1. (c)

Required area $\int_{-1}^{0}\{(3+x)-(-x+1)\}$

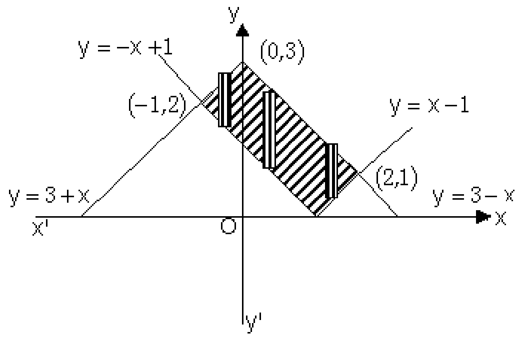
$+\int_{-1}^{0}\{(3-x)-(-x+1)\}dx+\int_{1}^{2}\{(3-x)-(x-1)\}dx$

$=\int_{-1}^{0}(2+2x)dx+\int_{0}^{1}2dx+\int_{1}^{2}(4-2x)dx$

$=\left[2x+x^2\right]_{-1}^{0}\;+\left[2x\right]_0^1+\left[4x-x^2\right]_1^2$

$=0-(-2+1)+(2-0)+(8-4)-(4-1)$

$=1+2+4-3=4$



2. (d)

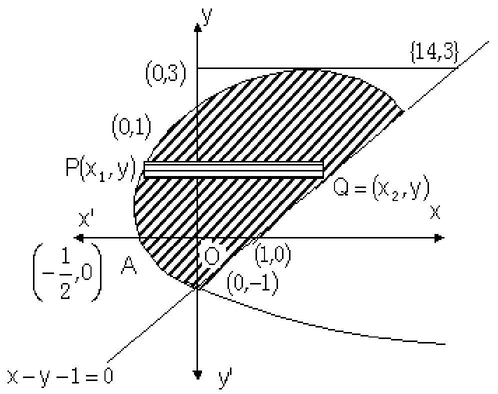
Required area

$=\int_{-1}^{3}(x_2-x_1)dy=\int_{-1}^{3}\left\{(y+1)-\left(\frac{y^2-1}{2}\right)\right\}dy$

$=\frac{1}{2}\int_{-1}^{3}\left(2y+3-y^2\right)dy=\frac{1}{2}\left[y^2+3y-\frac{y^3}{3}\right]^3$

$=\frac{1}{2}\left[(9+9-9)-\left(1-3+\frac{1}{3}\right)\right]$

$=\frac{1}{2}\left[9+\frac{5}{3}\right]=\frac{16}{3}$



3. (c)

$y = x\ \sin x$ is positive for $0<x<\pi$ and negative for $\pi\ x<2\pi$.

$\therefore$ Required area $=\left|\int_0^{\pi} x\sin x\,dx\ \right|\,x\,\left|\int_{\pi}^{2\pi} x\sin x\ dx\ \right|$

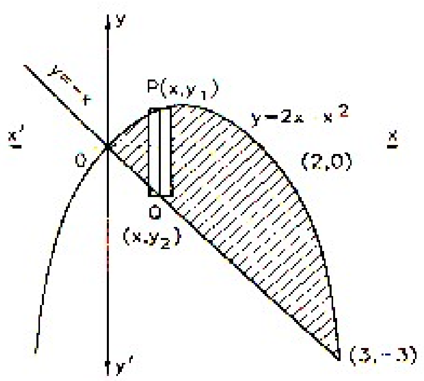
$=\pi+3\pi=4\pi$

4. (a)

Required Area $=\int_0^3 (y_1-y_2)dx$

$=\int_0^3\left(2x-x^2\right)-(-x)dx=\int_0^3\left(3x-x^2\right)dx$

$=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3=\frac{27}{2}-9=\frac{9}{2}$

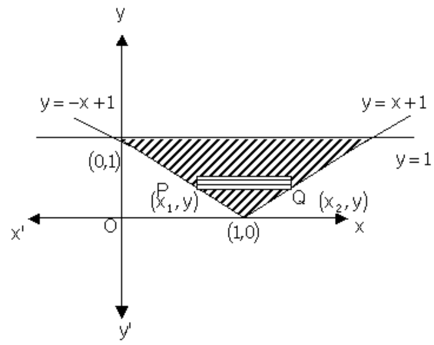


5. (a)

