1. (a)

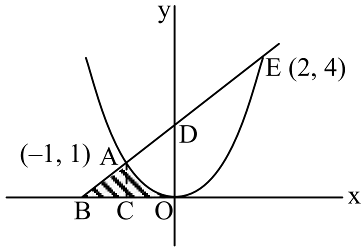
Intersection of $x^2 = y$ and $y = x + 2$ is $x^2 = x + 2$

$\Rightarrow\ x^2 - x - 2 = 0 \Rightarrow\ (x - 2)(x + 1) = 0$

$\Rightarrow x = -1, x = 2 \ \Rightarrow\ y = 1, y = 4$

$\Rightarrow\ A(-1, 1), B(2, 4)$

Required area = $A = (A_1 + A_2)$,



where A_1 = Area of

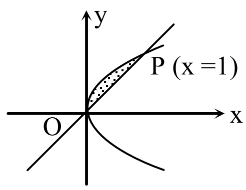
$\Delta ABC = \frac{1}{2} AC \times BC = \frac{1}{2}(1)(1) = \frac{1}{2}$

as OB = -2, CO = -1,

$$\text{Area} = A_2 = \left|\int_{-1}^{0} y\,dx\right| = \left|\int_{-1}^{0} x^2 dx\right| = \frac{1}{3}$$

By (1), $A = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

2. (b)

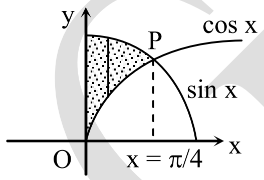


Solving the given equation for x, we get $4x^2 = 4x$

$$\Rightarrow x = 0, 1 \ \therefore \text{required area} = \int_0^1 \left(2\sqrt{x} - 2x\right) dx$$

$$= \left[2.\frac{2}{3}x^{3/2} - x^2\right]_0^1 = \frac{4}{3} - 1 = \frac{1}{3} \text{ unit}$$

3. (a)



In first quadrant sin x and cos x meet at $x = \pi/4$. The required area is a shown in the diagram. So required area

$$= \int_0^{\pi/4} (\cos x - \sin x)\,dx = [\sin x + \cos x]_0^{\pi/4}$$

$= (1/\sqrt{2} + 1/\sqrt{2}) - (0 + 1) = \sqrt{2} - 1$

4. (b)

$$\text{First area} = \int_0^{\pi/3} \cos x\,dx = [\sin x]_0^{\pi/3} \ z= \sqrt{3}/2$$

$$\text{second area} = \int_0^{\pi/4} \cos 2x\,dx + \left|\int_{\pi/4}^{\pi/3} \cos 2x\,dx\right|$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{\sqrt{3}}{4} = \frac{4-\sqrt{3}}{4}$$

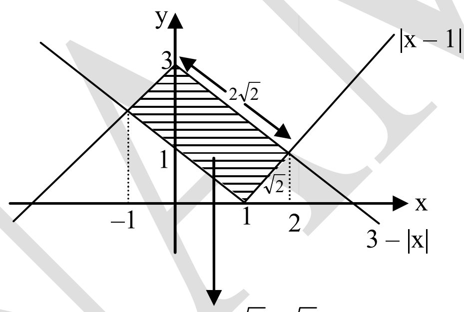
5. (b)

$$\int_0^{4a/m^2} (2\sqrt{a}\sqrt{x} - mx)\,dx = \frac{a^2}{3}$$

$$\Rightarrow \left[\frac{4}{3}\sqrt{a}x^{3/2} - m\frac{x^2}{2}\right]_0^{4a/m^2} = a^2/3$$

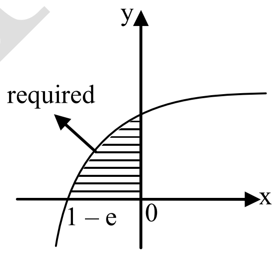
$$\Rightarrow \frac{8a^2}{3m^3} = \frac{a^2}{3} \Rightarrow m = 2$$

6. (d)



This area = $2\sqrt{2}\,.\sqrt{2}$ = 4 sq. unit

7. (a)



$$\Rightarrow \int_{1-e}^{0} \log_e(x+e)\,dx = \int_1^e \log_e x\,dx = 1$$

8. (b)

$$\text{Area} = \int_1^2 x^3 dx = \frac{15}{4}$$

9. (c)

$$xy - 3x - 2y - 10 = 0 \Rightarrow y = \frac{3x+10}{x-2}$$

$$\text{area} = \int_3^4 y\,dx \ \therefore \text{area} = \int_3^4 \frac{3x+10}{x-2}\,dx$$

area = $\int_3^4 \frac{3(x-2)+16}{x-2}\,dx$ area = $\int_3^4 \frac{3(x-2)}{x-2}\,dx + \int_3^4 \frac{16}{x-2}\,dx$

area = $3\int_3^4 dx + 16\int_3^4 \frac{1}{x-2}\,dx$

area = $3\,(x)_3^4 + 16\,(\log(x-2))_3^4$

Area = 3 + 16 log 2

10. (a)

