1. (d)

It is given that $T_2 = 240, T_3 = 720, T_4 = 1080$

Now, $T_2 = 240 \Rightarrow T_2 = {}^nC_1 x^{n-1} a^1 = 240$ .....(i)

and $T_3 = 720 \Rightarrow T_3 = {}^nC_2 x^{n-2} a^2 = 720$ .....(ii)

$T_4 = 1080 \Rightarrow T_4 = {}^nC_3 x^{n-3} a^3 = 1080$ .....(iii)

To eliminate x, $\frac{T_2.T_4}{T_3^2} = \frac{240.1080}{720.720} = \frac{1}{2} \Rightarrow \frac{T_2}{T_3}.\frac{T_4}{T_3} = \frac{1}{2}$.

Now $\frac{T_{r+1}}{T_r} = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$. Putting $r = 3$ and 2 in above expression, we get $\frac{n-2}{3}.\frac{2}{n-1} = \frac{1}{2}$

$\Rightarrow n = 5$

2. (c)

We know that, $(1+x)^n = {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \dots\dots + {}^nC_n x^n$

$\left(1+\frac{1}{x}\right)^n = {}^nC_0 + {}^nC_1 \frac{1}{x^1} + {}^nC_2 \frac{1}{x^2} + \dots + {}^nC_n \frac{1}{x^n}$ Obviously, the term independent of x will be

${}^nC_0.{}^nC_0 + {}^nC_1{}^nC_1 + \dots + {}^nC_n.{}^nC_n = C_0^2 + C_1^2 + \dots + C_n^2$

Trick : Put $n = 1$ in the expansion of

$(1+x)^1\left(1+\frac{1}{x}\right)^1 = 1 + x + \frac{1}{x} + 1 = 2 + x + \frac{1}{x}$ .....(i)

We want coefficient of x^0. Comparing to equation (i).

Then, we get 2 *i.e.,* independent of x.

Option (c) : $C_0^2 + C_1^2 + \dots C_n^2$; Put $n = 1$;

Then ${}^1C_0^2 + {}^1C_1^2 = 1 + 1 = 2$.

3. (b)

Given, total number of terms = $\frac{(n+1)(n+2)}{2} = 45$

$\Rightarrow (n+1)(n+2) = 90 \Rightarrow n = 8$.

4. (d)

$(1-x)^n = C_0 - C_1 x + C_2 x^2 - C_3 x^3 + \dots\dots$

$\Rightarrow x(1-x)^n = C_0 x - C_1 x^2 + C_2 x^3 - C_3 x^4 + \dots$

$\Rightarrow \int_0^1 x(1-x)^n dx = C_0\left[\frac{x^2}{2}\right]_0^1 - C_1\left[\frac{x^3}{3}\right]_0^1 + C_2\left[\frac{x^4}{4}\right]_0^1 - \dots\dots$ (i)

The integral on L.H.S. of (i) = $\int_1^0 (1-t)t^n(-dt)$ by putting

$1 - x = t$, $\Rightarrow \int_0^1 (t^n - t^{n+1})dt = \frac{1}{n+1} - \frac{1}{n+2}$

Whereas the integral on the R.H.S. of (i)

$= C_0\left[\frac{1}{2}\right] - C_1\left[\frac{1}{3}\right] + \frac{C_2}{4} - \dots\dots = \frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \dots\dots$ to $(n+1)$ terms $= \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$

Trick : Put $n = 1$ in given series = $\frac{{}^1C_0}{2} - \frac{{}^1C_1}{3} = \frac{1}{6}$.

Which is given by option (d).