Required Area

$=\int_0^1 (x_2-x_1)dy=\int_0^1\{(x+1)-(-x+1)\}dx$

$=\int_0^1 2x\ dx=\left[x^2\right]_0^1=1$



6. (b)

$A=\left|\int_0^2 (3x-x^2-x^2+x)\ dx\right|$

7. (b)

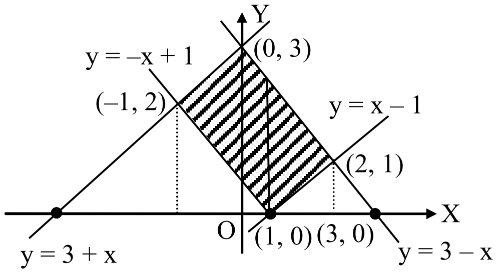
$x^2+2 = 2\,|\,x\,|$ - $\cos\pi x$

$\Rightarrow x^2 - 2\,|\,x\,| + 2 = -\cos\pi x$

$\Rightarrow (|x| - 1)^2 + 1 = -\cos \pi x$

$\Rightarrow x = \pm 1 \therefore$ Area $= \int_{-1}^{1} (x^2 + 2 - 2|x| + \cos \pi x)\, dx = 8/3$

8. (c)



Required area

$= \int_{-1}^{0} 3 + x - (-x + 1)\, dx + \int_{0}^{1} 3 - x - (-x + 1) dx +$

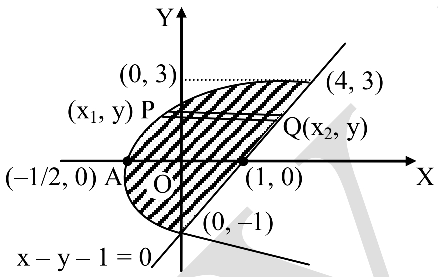
$\int_{1}^{2} 3 - x - (x - 1) dx = \int_{-1}^{0} (2 + 2x)\, dx + \int_{0}^{1} 2\, dx + \int_{1}^{2} (4 - 2x)\, dx$

= 4 sq. units..

9. (d)

Required area $= \int_{-1}^{3} (x_2 - x_1)\, dy$

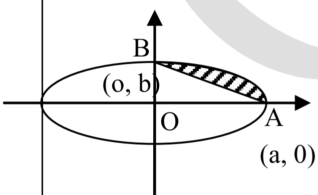
$= \int_{-1}^{3} \left[y + 1 - \left(\frac{y^2 - 1}{2} \right) \right] dy$



$= \frac{1}{2} \int_{-1}^{3} (2y + 3 - y^2) dy$

$= \frac{1}{2} \left[y^2 + 3y - \frac{y^3}{3} \right]_{-1}^{3} = \frac{16}{3}$ sq. units.

10. (b)



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots (1) \Rightarrow \quad y = \pm \frac{b}{a} \sqrt{a^2 - x^2}$

Equation of chord AB is $y - 0 = \frac{b - 0}{0 - a}$ (x - a) or

$y = -\frac{b}{a}(x - a) \quad \ldots (2)$

$\therefore$ Required

area$= \int_{0}^{a} \left\{ \frac{b}{a}\sqrt{a^2 - x^2} - \left(-\frac{b}{a} \right)(x - a) \right\} dx$

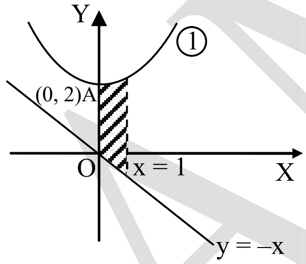
$= \frac{b}{a} \left[\frac{1}{2} x\sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a} + (x - a)^2 \right]_{0}^{a}$

$= \frac{ab}{4} (\pi - 2). \quad \left(\because \sin^{-1} 1 = \frac{\pi}{2} \right)$

11. (a)

$x^2 = y - 2 \quad \ldots (1)$

$y = -x \quad \ldots (2)$



Required area $= \int_{0}^{1} (x^2 + 2)\, dx + \left| \int_{0}^{1} -x\, dx \right| = \frac{7}{16}$.

