1. (d) Here $\overrightarrow{AB} = -2\mathbf{j}$, $\overrightarrow{BC} = (a-1)\mathbf{i} + (b+1)\mathbf{j} + c\mathbf{k}$

The points are collinear, then $\overrightarrow{AB} = k(\overrightarrow{BC})$

$$-2\mathbf{j} = k\{(a-1)\mathbf{i} + (b+1)\mathbf{j} + c\mathbf{k}\}$$

On comparing, $k(a-1) = 0$, $k(b+1) = -2$, $kc = 0$.

Hence $c = 0$, $a = 1$ and b is arbitrary scalar.

2. (c) If given points be A, B, C then $\overrightarrow{AB} = k(\overrightarrow{BC})$ or $2\mathbf{i} - 8\mathbf{j} = k[(a-12)\mathbf{i} + 16\mathbf{j}] \Rightarrow k = \frac{-1}{2}$

Also, $2 = k(a-12) \Rightarrow a = 8$.

3. (b) Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Rightarrow \mathbf{r}.\mathbf{i} = x,\ \mathbf{r}.\mathbf{j} = y,\ \mathbf{r}.\mathbf{k} = z$

$\Rightarrow (\mathbf{r}.\mathbf{i})^2 + (\mathbf{r}.\mathbf{j})^2 + (\mathbf{r}.\mathbf{k})^2 = x^2 + y^2 + z^2 = r^2$.

4. (a) Let $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$, then $\mathbf{a}.\mathbf{b} = 0$

$\Rightarrow 4x - 3y = 0 \Rightarrow \frac{x}{3} = \frac{y}{4} \Rightarrow x = 3\lambda,\ y = 4\lambda,\ \lambda \in R$.

Now $|\mathbf{a}| = |\mathbf{b}| \Rightarrow x^2 + y^2 = 16 + 9 + 25$

$= 9\lambda^2 + 16\lambda^2 = 50$

$\Rightarrow \lambda = \pm\sqrt{2} \Rightarrow x = \pm 3\sqrt{2},\ y = \pm 4\sqrt{2}$

Hence, $\mathbf{a} = \pm\sqrt{2}(3\mathbf{i} + 4\mathbf{j})$.

5. (a) $|\mathbf{a} - \mathbf{b}| = \sqrt{1^2 + 1^2 - 2.1^2\cos\theta} = \sqrt{2(1-\cos\theta)}$

$= \sqrt{2} \times \sqrt{2}\sin\frac{\theta}{2} = 2\sin\frac{\theta}{2} \Rightarrow \sin\frac{\theta}{2} = \frac{|\mathbf{a} - \mathbf{b}|}{2}$.

6. (b) $x^2 + y^2 = 1$

Let vector be $x\mathbf{i} + y\mathbf{j}$, then $4x - 3y = 0$

$\Rightarrow 4x = 3y \Rightarrow x = \frac{3}{5},\ y = \frac{4}{5}$,

Hence the required vector is $\frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$.

7. (d) Required work done

$= (3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + 2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}).(5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} - \mathbf{i} - 2\mathbf{j} - \mathbf{k})$

$= (5\mathbf{i} + 6\mathbf{j} - \mathbf{k}).(4\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 20 + 12 - 1 = 31$.

8. (d) It is obvious.

9. (a) $(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) = \mathbf{a} \times \mathbf{a} - \mathbf{b} \times \mathbf{a} + \mathbf{a} \times \mathbf{b} - \mathbf{b} \times \mathbf{b}$

$= \mathbf{a} \times \mathbf{b} - \mathbf{b} \times \mathbf{a} = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{b} = 2(\mathbf{a} \times \mathbf{b})$.

10. (c) It is obvious.

11. (b) $(\mathbf{a} \times \mathbf{b})^2 = a^2b^2 - (\mathbf{a}.\mathbf{b})^2 = \begin{vmatrix} \mathbf{a}.\mathbf{a} & \mathbf{a}.\mathbf{b} \\ \mathbf{a}.\mathbf{b} & \mathbf{b}.\mathbf{b} \end{vmatrix}$.

12. (c) $\mathbf{a}.(\mathbf{b} - \mathbf{c}) = 0$, $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{0} \Rightarrow \mathbf{b} = \mathbf{c}$ or $\mathbf{a} = \mathbf{0}$, but $\mathbf{a} \neq \mathbf{0}$. Hence $\mathbf{b} - \mathbf{c} = \mathbf{0}$. *i.e.*, $\mathbf{b} = \mathbf{c}$.

