1. (c)

We know that α is a root of f(x) = 0. This means (x - α) is a factor of f(x). If α is a repeated root of f(x) = 0 then f(x) has a repeated factor (x - α), i.e., $(x - \alpha)^2$ is a factor of f(x). But here f(x) is quadratic

$\therefore f(x) = \lambda(x - \alpha)^2$ …(1)

Where λ is a constant.

Let $\Delta(x) = \begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$

Which is of the degree 5 at most and 3 at least.

Clearly, $\Delta(\alpha) = 0$

…(2)

Differentiating $\Delta(x)$ w.r.t. x,

$$\Delta'(x) = \begin{vmatrix} A'(x) & B'(x) & C'(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} + \begin{vmatrix} A(x) & B(x) & C(x) \\ 0 & 0 & 0 \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

$$+ \begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ 0 & 0 & 0 \end{vmatrix}$$

because derivatives of constants = 0

$$= \begin{vmatrix} A'(x) & B'(x) & C'(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

$$'\Delta'(\alpha) = \begin{vmatrix} A'(\alpha) & B'(\alpha) & C'(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} = 0 \ldots(3)$$

because $R_1 \equiv R_3$. We know that

$\phi(\alpha) = 0 \Rightarrow (x - \alpha)$ is a factor of ϕ (x) and $\phi'(\alpha) = 0$

$\Rightarrow (x - \alpha)^2$ is a factor of $\phi(x)$

$\therefore$ from (2) and (3), $\Delta(x)$ has a factor $(x - \alpha)^2$. $\therefore \Delta(x)$

$= (x - \alpha)^2 \,.\, F(x) = \frac{1}{\lambda}\lambda(x - \alpha)^2 \,.\, F(x) = \frac{1}{\lambda} f(x).$

F(x), using (1) $\therefore \Delta(x)$ is divisible by f(x).

2. (d)

Put a = 1; b = 1; c = 2

$\Rightarrow \begin{vmatrix} -1 & 1 & 2 \\ 1 & -1 & 2 \\ 2 & 2 & -4 \end{vmatrix} = \lambda.4 \Rightarrow 16 = 4\lambda \Rightarrow \lambda = 4$

3. (d)

$\because$ Elements of all C_1, C_2, C_3 are in AP

$\therefore \Delta = 0$

4. (b)

$R_1 \to R_1 + R_2 + R_3$ then $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$ Δ =3

$(a + b) \times 3b^2 = ab^2 (a + b)$

5. (d)

Put a = 1; b = 1; c = 2

$\Rightarrow \begin{vmatrix} -1 & 1 & 2 \\ 1 & -1 & 2 \\ 2 & 2 & -4 \end{vmatrix} = \lambda.4 \Rightarrow 16 = 4\lambda \Rightarrow \lambda = 4$

6. (b)

short trick put a = 1, b = 2, c = 3

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 1(5) - 2(1)(1) + 3(-7) = 5 - 2 - 21 = -$ ive

7. (c)

$$\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix} = 10!\,11!\,12! \begin{vmatrix} 1 & 11 & 11\times 12 \\ 1 & 12 & 12\times 13 \\ 1 & 13 & 13\times 14 \end{vmatrix}$$

$= 10!\,11!\,12! \begin{vmatrix} 1 & 11 & 11\times 12 \\ 0 & 1 & 24 \\ 0 & 2 & 50 \end{vmatrix}$ applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= 10!\,11!\,12!\,(50 - 48) = 2\,10!\,11!\,12!$

8. (b)

The system has a non-trivial solution is

$\Delta = \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$

$\Rightarrow 7 \sin 3\theta + 7 \cos 2\theta - 6 + 7 \cos 2\theta - 8 = 0$

$\Rightarrow \sin 3\theta + 2 \cos 2\theta = 2 \Rightarrow 3 \sin \theta - 4 \sin^3\theta + 2 (1 - 2\sin^2\theta) = 2$

$\Rightarrow \sin \theta (- 4 \sin^2\theta - 4 \sin \theta + 3) = 0 \Rightarrow \sin \theta = 0,$

$\frac{1}{2}$ $\sin \theta \neq -\frac{3}{2}$

Int $[0, \pi]$, $\theta = 0, \pi, \pi/6, 5\pi/6$

9. (a)

$f(x) = \begin{vmatrix} 2^{-x} & 2^{x} & x^2 \\ 2^{-3x} & 2^{3x} & x^4 \\ 2^{-5x} & 2^{5x} & 1 \end{vmatrix}$ $f(-x) = \begin{vmatrix} 2^{x} & 2^{-x} & x^2 \\ 2^{3x} & 2^{-3x} & x^4 \\ 2^{5x} & 2^{-5x} & 1 \end{vmatrix} = - f(x)$

10. (c)

$2^{a_1}, 2^{a_2}, 2^{a_3}$ ….. are in G.P $\Rightarrow a_2 - a_1 = a_3 - a_2 = a_4 - a_3$ =….. $\Rightarrow a_1, a_2, a_3$ ….. are in A.P.