(on calculation)

12. (d)

Since $|x| = 1, x = \pm 1$

$\therefore y = x e^{|x|} = \begin{cases} x e^{-x}, & -1 < x < 0 \\ x e^{x}, & 0 \le x < 1 \end{cases}$

$\therefore$ Required area $= \left| \int_{-1}^{0} x e^{-x} dx \right| + \left| \int_{0}^{1} x e^{x} dx \right|$

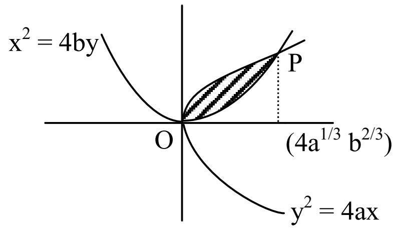
$= \left| \left[-xe^{-x} - e^{-x} \right]_{-1}^{0} \right| + \left| \left[xe^{x} - e^{x} \right]_{0}^{1} \right|$

= |-1 - e + e| + |e - e - 0 + 1| = 2 sq. units.

13. (b)

$y^2 = 4ax \ldots (1)$

$x^2 = 4by \quad \ldots (2)$



Using (2) in (1) $\frac{x^4}{16b^2} = 4ax \Rightarrow x = 0$ or $x^3 = 64ab^2$ i.e.

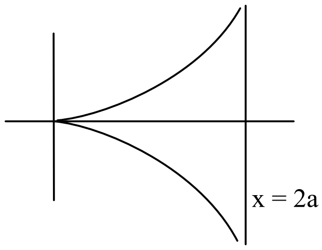
$x = 0, 4a^{1/3} b^{2/3}$.

$\therefore$Required area$= \int_{0}^{4a^{1/3}b^{2/3}} \sqrt{4ax} - \frac{x^2}{4b} dx$

$= \frac{16}{3}$ ab. (on calculation)

14. (a)

$y^2(2a - x) = x^3$ is symmetrical about x-axis because even power of y is present. It also passes through (0, 0). The line x = 2a is asymptote as $y \to \infty$ when x = 2a.



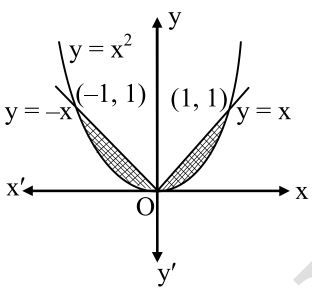
$\therefore$ Area $= 2\int_0^{2a} y\,dx = 2\int_0^{2a}\sqrt{\frac{x^3}{2a-x}}\,dx$ Put $x = 2a\sin^2\theta$

$\Rightarrow dx = 4a\sin\theta\cos\theta\,d\theta$

$\therefore$ Area $= 2\int_0^{\pi/2} 2a\frac{\sin^3\theta}{\cos\theta}\,.\,4a\sin\theta\cos\theta\,d\theta$

$= 16a^2\int_0^{\pi/2}\sin^4\theta\,d\theta = 16a^2\,.\,\frac{3.1}{4.2}\;\frac{\pi}{2} = 3\pi a^2.$

15. (a)



Hence, the required region R is the shaded region shown in figure. Since both the curves are symmetrical about y-axis. So, required area = 2(Shaded area in first quadrant)

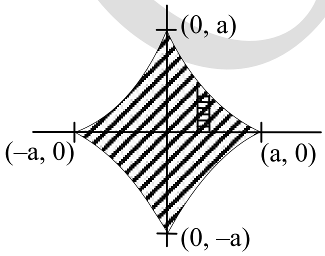
$= 2\int_0^1 (x - x^2)dx = 2\times\frac{1}{6} = \frac{1}{3}$ sq. units.

16. (a)

Eliminating t, we have

$x^{2/3} + y^{2/3} = a^{2/3}$ $x = 0 \Rightarrow y = \pm a$

$y = 0 \Rightarrow x = \pm a$ Symmetric about both the axis.

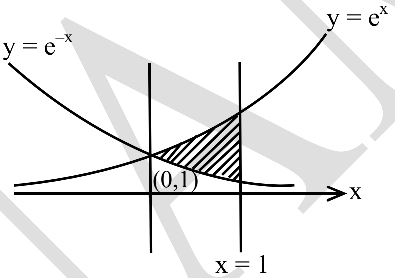


Required area $= 4\int_0^a y\,dx = 4\int_{\pi/2}^0 y.\frac{dx}{dt}dt$ $y = a\sin^3 t,\ x = a\cos^3 t \Rightarrow dx/dt = -3a\cos^2 t\sin t = 4\int_{\pi/2}^0 a\sin^3 t\,.\,(-3a\cos^2 t\,.\,\sin t)\,dt = 12a^2\int_0^{\pi/2}\sin^4 t\,.\,\cos^2 t\,dt = \frac{12a^2\,\Gamma\left(\frac{5}{2}\right).\Gamma\left(\frac{3}{2}\right)}{2\,\Gamma\left(\frac{8}{2}\right)}$