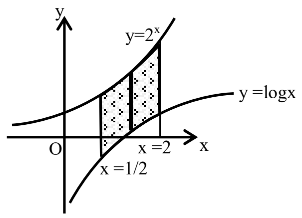
$$\int_{1/2}^{\sqrt{3}/2} (f(x)-g(x))\,dx = \int_{1/2}^{\sqrt{3}/2}\left[(1-x)-(x-\frac{1}{2})^2\right]dx = \frac{\sqrt{3}}{4}-\frac{1}{3}$$

11. (a)

Curves meet at $x = 0$ & $x = 1$

$$\therefore \text{Req. area} = \int_0^1 (\sqrt{x}-x^3)\,dx = \left[\frac{2}{3}x^{3/2}-\frac{x^4}{4}\right]_0^1 = \frac{5}{12}$$

12. (a)

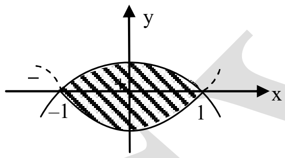


$$\text{Area } \int_{1/2}^{2} (2^x - \log x)\,dx =$$

$$\left[\frac{2^x}{\log 2} - x\log\left(\frac{x}{e}\right)\right]_{1/2}^{2} = \frac{4-\sqrt{2}}{\log 2}-\frac{5}{2}\log 2+\frac{3}{2}$$

13. (c)

$y = (1 - x^2) = -(x - 1)(x + 1)$



$$\text{Required area} = 2\int_{-1}^{1}(1-x^2)\,dx$$

14. (a)

$$\text{Required Area } = \int_0^1 (x-x^2)\,dx = \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1 = \frac{1}{2}-\frac{1}{3}=\frac{1}{6}.$$

15. (a)

Clearly the given equation are the parametric equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Curve meet the x-axis in the first quadrant at $(a,0)$

$$\therefore \text{ Required area } = \int_0^a y\,dx = \int_{\pi/2}^{0}(b\sin t)(-a\cos t)dt$$

$$= ab\int_0^{\pi/2}\sin^2 t\,dt = \left(\frac{\pi ab}{4}\right)$$

$(\because \text{At } x = 0, t = \pi/2 \text{ and } x = a, t = 0)$

16. (a)

Since the curve is symmetrical about x-axis, therefore the required area

$$= 2\int_0^1 y\,dx = 2\int_0^1 \sqrt{4x}\,dx$$

$$= 4.\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1 = \frac{8}{3}$$

17. (b)

$$\text{Required area} = 2\left[\int_0^1 x\,dx - \int_0^1 x^2\,dx\right] = 2\left[\left\{\frac{x^2}{2}\right\}_0^1 - \left\{\frac{x^3}{3}\right\}_0^1\right]$$

$$= 2\left[\left(\frac{1}{2}-0\right)-\left(\frac{1}{3}-0\right)\right] = 2\left[\frac{1}{2}-\frac{1}{3}\right] = 2\left[\frac{1}{6}\right] = \frac{1}{3}.$$

18. (a)

$$\text{Required area} = \int_1^2 (x^3-x^2)\,dx =$$

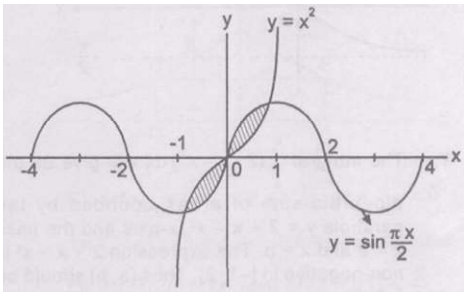
$$\left[\frac{x^4}{4}-\frac{x^3}{3}\right]_1^2 = \left(4-\frac{8}{3}\right)-\left(\frac{1}{4}-\frac{1}{3}\right) = \frac{4}{3}+\frac{1}{12} = \frac{16+1}{12} = \frac{17}{12}.$$

19. (d)

Bounded area,

$$\Delta = 2\int_0^1\left(\sin\frac{\pi x}{2}-x^3\right)dx$$

$$= 2\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}-\frac{x^4}{4}\right)_0^1 = 2\left(\frac{2}{\pi}-\frac{1}{4}\right) = \frac{8-\pi}{2\pi}$$



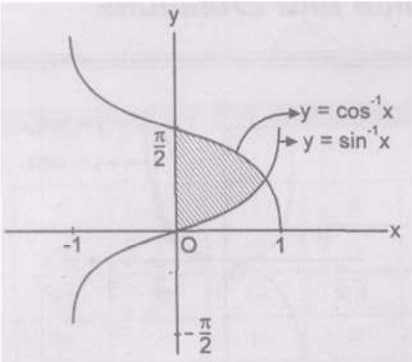
20. (b)

Required area,

$$\Delta = \int_0^{\pi/4} \sin y\, dy + \int_{\pi/4}^{\pi/2} \cos y\, dy$$

$$= -\cos y\Big|_0^{\pi/4} + \sin y\Big|_{\pi/4}^{\pi/2}$$

$$= -\frac{1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}} = (2-\sqrt{2}) \text{ sq. units.}$$



