^2s^2} + \tan^{-1}\sqrt{b^2s^2} + \tan^{-1}\sqrt{c^2s^2}$

$\Rightarrow \theta = \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs) \Rightarrow$

$\theta = \tan^{-1}\left[\frac{as + bs + cs - abcs^3}{1 - abs^2 - bcs^2 - cas^2}\right] \Rightarrow$

$\tan\theta = s\left[\frac{(a+b+c) - abcs^2}{1 - (ab+bc+ca)s^2}\right] = 0$ $\quad [\because abcs^2 = (a+b+c)]$

Trick : Since it is an identity so it will be true for any value of a, b, c. Let $a = b = c = 1$ then

$\theta = \tan^{-1}\sqrt{3} + \tan^{-1}\sqrt{3} + \tan^{-1}\sqrt{3} = \pi$, $\tan\theta = 0$

18. (b)

$\sin\left\{\left(\frac{\pi}{2} - 2\tan^{-1} x\right) + 2\tan^{-1} x\right\} = \sin\frac{\pi}{2} = 1$

19. (b)

$\sin\left[2\tan^{-1}\left(\frac{1}{3}\right)\right] + \cos[\tan^{-1}(2\sqrt{2})] =$

$\sin\left[\tan^{-1}\frac{\frac{2}{3}}{1 - \frac{1}{9}}\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \sin\left[\tan^{-1}\frac{3}{4}\right] + \cos[\tan^{-1} 2\sqrt{2}]$

$= \sin\left[\sin^{-1}\frac{3}{5}\right] + \cos\left[\cos^{-1}\frac{1}{3}\right] = \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$

20. (b)

$\tan^{-1} x + \tan^{-1}\frac{1}{y} = \tan^{-1} 3$ or $\tan^{-1}\frac{1}{y} = \tan^{-1} 3 - \tan^{-1} x$ or $\tan^{-1}\frac{1}{y} = \tan^{-1}\frac{3-x}{1+3x} \Rightarrow y = \frac{1+3x}{3-x}$

As x, y are positive integers, $x = 1, 2$ and corresponding $y = 2, 7$

$\therefore$ Solutions are $(x, y) = (1, 2), (2, 7)$

21. (c)

The given expression is equal to

$2\tan^{-1}\left(\text{cosec}\frac{\pi}{3} - \tan\frac{\pi}{6}\right)$

$= 2\tan^{-1}\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right) = 2\tan^{-1}\frac{1}{\sqrt{3}} = 2 \text{ x } \frac{\pi}{6} = \frac{\pi}{3}$

22. (d)

The given expression is equal to

$2\left[\pi + \tan^{-1}\frac{1+2}{1-2} + \tan^{-1} 3\right]$

$= 2\left(\pi - \tan^{-1} 3 + \tan^{-1} 3\right) = 2\pi$

23. (d)

Taking $x = \tan\theta, \tan^{-1}\frac{\sqrt{1-x^2} - 1}{x} = \tan^{-1}\frac{\sec\theta - 1}{\tan\theta}$

$= \tan^{-1}\frac{1 - \cos\theta}{\sin\theta} = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \left(\frac{1}{2}\right)\theta = \left(\frac{1}{2}\right)\tan^{-1} x$

So that according to the given condition

$\left(\frac{1}{2}\right)\tan^{-1} x = 4 \Rightarrow \tan^{-1} x = 8$ or $x = \tan 8$

24. (d)

The given expression is equal to

$$1+\left[\tan\left(\tan^{-1}2\right)\right]^2+1+\left[\cot\left(\cot^{-1}3\right)\right]^2$$

$= 1+ 4 +1 9 = 15$

25. (b)

Let $f(x) = \pi \cot^{-1}(x-1) + (\pi - 1) \cot^{-1} x$

$$f'(x) = \frac{-\pi}{(x-1)^2+1} - \frac{(\pi-1)}{x^2+1} < 0 \ \forall \ x \in R$$

Hence $f(x)$ is decreasing function and $0 < f(x) < \pi (2\pi - 1)$

$\therefore f(x) = 2\pi - 1$ has only one solution

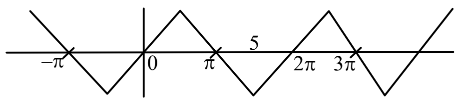
26. (d)

$\therefore \cos^{-1}(2x^2-1) = 2\pi - 2\cos^{-1}x \quad$ if $x < 0$

So, $2\pi - 2\cos^{-1}x - 2\sin^{-1}x = \pi$

27. (a)

Graph of $\sin^{-1} \sin(x) = f(x)$



$f(x) = \sin^{-1} (\sin x) = x - 2\pi \quad 3\pi/2 \le x \le 5\pi/2$

$f(5) = \sin^{-1} (\sin 5) = 5 - 2\pi$

$\log_2(x) < 5 - 2\pi$

$x > 0$

$x < 2^{5-2\pi}$

So, $(0, 2^{5-2\pi})$

28. (c)