$= \frac{6a^2\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}\,.\,\frac{1}{2}\sqrt{\pi}}{3\times 2}$

$= \frac{3}{8}\pi a^2$

17. (b)



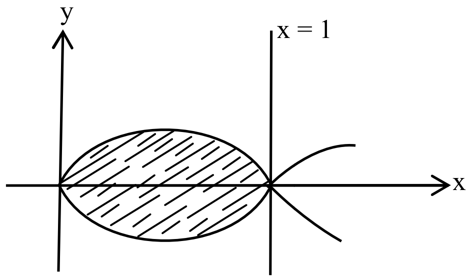
$\int_0^1 (e^x - e^{-x})\,dx = (e^x + e^{-x})_0^1$

$= (e + e^{-1}) - (1 + 1)$

$= e + \frac{1}{e} - 2$

18. (a)

curve is symmetric about x - axis



Reqd. Area $= 2\int_0^1 \sqrt{x}\,(1 - x)\,dx = 2\int_0^1 \sqrt{x} - x^{3/2}\,dx$

$= 2\left[\frac{2x^{3/2}}{3} - \frac{2x^{5/2}}{5}\right]_0^1$

$= 2\left(\frac{2}{3} - \frac{2}{5}\right) = 2\left(\frac{10-6}{15}\right) = 8/15$

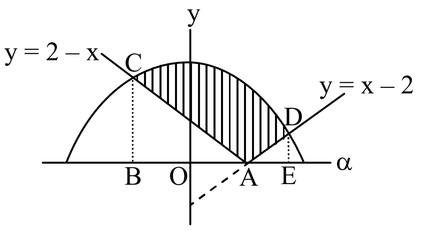
19. (b)

$3x^2 + 5y = 32$

$\Rightarrow x^2 = -\frac{-5}{3}\left(y - \frac{32}{5}\right)$ $y = |x - 2| = \begin{cases} x-2, & \text{if } x > 2 \\ 2-x, & \text{if } x < 2 \end{cases}$

$$\Rightarrow \begin{cases} \frac{x}{2}+\frac{y}{-2}=1 \\ \frac{x}{2}+\frac{y}{2}=1 \end{cases}$$

Here C (-2, 4), D(3, 1), A (2, 0)



E(3, 0), B (-2, 0)

Required area = $\int_{-2}^{3} y\,dx$ - ΔABC - ΔADE

$= \int_{-2}^{3} \frac{1}{5}(32 - 3x^2)\,dx - \frac{1}{2}(BC \times AB) - \frac{1}{2}(DE \times AE)$

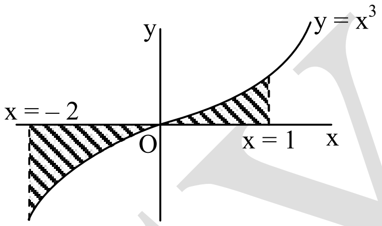
$= \frac{1}{5}(32x - x^3)_{x=-2}^{3} - \frac{1}{2}(4 \times 4) - \frac{1}{2}(1 \times 1)$

$= \frac{1}{5}(96 - 27 + 64 - 8) - \frac{17}{2} = \frac{33}{2}$

20. (d)

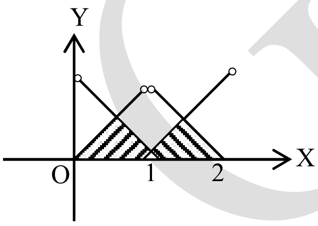
Required area = $\left|\int_{0}^{1} y\,dx\right| + \left|\int_{-2}^{0} y\,dx\right|$

$= \left|\int_{0}^{1} x^3\,dx\right| + \left|\int_{-2}^{0} x^3\,dx\right| = \frac{1}{4} + 4 = \frac{17}{4}$



21. Ans. 1

Sol.