5. (a)

$\frac{1}{(1-x)(3-x)} = (1-x)^{-1}(3-x)^{-1} = 3^{-1}(1-x)^{-1}\left(1-\frac{x}{3}\right)^{-1}$

$= \frac{1}{3}[1 + x + x^2 + \dots x^n]\left[1 + \frac{x}{3} + \frac{x^2}{3^2} + \dots + \frac{x^{n-1}}{3^{n-1}} + \frac{x^n}{3^n}\right]$

Coefficient of $x^n = \frac{1}{3^{n+1}} + \frac{1}{3^n} + \frac{1}{3^{n-1}} + \dots (n+1)$ terms

$= \frac{1}{3^{n+1}}\frac{[3^{n+1}-1]}{3-1} = \frac{3^{n+1}-1}{2.3^{n+1}}$.

Trick: Put $n = 1, 2, 3\dots\dots$ and find the coefficients of $x, x^2, x^3\dots\dots$ and comparing with the given option as :

Coefficient of x^2 is $= \frac{1}{3^3} + \frac{1}{3^2} + \frac{1}{3^1} = \frac{1}{3^3}\frac{[3^3-1]}{3-1} = \frac{13}{27}$;

Which is given by option (a) $\frac{3^{n+1}-1}{2.(3^{n+1})} = \frac{3^3-1}{2.3^3} = \frac{13}{27}$.

6. (c)

$(1+100)^{100} = 1 + 100.100 + \frac{100.99}{1.2}(100)^2 + \dots$

$\Rightarrow 101^{100} - 1 = 100.100\left[1 + \frac{100.99}{1.2} + \frac{100.99.98}{3.2.1}100 + \dots\right]$

From above it is clear that, $101^{100} - 1$ is divisible by $(100)^2 = 10000$

7. (d)

General term in the expansion is ${}^9C_r\left(x^2\right)^r\left(\frac{-1}{x}\right)^{9-r}$

$= {}^9C_r x^{3r-9}(-1)^{9-r}$

For constant term, put r= 3

8. (b)

Coefficient will be ${}^{2n}C_n = \frac{2n!}{(n!)^2}$

$= \frac{1.3.5\cdots(2n-1).\ 2.4.6.8\cdots 2n}{(n!)^2} = \frac{1.3.5\cdots(2N-1)}{N!}\cdot 2^n$

9. (b)

Numerically greatest term in the expansion of $(8 - 5x)^{18}$

= Numerically greatest term in the expansion of $(8 + 5x)^{18}$

Now, $\frac{(18+1)\times 5x}{8+5x} = \frac{19\times 5\times 2/5}{8+5\times 2/5} = \frac{38}{10} = 3.8$

$\therefore$ Greatest term is $t_{3+1} = {}^{18}C_3\ 8^3 \times \left(5\times\frac{2}{5}\right)^{15}$

$= \frac{18\times 17\times 16}{3\times 2}\times 2^{9+15} = 816 \times 2^{24} = 1632 \times 2^{23}$

$\therefore$ (B) is correct.

10. (a)

$I + f + F = (7 + 2\sqrt{5})^{2n+1} + (7 - 2\sqrt{5})^{2n+1}$

$= 2\left[{}^{2n+1}C_0 7^{2n+1} + {}^{2n+1}C_2 7^{2n-1}(2\sqrt{5})^2 + \dots\right]$

= Even integer

F = -k (because F < 0)

So $I + f - k$ = even integer [$\because f - k \Rightarrow 0$]

So $I \Rightarrow$ even integer

11. (b)

We have,

$\{(1 + x)(1 + y)(x + y)\}^n$

$= (C_0 + C_1x + C_2x^2 + ... + C_nx^n) \times (C_0y^n + C_1y^{n-1} + ... + C_{n-1}y + C_n) \times (C_0x^n + C_1x^{n-1}y + ... + C_{n-1}xy^{n-1} + C_ny^n)$

Clearly, (Coefficient of $x^n y^n$ on RHS)

$$= C_0^3 + C_1^3 + C_2^3 + ... + C_n^3 = \sum_{r=0}^{n} C_r^3$$

Hence (B) is correct answer.