We have $A = 2 \tan^{-1}(2\sqrt{2} - 1) = 2 \tan^{-1} (1.828) \Rightarrow A > 2 \tan^{-1} \sqrt{3} \Rightarrow A > \frac{2\pi}{3}$

Next $\sin^{-1}\left(\frac{1}{3}\right) < \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{6}$

$\Rightarrow \sin^{-1}\frac{1}{3} < \frac{\pi}{2}$

Also $3 \sin^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left[3.\frac{1}{3} - 4\left(\frac{1}{3}\right)^3\right]$

$= \sin^{-1}\left(\frac{23}{27}\right) = \sin^{-1} (0.852)$

$\Rightarrow 3 \sin^{-1}\left(\frac{1}{3}\right) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow 3 \sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{3}$

Further $\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1} (0.6) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{3}$

Hence, $B = 3 \sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

$< \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$. Hence $A > B$

29. (b)

L.H.S. of choice (B) is a negative number and R.H.S. is a positive number.

30. (c)

$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

$\therefore \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

$\Leftrightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$

$\Leftrightarrow x = y = z = 1$

Also $f(p + q) = f(p). f(q) \ \forall \ p, q \in R \qquad \ldots(1)$

Given $f(1) = 1$

from (1),

$F(1 + 1) = f(1). F(1) \Rightarrow f(2) = 1^2 = 1$

from (2), $f(2 + 1) = f(2) . f(1)$

$\Rightarrow f(3) = 1^2. 1 = 1^3 = 1$

Now given expression $= 3 - \frac{3}{3} = 2$

31. (a)

$1 \text{ rad} > 45^0 \tan 1 > \tan 45^0$

$\tan 1 > 1$

Also $\tan^{-1}(1) = \frac{\pi}{4} < 1,$

Hence, $\tan 1 > \tan^{-1}(1)$

32. (d)

$\because$ The given equation can be written as

$\frac{x+\frac{1}{x}}{2} = -\sin (\cos^{-1}y) \ (x \ne 0)$

$\therefore x + \frac{1}{x} \ge 2$ or $x + \frac{1}{x} \le -2$

$\therefore$ L.H.S. = R.H.S. if $\sin (\cos^{-1} y) = \pm 1$ i.e. $x = \pm 1$

When $x = -1 \qquad \sin (\cos^{-1}y) = 1$

$\Rightarrow y = 0 \qquad$ as $0 \le \cos^{-1} y \le \pi$

When $x = 1 \quad \sin(\cos^{-1}y) = -1$

Not possible as $\qquad 0 \le \cos^{-1} y \le \pi$

$\therefore \quad x = -1, y = 0$ is only solution.

33. (b)

$\cos^{-1}\sqrt{x} > \frac{\pi}{2} - \cos^{-1}\sqrt{x} \quad [\because x \ge 0]$

$\cos^{-1}\sqrt{x} > \frac{\pi}{4}$

$\Rightarrow \frac{\pi}{4} < \cos^{-1}\sqrt{x} \le \frac{\pi}{2}$

$0 \le \sqrt{x} < \frac{1}{\sqrt{2}}$

$0 \le x < \frac{1}{2}$

34. (c)

$\because$ Sum of roots = $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = m$

Product of roots = $\tan^{-1}\alpha \tan^{-1}\beta \tan^{-1}\gamma = m$

Using formulae of tan (A + B + C)

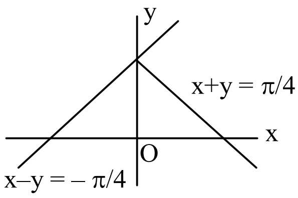
$\tan(\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma)$

$$= \tan\left(\frac{\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma - \tan^{-1}\alpha\ \tan^{-1}\beta \tan^{-1}\gamma}{(1 - \sum \tan^{-1}\alpha \tan^{-1}\beta)}\right)$$

$$= \tan\left(\frac{(m - m)}{1 - \sum \tan^{-1}\alpha \tan^{-1}\beta}\right) = 0$$

$\Rightarrow \tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma) = n\pi$

35. **(a)** a = 0



equation $x + y = \frac{\pi}{2} - \frac{\pi}{4} \Rightarrow x + y = \frac{\pi}{4}$

image $\quad x - y + \frac{\pi}{4} = 0$

36. **(a)**

1 rad > 45^0 tan 1 > tan 45^0

$\Rightarrow$ tan 1 > 1

Also $\tan^{-1}(1) = \frac{\pi}{4} < 1$,

Hence, tan 1 > $\tan^{-1}(1)$

37. **(a)**

Let $I = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$= (\sin^{-1} x + \cos^{-1} x)\ [(\sin^{-1} x)^2 + (\cos^{-1} x)^2$