13. (c) Area of triangle $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$= \frac{1}{2}\left\|\begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{matrix}\right\|$$

Here, $(x_1, y_1, z_1) \equiv (1, 2, 3)$, $(x_2, y_2, z_2) \equiv (2, 5, -1)$, $(x_3, y_3, z_3) \equiv (-1, 1, 2)$

$$= \frac{1}{2}\left\|\begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -4 \\ -2 & -1 & -1 \end{matrix}\right\| = \frac{1}{2}|(-7\mathbf{i} + 9\mathbf{j} + 5\mathbf{k})|$$

$= \frac{1}{2}\sqrt{49 + 81 + 25} = \frac{\sqrt{155}}{2}$ *sq. unit.*

14. (a) $\mathbf{a}.(\mathbf{c} \times \mathbf{b}) = \mathbf{c}.(\mathbf{b} \times \mathbf{a}) = 0$, (Since $\mathbf{a}$ and $\mathbf{b}$ are parallel)

15. (a) $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & p & 5 \end{vmatrix} = 0 \Rightarrow p = -6$.

16. (b) $\mathbf{i}.(\mathbf{j} \times \mathbf{k}) + \mathbf{j}.(\mathbf{k} \times \mathbf{i}) + \mathbf{k}.(\mathbf{i} \times \mathbf{j}) = \mathbf{i}.\mathbf{i} + \mathbf{j}.\mathbf{j} + \mathbf{k}.\mathbf{k} = 3$.

17. (c) options (a), (b) and (d) = $[\mathbf{u}, \mathbf{v}, \mathbf{w}]$ while option (c) $= -[\mathbf{u}, \mathbf{v}, \mathbf{w}]$.

18. (a) $[\mathbf{a} - \mathbf{b}\ \ \mathbf{b} - \mathbf{c}\ \ \mathbf{c} - \mathbf{a}] = \{(\mathbf{a} - \mathbf{b}) \times (\mathbf{b} - \mathbf{c})\}.(\mathbf{c} - \mathbf{a})$

$= (\mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{c} - \mathbf{b} \times \mathbf{b} + \mathbf{b} \times \mathbf{c}).(\mathbf{c} - \mathbf{a})$

$= (\mathbf{a} \times a\mathbf{b} + \mathbf{c}\mathbf{a} \times \mathbf{a} + \mathbf{b} \times \mathbf{c}).(\mathbf{c} - \mathbf{a})$

$= (\mathbf{a} \times \mathbf{b}).\mathbf{c} - (\mathbf{a} \times \mathbf{b}).\mathbf{a} + (\mathbf{c} \times \mathbf{a}).\mathbf{c} - (\mathbf{c} \times \mathbf{a}).\mathbf{a}$

$= (\mathbf{a} \times \mathbf{b}).\mathbf{c} - (\mathbf{a} \times \mathbf{b}).\mathbf{a} + (\mathbf{c} \times \mathbf{a}).\mathbf{c} - (\mathbf{c} \times \mathbf{a}).\mathbf{a}$
$+ (\mathbf{b} \times \mathbf{c}).\mathbf{c} - (\mathbf{b} \times \mathbf{c}).\mathbf{a}$

$= [\mathbf{a}\,\mathbf{b}\,\mathbf{c}] - [\mathbf{a}\,\mathbf{b}\,\mathbf{a}] + [\mathbf{c}\,\mathbf{a}\,\mathbf{c}] - [\mathbf{c}\,\mathbf{a}\,\mathbf{a}] + [\mathbf{b}\,\mathbf{c}\,\mathbf{c}] - [\mathbf{b}\,\mathbf{c}\,\mathbf{a}] = 0$.

19. (a) $[\mathbf{a} \times \mathbf{b}\ \ \mathbf{b} \times \mathbf{c}\ \ \mathbf{c} \times \mathbf{a}] = (\mathbf{a} \times \mathbf{b}).[(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})]$

$= (\mathbf{a} \times \mathbf{b}).([\mathbf{b}\,\mathbf{c}\,\mathbf{a}]\mathbf{c} - [\mathbf{b}\,\mathbf{c}\,\mathbf{c}]\mathbf{a}) = (\mathbf{a} \times \mathbf{b}).([\mathbf{b}\,\mathbf{c}\,\mathbf{a}]\mathbf{c} - 0)$

$= [\mathbf{b}\,\mathbf{c}\,\mathbf{a}][\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = [\mathbf{a}\,\mathbf{b}\,\mathbf{c}][\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = 4.4 = 16$.