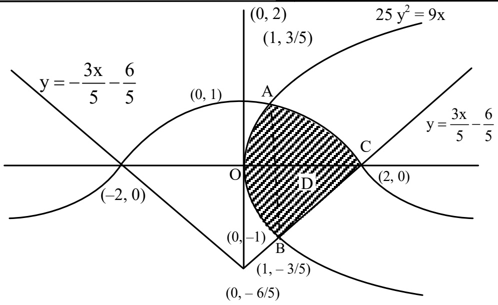


f(x) = min. ({x}, {-x}) which is even function

$I = \int_{-2}^{2} f(x)\,dx = 2\int_{0}^{2} f(x)\,dx$

22. Ans.0014

Sol.



Required area = area of region OABO + Area of region DBCA

= 2 (Area of the region OADO) + Area of the region ADBCA

$= 2\int_0^1 y\,dx + \int_1^2 (y_1 - y_2)\,dx$

$= 2\int_0^1 \frac{3}{5}x\,dx + \int_1^2 \left\{\frac{4-x^2}{4+x^2} - \frac{3x-6}{5}\right\}dx$

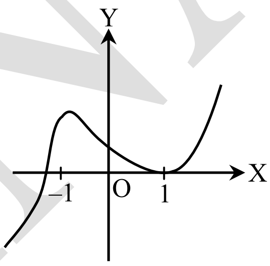
$= \left\{\pi - 4\tan^{-1}\frac{1}{2} - \frac{1}{10}\right\}$ sq. units

A + B = 14

23. Ans. 6

Sol. $\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 3 = 0$ or $x = \pm 1$

$x = \pm 1$ is point of extremum



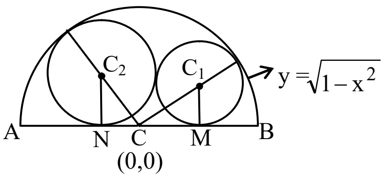
Area = $\int_{-1}^{1} |x^3 - 3x + 4|\,dx$

$= \int_{-1}^{1} (x^3 - 3x + 4)\,dx$

$= \int_{-1}^{1} 4\,dx = 8$ Þ $3A = 24 = 2^3 3^1$

24. Ans. 0004

Sol.



AC = BC = 1

Let $r_1 = a$, $r_2 = b$

Now $CC_1 = 1 - a$, $C_1M = a$

$\Rightarrow CM = \sqrt{(1-a)^2 - a^2} = \sqrt{1-2a}$

$CC_2 = 1 - b$, $C_2N = b \Rightarrow CN = \sqrt{1-2b}$

$\therefore$ Coordinate of C_1 ($\sqrt{1-2a}$, a)

Coordinate of C_2 ($\sqrt{1-2b}$, b) = (0, 1/2)

Use $C_1C_2 = r_1 + r_2$

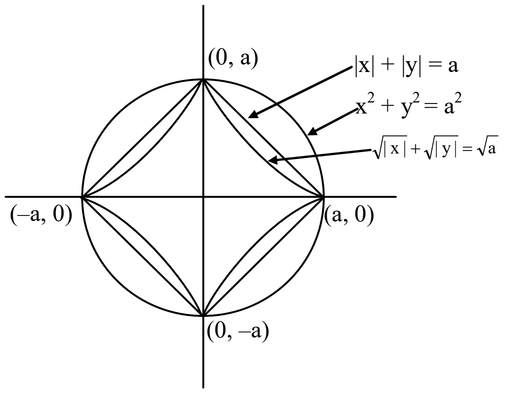
$$\sqrt{1-2a+\left(a-\frac{1}{2}\right)^2} = a + b$$

$$\sqrt{1-2a+\left(a-\frac{1}{2}\right)^2} = a + \frac{1}{2}$$

$$\Rightarrow \boxed{a = \frac{1}{4}}$$

25. Ans. 0010

Sol.



$A_1 \Rightarrow$ area included between the line and circle in first quadrant $= \frac{\pi a^2}{4} - \frac{1}{2}a^2 = \frac{(\pi-2)a^2}{4}$

A_2 = area included between the line and $\sqrt{|x|} + \sqrt{|y|} = \sqrt{a}$

$$= \frac{1}{2}a^2 - \int_0^a (\sqrt{a}-\sqrt{x})^2\, dx$$

$$= (9\pi - 2)\ \frac{a^2}{12}$$

Now $\frac{A_2}{A_1} = \frac{1}{3}\left(\frac{9\pi-2}{\pi-2}\right)$, A = 9, B = 1

A + B = 10