$-(\sin^{-1} x)(\cos^{-1} x)]$

$$= \frac{\pi}{2}\left[(\sin^{-1} x + \cos^{-1} x)^2 - 3\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)\right]$$

$$= \frac{\pi}{2}\left[\left(\frac{\pi^2}{4} - \frac{3\pi}{2}\sin^{-1} x + 3(\sin^{-1} x)^2\right)\right]$$

$$= \frac{\pi}{2}\left[3\left(\sin^{-1} x)^2 - \frac{\pi}{2}(\sin^{-1} x) + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right) + \frac{\pi^2}{4}\right]$$

$$= \frac{\pi}{2}\left(\frac{\pi^2}{16} + 3\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right)$$

Now, $\frac{\pi}{4} \geq \sin^{-1} x - \frac{\pi}{4} \geq -\frac{3\pi}{4}$

$$\Rightarrow \frac{9\pi^2}{16} \geq \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \geq 0$$

$$\Rightarrow I \geq \frac{\pi}{2} \cdot \frac{\pi^2}{16} = \frac{\pi^3}{32}$$

Hence (A) is correct answer.

38. **(b)**

We have $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$

or, $x\sqrt{(1 - y^2)} + y\sqrt{(1 - x^2)} = z$

or, $x^2(1 - y^2) = z^2 + y^2(1 - x^2) - 2yz\sqrt{(1 - x^2)}$

or, $(x^2 - z^2 - y^2)^2 = 4y^2 z^2 (1 - x^2)$

or, $x^4 + y^4 + z^4 - 2x^2 z^2 + 2y^2 z^2 - 2x^2 y^2 + 4x^2 y^2 z^2 - 4y^2 z^2 = 0$

or, $x^4 + y^4 + z^4 + 4x^2 y^2 z^2 = 2(x^2 y^2 + y^2 z^2 + z^2 x^2)$

k = 2.

39. **(d)** $\cos^{-1} 1/2 + 2\sin^{-1} 1/2 = \pi/3 + 2\pi/6 = 2\pi/3$

40. **(d)** $\sin^{-1}[\sin(\pi - 2\pi/3)] = \sin^{-1}\sin(\pi/3) = \pi/3$

41. **(b)** $\tan\left[\tan^{-1}\frac{\sqrt{(1 - 16/25)}}{4/5} + \tan^{-1} 2/3\right.$

$= \tan . \tan^{-1} 17/6 = 17/6$

42. **(d)** $\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = \cos^{-1}(-1)$

$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}(-1) - \cos^{-1}(z)$

$\Rightarrow \cos^{-1}(xy - \sqrt{1 - x^2}\ \sqrt{1 - y^2}) = \cos^{-1}\{(-1)(z)\}$

$\Rightarrow xy - \sqrt{(1 - x^2)(1 - y^2)} = -z$

squaring both sides we get

$x^2 + y^2 + z^2 + 2xyz = 1$

Trick : Put $x = y = z = \frac{1}{2}$

so $\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} = \pi$

Obviously (D) holds for these values of x, y, z.

43. **(d)** Putting a = tan θ, b = tan ϕ so,

$\sin^{-1}(2\tan\theta/1 + \tan^2\theta) + \sin^{-1}(2\tan\phi/1 + \tan^2\phi) = 2\tan^{-1} x$

$\Rightarrow 2(\theta + \phi) = 2\tan^{-1} x.$

Hence $x = \tan(\theta + \phi)$

$\Rightarrow x = (\tan\theta + \tan\phi/1 - \tan\theta\tan\phi)$

Substitute values $\boxed{x = a + b/1 - ab}$

44. **(c)** The given equation may be written as

$\tan^{-1} x + \cot^{-1} x + \cot^{-1} x = 2\pi/3$

$\Rightarrow \cot^{-1} x = 2\pi/3 - \pi/2 = \pi/6$

$\Rightarrow \boxed{x = \sqrt{3}}$

45. **(b)**

$f(x) = \sin^{-1}\left(\sin\left(\theta - \frac{\pi}{6}\right)\right)$ (when x = sinθ)

$\theta - \pi/6 \Rightarrow \sin^{-1}x - \pi/6$

46. **(d)** $\sin^{-1} x/5 + \cos^{-1} 3/5 = \pi/2 \quad \Rightarrow \frac{x}{5} = \frac{3}{5}\ x = 3$

47. **(b)** Let $\theta = \tan^{-1} x \Rightarrow x = \tan\theta$

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}} = \frac{1}{\sqrt{1 + x^2}}$$

Hence $\cos\theta = \cos(\tan^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$

48. **(c)**

$\sin^{-1} x = \sin^{-1} \frac{2x}{1+x^2} \Rightarrow \frac{2x}{1+x^2} = x, x^3 - x = 0 \quad x = -1, 1, 0$

49. (a)