20. (d) $[\lambda(\mathbf{a} + \mathbf{b})\ \ \lambda^2\mathbf{b}\ \ \lambda\mathbf{c}] = [\mathbf{a}\ \ \mathbf{b} + \mathbf{c}\ \ \mathbf{b}]$

$\Rightarrow \lambda(\mathbf{a} + \mathbf{b}).(\lambda^2\mathbf{b} \times \lambda\mathbf{c}) = \mathbf{a}.((\mathbf{b} + \mathbf{c}) \times \mathbf{b})$

$\Rightarrow \lambda(\mathbf{a}+\mathbf{b}).\lambda^3(\mathbf{b}\times\mathbf{c}) = \mathbf{a}.(\mathbf{b}\times\mathbf{b}+\mathbf{c}\times\mathbf{b})$

$\Rightarrow \lambda^4[\mathbf{a}.(\mathbf{b}\times\mathbf{c})+\mathbf{b}.(\mathbf{b}\times\mathbf{c})] = \mathbf{a}.(\mathbf{c}\times\mathbf{b})$

$\Rightarrow \lambda^4[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = -[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \Rightarrow [\mathbf{a}\ \mathbf{b}\ \mathbf{c}](\lambda^4+1)=0$

Since **a, b, c** are non-coplanar, so $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \neq 0$

$\therefore \lambda^4 = -1$. Hence no real value of λ.

21. (a) $\mathbf{a}.\mathbf{c}=1$ and $\mathbf{b}.\mathbf{c}=1$

Given that $(\mathbf{a}\times\mathbf{b})\times\mathbf{c} = (\mathbf{c}.\mathbf{a})\mathbf{b}-(\mathbf{c}.\mathbf{b})\mathbf{a} = \mu\mathbf{b}+\lambda\mathbf{a}$

where $\mu = \mathbf{c}.\mathbf{a} = 1,\ \lambda = -(\mathbf{c}.\mathbf{b}) = -1$

$\Rightarrow \mu+\lambda = 1-1 = 0$.

22. (b) The equation of a plane parallel to the plane $\mathbf{r}.(4\mathbf{i}-12\mathbf{j}-3\mathbf{k})-7=0$ is $\mathbf{r}.(4\mathbf{i}-12\mathbf{j}-3\mathbf{k})+\lambda=0$.

This passes through $2\mathbf{i}-\mathbf{j}-4\mathbf{k}$.

Therefore, $(2\mathbf{i}-\mathbf{j}-4\mathbf{k}).(4\mathbf{i}-12\mathbf{j}-3\mathbf{k})+\lambda=0$

$\Rightarrow 8+12+12+\lambda=0 \Rightarrow \lambda=-32$

So, the required plane is $\mathbf{r}.(4\mathbf{i}-12\mathbf{j}-3\mathbf{k})-32=0$.

23. (c) The given plane passes through $\mathbf{a}$ and is parallel to the vectors $\mathbf{b}-\mathbf{a}$ and $\mathbf{c}$. So it is normal to $(\mathbf{b}-\mathbf{a})\times\mathbf{c}$. Hence, its equation is $(\mathbf{r}-\mathbf{a}).((\mathbf{b}-\mathbf{a})\times\mathbf{c})=0$

or $\mathbf{r}.(\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}) = [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$

The length of the perpendicular from the origin to this plane is $\dfrac{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}{|\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}$.

24. (d) The required line passes through the point $\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and is perpendicular to the lines

$$\mathbf{r} = (\mathbf{i}+2\mathbf{j}-\mathbf{k})+\lambda(2\mathbf{i}+\mathbf{j}+\mathbf{k})$$

and $\mathbf{r} = (2\mathbf{i}+6\mathbf{j}+\mathbf{k})+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$, therefore it is parallel to the vector

$\mathbf{b} = (2\mathbf{i}+\mathbf{j}+\mathbf{k})\times(\mathbf{i}+2\mathbf{j}+3\mathbf{k}) = (\mathbf{i}-5\mathbf{j}+3\mathbf{k})$

Hence, the equation of the required line is

$\mathbf{r} = (\mathbf{i}+3\mathbf{j}+2\mathbf{k})+\lambda'(\mathbf{i}-5\mathbf{j}+3\mathbf{k})$

$\Rightarrow \mathbf{r} = (\mathbf{i}+3\mathbf{j}+2\mathbf{k})+\lambda(-\mathbf{i}+5\mathbf{j}-3\mathbf{k})$, where $\lambda = -\lambda'$.

25. (a) A vector perpendicular to the plane P_1 of **a**, **b** is $\mathbf{a}\times\mathbf{b}$

A vector perpendicular to the plane P_2 of **c**, **d** is $\mathbf{c}\times\mathbf{d}$.

$\Rightarrow (\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=0 \Rightarrow (\mathbf{a}\times\mathbf{b}) \parallel (\mathbf{c}\times\mathbf{d})$

$\therefore$ The angle between the planes is $0°$.