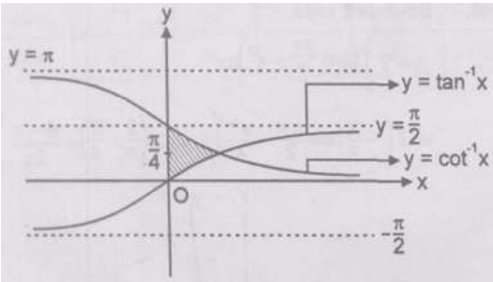
21. (d)

Required area,

$$\Delta = \int_0^{\pi/4} \tan y\, dy + \int_{\pi/4}^{\pi/2} \cot y\, dy$$

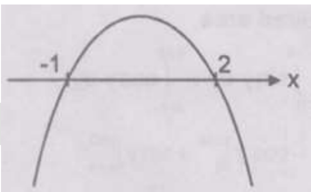
$$= \ell n \sec y\Big|_0^{\pi/4} + \ell n \sin y\Big|_{5\pi/4}^{\pi/2}$$

$$= \ell n\sqrt{2} - \ell n\frac{1}{\sqrt{2}} = \ell n\, 2$$



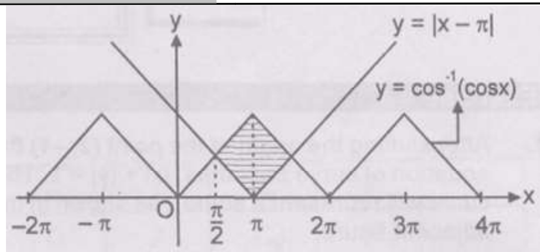
22. (a)

The integral $\int_a^b (2+x-x^2)\,dx$ will give us the algebraic sum of areas bounded by the parabola $y = 2 + x - x^2$, x-axis and the lines x = a and x = b. The expression $2 + x - x^2$ is non-negative in [–1, 2]. Thus [a, b] should be [-1, 2].



23. (c)

$$\cos^{-1}(\cos x) = \begin{cases} x &, \; x \in [0, \pi] \\ 2\pi - x &, \; x \in [\pi, 2\pi] \end{cases}$$



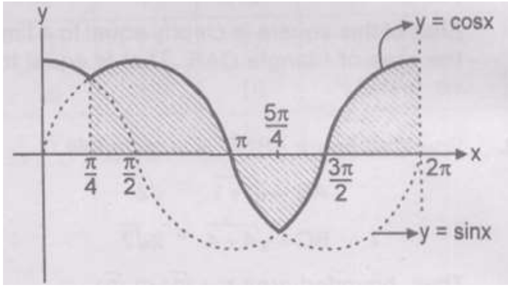
Required area,

$$\Delta = 2\int_{\pi/2}^{\pi} (x - (\pi - x))\, dx$$

$$= 2\left(x^2 - \pi x\right)_{\pi/2}^{\pi} = \frac{\pi^2}{2} \text{ sq. units.}$$

24. (a)

Bold lines represents the graph of y = maxi.{sinx, cosx}.



Required area,

$$\Delta = \int_{\pi/4}^{\pi} \sin x\, dx - \int_{\pi}^{5\pi/4} \sin x\, dx - \int_{5\pi/4}^{3\pi/2} \cos x\, dx + \int_{3\pi/2}^{2\pi} \cos x\, dx$$

$$= -\cos x\Big|_{\pi/4}^{\pi} + \cos x\Big|_{\pi}^{5\pi/4} - \sin x\Big|_{5\pi/4}^{3\pi/2} + \sin x\Big|_{3\pi/2}^{2\pi}$$

$$= \frac{(4\sqrt{2}-1)}{\sqrt{2}} \text{ sq. units.}$$

25. (c)

Let the drawn tangents be PA and PB. AB is clearly the chord of contact of point 'P', thus equation of AB is $\frac{1}{2}.(y + 0) = x.1 - (x+ 1) + 3$ i.e. y = 4

x coordinates of points A and B will be given by, $x^2 - 2x + 3 = 4$ i.e. $x^2 - 2x - 1 = 0$

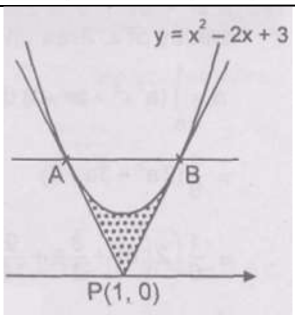
$\Rightarrow x = 1 \pm \sqrt{2}.$

Thus AB = $2\sqrt{2}$ units.

Hence $\Delta_{PAB} = \frac{1}{2}(2\sqrt{2}).4 = 4\sqrt{2}$

Now area bounded by line AB and parabola is equal to $\int_{1-\sqrt{2}}^{1+\sqrt{2}} (4-(x^2-2x+3))\,dx$ i.e equal to $\frac{4\sqrt{2}}{3}$ sq. units.

Thus required area = $4\sqrt{2} - \frac{4\sqrt{2}}{3} = \frac{8\sqrt{2}}{3}$ sq. units.


y = x² – 2x + 3
A
B
P(1, 0)