$\cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha} = x$

$\frac{\pi}{2} - 2\tan^{-1}\sqrt{\cos\alpha} = x \Rightarrow \frac{\pi}{2} - \tan^{-1}\left(\frac{2\sqrt{\cos\alpha}}{1-\cos\alpha}\right) = x$

$\cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha} \Rightarrow \operatorname{cosec} x = \sqrt{1+\cot^2 x}$

$= \frac{1+\cos\alpha}{1-\cos\alpha} = \cot^2\left(\frac{\alpha}{2}\right) \Rightarrow \because \sin x = \tan^2\left(\frac{\alpha}{2}\right)$

50. (d)

$\sin^{-1}\frac{x}{5} + \cos^{-1}\frac{3}{5} = \frac{\pi}{2} \Rightarrow \frac{x}{5} = \frac{3}{5} \qquad \therefore x = 3$

51. (b) Put $x = \tan\theta$ solution $= \frac{1}{\sqrt{3}}$

52. (d)

As LHS is +ive, RHS is -ive

Hence in ans. $+\pi$ which is not in any option

53. (a)

$\sin^{-1}\sin 22 = 7\pi - 22$, $\cos^{-1}\cos 33 = 33 - 10\pi$

$\tan^{-1}\tan 44 = 44 - 14\pi$

Hence $7\pi - 22 + 33 - 10\pi + 44 - 14\pi$, $55 - 17\pi$

54. (d)

$\cos^{-1}\left[\cos\left(\frac{\pi}{4}+\frac{9\pi}{10}\right)\right] \Rightarrow \cos^{-1}\left[\cos\frac{46\pi}{40}\right]$

$2\pi - \frac{46\pi}{40} = \frac{34\pi}{40} = \frac{17\pi}{20}$

55. (c)

RHS : $\cos^{-1}\frac{x}{2} - \cos^{-1}2$

$\cos^{-1}\left\{\frac{1}{2}x^2 + \sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right\}$

considering $\cos^{-1}\frac{x}{2} \geq \cos^{-1}x \Rightarrow \frac{x}{2} \leq x$

True for all positive x also for $\cos^{-1}x$ $-1 \leq x \leq 1$

Hence : $0 \leq x \leq 1$

56. (b) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

$\sin^{-1}\frac{5}{x} = \frac{\pi}{2} - \sin^{-1}\frac{12}{x}$

$\frac{5}{x} = \sin\left(\frac{\pi}{2} - \sin^{-1}\frac{12}{x}\right)$

$\Rightarrow \frac{5}{x} = \cos\sin^{-1}\left(\frac{12}{x}\right) \frac{5}{x} = \frac{\sqrt{x^2-144}}{x}$, $x \neq 0$ $25 = x^2 - 144$

$\Rightarrow x^2 = 169 \Rightarrow x = 13$

57. (b) $\tan\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right] = \tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right] = 17/6$

58. (d) Maximum value of $(\sin^{-1}x + \cos^{-1}x)^3 = \frac{7\pi^3}{8}$

Minimum value $= \frac{\pi^3}{32}$

So $\alpha \not< \frac{1}{32}$

59. (c)

$A = 2\tan^{-1}(2\sqrt{2} - 1) > 2\tan^{-1}\sqrt{3} = 120^0$

$B = 3\tan^{-1}\frac{1}{2\sqrt{2}} + \cos^{-1}\frac{4}{5} < 3\tan^{-1}(\sqrt{2}-1)$

$+ \cos^{-1}\frac{\sqrt{5}+1}{4} = 103\frac{1}{2}$

60. (c)

$\frac{\pi}{2} - \sin^{-1}\left(-\sin\left(\frac{7\pi}{6}\right)\right) = \frac{\pi}{2} + \sin^{-1}\sin\left(\frac{7\pi}{6}\right)$

$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

61. (c)

For $\alpha \in \left(-\frac{3\pi}{2}, -\pi\right)$, $\tan\alpha < 0$

$\Rightarrow \tan^{-1}(\cot\alpha) - \cot^{-1}(\tan\alpha) = -\pi$

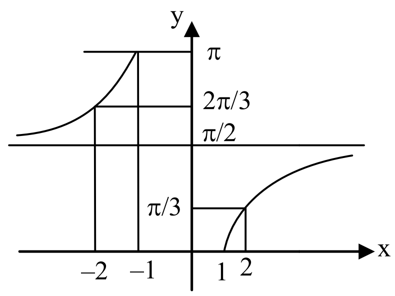
Also for points in 2nd quadrant

$\sin^{-1}(\sin\alpha) + \cos^{-1}(\cos\alpha) = \pi$.

62. (d)

$xy > 0 \Rightarrow$ x & y are of same sign

$x + \frac{1}{x} \geq 2$ or ≤ -2



$\sec^{-1}\left(x+\frac{1}{x}\right) \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

$z \in \left[\frac{2\pi}{3}, \pi\right) \cup \left(\pi, \frac{4\pi}{3}\right]$

Hence, value of z (among the given) which does not lie in the set is $\frac{5\pi}{3}$.