$\Rightarrow a_{2n+1} + a_1 = 2a_{n+1}$, $a_2 + a_{2n+2} = 2a_{n+2}$, $a_3 + a_{2n+3} = 2a_{n+3}$

$\Rightarrow \Delta = \frac{1}{2}\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ a_{2n+1} & a_{2n+2} & a_{2n+3} \end{vmatrix} = 0$

11. (a)

$\Delta = pqr(a^3 + b^3 + c^3) - abc (p^3 + q^3 + r^3)$ $p + q + r = a + b + c = 0 \Rightarrow p^3 + q^3 + r^3 = 3pqr$, $a^3 + b^3 + c^3 = 3abc$

$\Delta = 3pqr\ abc - 3abc\ pqr = 0$

12. (c)

$z = e^{-2iA}(e^{-2i(B+C)} - e^{2iA}) - e^{+ic}(e^{-ic} - e^{i(A+B)}) + e^{iB}(e^{i(A+C)} - e^{-iB})$

$z = e^{-2\pi i} - 1 - 1 + e^{\pi i} + e^{i\pi} - 1 = 1 - 1 - 1 - 1 - 1 - 1 = -4$

$\Rightarrow \arg(z) = 11$

13. (c)

The system has a non trivial solution if

$$\Delta = \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$$

$\Rightarrow 7\sin 3\theta + 7\cos 2\theta - 6 + 7\cos 2\theta - 8 = 0$

$\Rightarrow \sin 3\theta + 2\cos 2\theta = 2 \Rightarrow 3\sin\theta - 4\sin^3\theta + 2 - 2\sin^2\theta = 2$

$\Rightarrow \sin\theta\,(4\sin^2\theta + 2\sin\theta - 3) = 0$

$\Rightarrow \sin\theta\,(2\sin\theta + 3)(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0, \frac{1}{2}$ since $\sin\theta \neq -\frac{3}{2}$

$\therefore$ Int $[0, \pi]$ $\theta = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

14. (d)

$R_2 \to R_2 - 2R_1;\ R_3 \to R_3 - 5R_1$

$$= \begin{vmatrix} 18 & 40 & 89 \\ 4 & 9 & 20 \\ -1 & -2 & -5 \end{vmatrix} \Rightarrow \Delta = \begin{vmatrix} 18 & 40 & 9 \\ 4 & 9 & 2 \\ -1 & -2 & -1 \end{vmatrix}$$

apply $C_3 - 2C_2$

$$\Delta = \begin{vmatrix} 0 & 40 & 9 \\ 0 & 9 & 2 \\ 1 & -2 & -1 \end{vmatrix} C_1 \to C_1 - 2C_3 = \begin{vmatrix} 40 & 9 \\ 9 & 2 \end{vmatrix}$$

$= 80 - 81 = -1$

15. (c)

On differentiating with respect to x

$4ax^3 + 3bx^2 + 2cx + 50$

$$= \begin{vmatrix} 3x^2 - 28x & -1 & 3 \\ 4x+1 & 3x & x-4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} x^3 - 14x^2 & -x & 3x+\lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

Put x = 0

$$50 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & \lambda \\ 4 & 3 & 1 \\ -3 & 4 & 0 \end{vmatrix}$$

$50 = 25\lambda \Rightarrow \lambda = 2$

16. (b)

Apply $R_2 \to R_2 - xR_1$, $R_3 \to R_3 - xR_2$ we get

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & 0 & a+x \end{vmatrix} = a\,(a+x)^2$$

$f(2x) - f(x) = a(a+2x)^2 - a\,(a+x)^2 = ax\,(2a + 3x)$

17. (c)

$$\Delta = 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 1 & a & a+b \\ 1 & a+2b & 0 \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & b & -2b \end{vmatrix}$$

$= 9b^2\,(a+b)$

$\therefore m = 9, n = 2$

18. (b)

Value $= \Delta^{n-1} \Rightarrow \Delta^2 = 121$

19. (b)

$C_2 \to C_2 - C_1;\ C_3 \to C_3 - C_1$

$$\Delta = \begin{vmatrix} a+b & b & 2b \\ a+2b & b & 2b \\ a+4b & b & 2b \end{vmatrix} = 0$$

20. (c)

By putting x = 0

all the three columns are identical so x^2 is a factor

$C_1 \to C_1 + C_2 + C_3$ then $(x + \alpha + \beta + \gamma)$ is also a factor

21. (b)

$$\begin{vmatrix} 5 & 4 & 3 \\ 100x+51 & 100y+41 & 100z+31 \\ x & y & z \end{vmatrix} R_2 \to R_2 - 100 \times R_3$$

$$= \begin{vmatrix} 5 & 4 & 3 \\ 51 & 41 & 31 \\ x & y & z \end{vmatrix} = 0$$

22. (b)

$c_3 \to c_3 - c_1,\ c_2 \to c_2 - c_1$

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ bc & c(a-b) & b(a-c) \\ b+c & a-b & a-c \end{vmatrix} = (a-b)(a-c)\begin{vmatrix} c & b \\ 1 & 1 \end{vmatrix}$$

$= (a - b)(b - c)(c - a)$

23. (a)

$$\begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1 \end{vmatrix}$$

$$= \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

24. (b)

Since α, β, γ are the roots

$\therefore \alpha + \beta + \gamma = \frac{-b}{a} = 0$

$$\text{So } \begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

25. (b)

$C_1 \longrightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} 0 & \sin^2 77 & -1 \\ 0 & -1 & \sin^2 13 \\ 0 & \sin^2 13 & \cos^2 13 \end{vmatrix} = 0$$