63. (b)

By tan (A + B + C) formula $\tan(\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma)$

= tan
$$\frac{\tan^{-1}\alpha+\tan^{-1}\beta+\tan^{-1}\gamma-(\tan^{-1}\alpha)(\tan^{-1}\beta)\tan^{-1}\gamma}{1-\sum\tan^{-1}\alpha\tan^{-1}\beta}$$

$= \tan\frac{(-m)-(-m)}{1-3} = \tan 0 = 0$

$\Rightarrow \tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$

64. (c)

For $x \in \left(\frac{3\pi}{2}, 2\pi\right)$, $\cos^{-1}(\cos x)$

$= \cos^{-1}(\cos(2\pi - (2\pi - x)))$

$= \cos^{-1}(\cos(2\pi - x)) = 2\pi - x$

and $\sin^{-1}(\sin x) = \sin^{-1}(\sin(2\pi + (x - 2\pi)))$

$= x - 2\pi$

$\Rightarrow \cos^{-1}(\cos x) + \sin^{-1}(\sin x)$

$= (2\pi - x) + (x - 2\pi) = 0.$

Therefore

$\sin^{-1}(\cos(\cos^{-1}(\cos x) + \sin^{-1}(\sin x))) = \sin^{-1} 0 = 0$

65. (c) Since $\frac{x}{y}\cdot\frac{x+y}{x-y} > 1$ .The given expression is equal to

$$\pi + \tan^{-1}\left[\frac{\frac{x}{y}+\frac{x+y}{x-y}}{1-\frac{x}{y}\times\frac{x+y}{x-y}}\right]$$

$= \pi + \tan^{-1}\frac{x^2+y^2}{-(x^2+y^2)} = \pi + \tan^{-1}(-1) = 3\pi/4.$

66. (c)

Let $\tan^{-1} 1/3 = \alpha$ and $\tan^{-1} 2\sqrt{2} = \beta$.

Then $\tan\alpha = 1/3$ and $\tan\beta = 2\sqrt{2}$, so that

$\sin(2\tan^{-1}(1/3)) + \cos(\tan^{-1} 2\sqrt{2})$

$= \sin 2\alpha + \cos\beta$

$= \frac{2\tan\alpha}{1+\tan^2\alpha} + \frac{1}{\sqrt{1+\tan^2\beta}}$ [$\because -\pi/2 < \beta < \pi/2$]

$= \frac{2.(1/3)}{1+(1/9)} + \frac{1}{\sqrt{1+8}} = \frac{2}{3}\cdot\frac{9}{10} + \frac{1}{3} = \frac{3}{5} + \frac{1}{3} = \frac{14}{15}.$

67. (d)

$[\cot^{-1} x] = 0, 1, 2, 3$, $[\tan^{-1} x] = -2, -1, 0, 1$

Case (i) $[\cot^{-1} x] = [\tan^{-1} x] = 1$

$x \in (\cot 2, \cot 1]$, & $x \in [\tan 1, \infty)$

$\therefore x \in \phi$ as $\cot 1 < \tan 1$

(ii) $[\cot^{-1} x] = 3$, $[\tan^{-1} x] = -1$

$x \in (-\infty, \cot 3]$, $x \in [-\tan 1, 0]$

$\therefore x \in \phi$ as $\cot 3 < -\tan 1$

(iii) $[\cot^{-1} x] = 2$, $[\tan^{-1} x] = 0$

$x \in (\cot 3, \cot 2]$, $x \in [0, \tan 1)$

$x \in \phi$ as $\cot 2 < 0$

So no solution

68. (a)

$\tan^{-1}\frac{2x+3x}{1-6x^2} = \frac{\pi}{4}$

$\Rightarrow 5x = 1 - 6x^2$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow x = -1, 1/6$

But sum of the -ve number cannot be $\pi/4$, hence $x = 1/6$ is only solution.

69. (c)

Since $\frac{3\pi}{2} < 5 < 2$,

We have $\sin 5 < 0$, so $\sin^{-1}(\sin 5) = 2\pi - 5$

Thus the given inequality can be written as

$2\pi - 5 > x^2 - 4x$ or $x^2 - 4x - (2\pi - 5) < 0$

$$\Rightarrow \left[x - \frac{4-\sqrt{16-4(2\pi-5)}}{2}\right]\left[x - \frac{4+\sqrt{16-4(2\pi-5)}}{2}\right] < 0$$

$\Rightarrow [x - 2 - \sqrt{9-2\pi}]\,[x - (2 + \sqrt{9-2\pi})] < 0$

$x \in (2 - \sqrt{9-2\pi}), (2 + \sqrt{9-2\pi}).$

70. (a)

$\because -1 \le x - 3 \le 1$

$\Rightarrow 2 \le x \le 4$ ....(i)

Also $-1 \le x - 1 \le 1$

$0 \le x \le 2$ ....(ii)

form (i) & (ii) $x = 2$

Also, $2 - x^2 \ne 0$

$x \ne \pm\sqrt{2}$

$\therefore$ At $x = 2$

$\sin^{-1}(2-3) + \cos^{-1}(2-1) + \tan^{-1}\left(\frac{2}{2-4}\right) = \cos^{-1}k - \pi$

$-\frac{\pi}{2} + 0 - \frac{\pi}{4} = \cos^{-1} k - \pi$

$\Rightarrow \cos^{-1} k = \pi/4$

$\Rightarrow k = \frac{1}{\sqrt{2}}$

71. (a) $\text{cosec}(\text{cosec}^{-1}x) = x\ \forall\ x \in R - (-1, 1)$

also range of $\text{cosec}^{-1}(\text{cosec } x) \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

so combining these two.

$x \in \left[-\frac{\pi}{2}, -1\right] \cup \left[1, \frac{\pi}{2}\right]$

72. (a)

$\sin\alpha = \cos(\sin^{-1} x) = \cos\left(\frac{\pi}{2} - \cos^{-1} x\right)$

$= \sin(\cos^{-1} x)$

$\therefore \quad \cos\beta = \sin(\cos^{-1} x)$

$\Rightarrow \quad \sin\alpha = \cos\beta$

$\Rightarrow \quad \tan\alpha = \cot\beta$

73. (c)

$$\cos^{-1}x = \tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{1+x^2}} \qquad \Rightarrow \qquad x^2 = \frac{-1+\sqrt{5}}{2}$$

$$\Rightarrow \quad \frac{x^2}{2} = \frac{\sqrt{5}-1}{4} \quad \cos^{-1}\left(\frac{x^2}{2}\right) = \frac{2\pi}{5}$$

74. (c)

since $\frac{x}{y} \times \frac{x+y}{x-y} > 1$

$$\text{given sum} = \pi + \tan^{-1}\left(\frac{\frac{x}{y}+\frac{x+y}{x-y}}{1-\frac{x}{y}.\frac{x+y}{x-y}}\right)$$

$= \pi + \tan^{-1}(-1) = 3\pi/4$

75. (b)

We have $\Sigma x_1 = \sin 2\beta$, $\Sigma x_1 x_2 = \cos 2\beta$, $\Sigma x_1 x_2 x_3 = \cos\beta$ and $x_1 x_2 x_3 x_4 = -\sin\beta$

$\therefore \ \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$

$$= \tan^{-1}\frac{\Sigma x_1 - \Sigma x_1 x_2 x_3}{1 - \Sigma x_1 x_2 + x_1 x_2 x_3 x_4}$$

$$= \tan^{-1}\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta} = \tan^{-1}\frac{(2\sin\beta - 1)\cos\beta}{(2\sin\beta - 1)\sin\beta}$$

$= \tan^{-1}(\cot\beta) = \tan^{-1}\tan(\pi/2 - \beta) = \pi/2 - \beta.$

76. (c)

Let $\cos^{-1}\left(\frac{3a}{b}\right) = \theta \quad \Rightarrow \cos\theta = \left(\frac{3a}{b}\right)$

Now $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$$= \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} + \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$$

$$= \frac{2\left(1+\tan^2\left(\frac{\theta}{2}\right)\right)}{1-\tan^2\left(\frac{\theta}{2}\right)} = \frac{2}{\cos\theta}$$

$$= \frac{2}{\left(\frac{3a}{b}\right)} = \frac{2}{3}\left(\frac{b}{a}\right).$$

77. (c)

$$\tan^{-1}\frac{2m}{m^4+m^2+2} = \tan^{-1}\frac{2m}{1+(m^2+m+1)(m^2-m+1)}$$

$\Rightarrow \tan^{-1}(m^2 + m + 1) - \tan^{-1}(m^2 - m + 1)$

So that $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$

$= (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) + \ldots. + \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)$

$\Rightarrow \tan^{-1}(n^2 + n + 1) - \tan^{-1}(1)$

$$\Rightarrow \tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$$

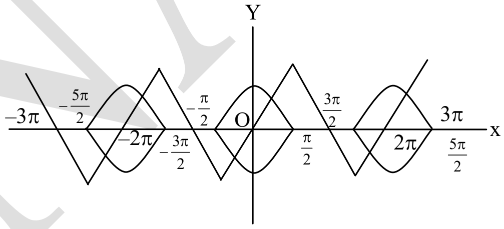
$= (\tan^{-1} 3 - \tan^{-1}) + (\tan^{-1} 7 - \tan^{-1} 3) + \ldots..+$
$(\tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1))$

$= \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)$

$$= \tan^{-1}\frac{n^2+n}{n^2+n+2}.$$

78. (c)

Graphs of $y = \sin^{-1}(\sin x)$ and $|y| = \cos x$ meet exactly six times in $[-3\pi, 3\pi]$.



79. (d)

$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2} \qquad \therefore \ \sin^{-1} x_i = \frac{\pi}{2}$

$\therefore \ x_i = 1, \qquad 1 < \iota < 1000$

$$\sum_{i=1}^{1000} x_i = 1000.$$

80. (c)

Clearly, $x(x + 1) \ge 0$ and $x^2 + x + 1 \le 1$. Together they imply $x(x + 1) = 0$.

$\therefore \ x = 0, -1.$

When $x = 0$,

L.H.S. $= \tan^{-1} 0 + \sin^{-1} 1 = \frac{\pi}{2}$.

When $x = -1$,

L.H.S. $= \tan^{-1} 0 + \sin^{-1}\sqrt{1-1+1} = 0 + \sin^{-1} 1 = \frac{\pi}{2}$,

Thus two solution.

81. (d) $\cos\theta = 1/x$, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-(1/x)^2}}{1/x}$

$= \sqrt{x^2 - 1}$

82. **(a)** Let $\cos^{-1} x = \theta$. Then $x = \cos\theta$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1} = 1/\sqrt{x^2-1} = \sqrt{1-x^2}\,/x$

83. **(a)**

Let $\cos^{-1} 4/5 = x \quad \Rightarrow \cos x = 4/5 \quad \ldots(i)$

Now $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \sin(x/2) \ldots(ii)$

From (i) $\cos x = \frac{4}{5}$

$\Rightarrow 1 - 2\sin^2 x/2 = \frac{4}{5}$

$\Rightarrow \sin x/2 = 1/\sqrt{10}$

84. **(c)**

$\tan\left[\tan^{-1}\frac{2/5}{1-1/25} - \tan^{-1}(1)\right]$

$= \tan\tan^{-1}\left[\frac{5/12-1}{1+5/12}\right] = 7/17$

85. **(a)**

$\Rightarrow \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right] = \pi$

$\Rightarrow x + y + z - xyz = 0$

$\Rightarrow x + y + z = xyz$

86. **(a)**

$\sin^{-1}\sin 22 = 7\pi - 22$; $\cos^{-1}\cos 33 = 33 - 10\pi$

$\tan^{-1}\tan 44 = 44 - 14\pi$

Hence $7\pi - 22 + 33 - 10\pi + 44 - 14\pi = 55 - 17\pi$

87. **(c)**

RHS $\cos^{-1}\frac{x}{2} - \cos^{-1} x$; $\cos^{-1}\left[\frac{x^2}{2} + \sqrt{1-\frac{x^2}{4}}\sqrt{1-x^2}\right]$

considering $\cos^{-1}\frac{x}{2} \geq \cos^{-1} x$; $\frac{x}{2} \leq x$

true for all positive x, also

for $\cos^{-1} x \rightarrow -1 \leq x \leq 1$, Hence $0 \leq x \leq 1$

88. **(a)**

$\theta = \tan^{-1}\dfrac{2\tan^2\theta - \frac{1}{3}\tan\theta}{1+\frac{2}{3}\tan^3\theta}$

$\tan\theta = \dfrac{6\tan^2\theta - \tan\theta}{3+2\tan^3\theta}$

$1 = \dfrac{6\tan\theta - 1}{3+2\tan^3\theta} \qquad \tan\theta = 0$

$2\tan^3\theta - 6\tan\theta + 4 = 0$; $(\tan\theta - 1)^2(\tan\theta + 2) = 0$

$\tan\theta = 1$; $\quad \tan\theta = -2$; $\quad \tan\theta = 0$

89. **(c)**

$\dfrac{3\sin 2\theta}{5+4\cos 2\theta} = \dfrac{3\times 2\tan\theta/(1+\tan^2\theta)}{5+4\dfrac{(1-\tan^2\theta)}{1+\tan^2\theta}} = \dfrac{6\tan\theta}{9+\tan^2\theta}$

Put $\tan\theta = 3\tan\phi \Rightarrow \dfrac{18\tan\phi}{9+9\tan^2\phi} = \dfrac{2\tan\phi}{1+\tan^2\phi} = \sin 2\phi$

Hence $\frac{1}{2}\cdot\sin^{-1}(\sin 2\phi) = \tan^{-1} x$; $\phi = \tan^{-1} x$

$\tan^{-1}\left(\frac{1}{3}\tan\theta\right) = \tan^{-1} x$; $x = \frac{1}{3}\tan\theta$

90. **(a)**

$A = 2\tan^{-1}(2\sqrt{2} - 1)$

$= 2\tan^{-1}(2 \times 1.414 - 1) = 2\tan^{-1}(1.828)$

$\Rightarrow A > 2\tan^{-1}\sqrt{3} = 2\pi/3$

For angle B, $\sin^{-1}\frac{1}{3} < \sin^{-1}\frac{1}{2} = \frac{\pi}{6} \Rightarrow 3\sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{2}$

$3\sin^{-1}\frac{1}{3} = \sin^{-1}\left(3.\frac{1}{8} - 4.\frac{1}{27}\right) = \sin^{-1}\left(\frac{23}{27}\right)$

$= \sin^{-1}(.851) < \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3} \Rightarrow 3\sin^{-1}\left(\frac{1}{3}\right) < \frac{\pi}{3}$

Also $\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}(0.6) < \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

so $\sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{3}$

Adding $B = 3\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{3}{5} < \frac{2\pi}{3}$ So $A > B$

91. **(d)**

Let $\frac{1}{5}\cos^{-1} x = \alpha \qquad$ As $0 \leq \cos^{-1} x \leq \pi$

so $0 \leq 5\alpha \leq \pi \qquad \Rightarrow 0 \leq \alpha \leq \frac{\pi}{5}$

For this α, $\sin\alpha$ can not be equal to 1.

So no solution.

92. **(d)**

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = \frac{\pi}{2}$

but will not lie in the domain of $\tan^{-1} x$. so, none of these is appropriate answer.

93. **(d)**

$\tan(A - B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

$= \dfrac{\left(\frac{x\sqrt{3}}{2k-x}\right) - \frac{2x-k}{k\sqrt{3}}}{1+\left(\frac{x\sqrt{3}}{2k-x}\right)\left(\frac{2x-k}{k\sqrt{3}}\right)} = \frac{1}{\sqrt{3}} \Rightarrow A - B = 30^0$

94. **(a)**

$\sin^{-1}x = \frac{\pi}{5}$, $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1} x$

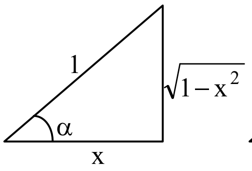
$\cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$

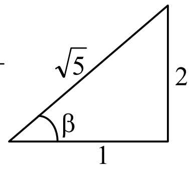
95. (b)

$\tan(\cos^{-1}x) = \sin(\cot^{-1}\frac{1}{2})$

let $\cos^{-1}x = \alpha$ and $\cos^{-1}\frac{1}{2} = \beta$

$\cos a = x \qquad \Rightarrow \cot\beta = \frac{1}{2}$





$\Rightarrow \tan\alpha = \sin\beta \Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow 5 - 5x^2 = 4x^2 \Rightarrow x = \pm\frac{\sqrt{5}}{3}$

96. (a)

$\sin(2\sin^{-1}0.8) = \sin\left[\sin^{-1}\left\{2(0.8)\sqrt{1-(0.8)^2}\right\}\right]$

$= (1.6)\sqrt{1-.64} = (1.6)(0.6) = 0.96$

97. (d)

$\sec^{-1}x = \text{cosec}^{-1}y$

$\cos^{-1}\frac{1}{x} = \sin^{-1}\frac{1}{y} \Rightarrow \cos^{-1}\frac{1}{x} = \frac{\pi}{2} - \cos^{-1}\frac{1}{y}$

$\Rightarrow \cos^{-1}\frac{1}{x} + \cos^{-1}\frac{1}{y} = \frac{\pi}{2}$

98. (c)

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

$\cos^{-1}\frac{1}{\sqrt{x^2+x+1}} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

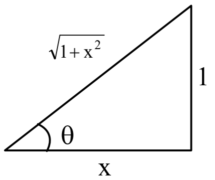
$\frac{1}{\sqrt{x^2+x+1}} = \sqrt{x^2+x+1}$

$\therefore x(x+1) = 0 \Rightarrow \qquad x = 0, x = -1$

99. (c)

$\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$

$\sqrt{1+x^2}\left[\left(\frac{x^2}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2}$



$= x\sqrt{1+x^2}$

100.(a)

$S = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{4}{33} + \ldots\infty$

$\tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} = \tan^{-1}\frac{2^n - 2^{n-1}}{1+2^n.2^{n-1}}$

$= \tan^{-1}2^n - \tan^{-1}2^{n-1}$

S_n = Sum of n term of the series

$\left(\tan^{-1}2 - \tan^{-1}1\right) + \left(\tan^{-1}2^2 - \tan^{-1}2\right) + \left(\tan^{-1}2^3 - \tan^{-1}2^2\right)$

$\ldots\ldots\ldots\ldots \left(\tan^{-1}2^n - \tan^{-1}2^{n-1}\right)$

$= \tan^{-1}2^n - \tan^{-1}1$

$S = \lim_{x\to\infty} S_n = \lim_{n\to\infty}\tan^{-1}2^n - \pi/4$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$