12. (b)

$T_{r+1} = {}^{6561}C_r\,(7^{1/3})^{6561-r}(11^{1/9})^r$

$= {}^{6561}C_r\, 7^{2187} . 7^{-r/3} . 11^{r/9}$

$0 \le r \le 6561$

For rational terms r is multiple of 9 0, 9, 81, 27,……6561

$T_n = 6561 = a + (n - 1)d = 0 + (n - 1)9$

$$n = 1 + \frac{6561}{9} = 1 + 729 = 730.$$

13. (c)

We have $I + f = (5 + 2\sqrt{6})^n$

Let $G = (5 - 2\sqrt{6})^n$

$0 < 5 - 2\sqrt{6} < 1 \Rightarrow 0 < (5 - 2\sqrt{6})^n < 1 \Rightarrow 0 < G < 1$

$I + f + G = (5 + 2\sqrt{6})^n + (5 - 2\sqrt{6})^n$

$= 2\,[{}^nC_0\, 5^n + {}^nC_2\, 5^{n-2}(2\sqrt{6})^2 + ...] \in Z$

$\Rightarrow f + G \in Z \qquad (\because I \in Z)$

Now $0 \le f < 1$, $0 < G < 1 \Rightarrow 0 < f + G < 2$

$\Rightarrow f + G = 1$

Now $I = (I + f) - f = (5 + 2\sqrt{6})^n - f$

$$= \frac{1}{(5 - 2\sqrt{6})^n} - f = \frac{1}{G} - f$$

$$= \frac{1}{1 - f} - f \quad (\because f + G = 1)$$

14. (b)

$${}^{47}C_4 + \sum_{r=1}^{5} {}^{(52-r)}C_3$$

$$\Rightarrow \underbrace{{}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3}$$

$${}^{48}C_4$$

$$\Rightarrow \underbrace{{}^{48}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3}$$

$${}^{49}C_4$$

Similarly proceeding we get ${}^{52}C_4$.

15. (c)

$$\frac{{}^nC_6}{{}^{n-3}C_3} = \frac{33}{4}$$

$$\Rightarrow \frac{n!/6!(n-6)!}{(n-3)!/3!(n-6)!} = \frac{33}{4}$$

$$\Rightarrow \frac{n(n-1)(n-2)}{6.5.4} = \frac{33}{4}$$

$\Rightarrow n(n - 1)(n - 2) = 30 \times 33$

$\Rightarrow n(n - 1)(n - 2) = 11(11 - 1)(11 - 2) \Rightarrow n = 11$

16. (a)

$$\int_0^2 (1+x)^{10}dx = \int_0^2 (C_0 + C_1x + C_2x^2 + ... + C_{10}x^{10})dx$$

$$= \frac{(1+2)^{11} - 1}{11} = \frac{C_0.(2)}{1} + \frac{C_1.2^2}{2} + \frac{C_2.2^3}{3}$$

$$+ .... + \frac{C_{10}.2^{11}}{11} = \frac{3^{11} - 1}{11}$$

17. (d)

$$r = \frac{n\alpha - m}{\alpha + \beta}$$

$$\Rightarrow r = \frac{6 \times 5/2 - 3}{5/2 + 3/2} = \frac{12}{4} = 3$$

$\therefore T_4 = {}^6C_3\,(3)^3 = 540$

18. (c)

$S = {}^nC_0 + 3.{}^nC_1 + 5.{}^nC_2 + ...+ (2n + 1){}^nC_n$

$+ S = (2n+1).{}^nC_0 + (2n-1).{}^nC_1 + ... + 1.{}^nC_n$

$2S = (2n+2)[{}^nC_0 + {}^nC_1 + ... + {}^nC_n]$

$S = (n + 1).(2)^n$

19. (d)

$$r = \frac{n\alpha - m}{\alpha + \beta}$$

$$\Rightarrow r = \frac{6 \times 5/2 - 3}{5/2 + 3/2} = \frac{12}{4} = 3$$

$\therefore T_4 = {}^6C_3\,(3)^3 = 540$

20. (d)

For sum of coefficients, Put x = 1 ;

$(1 + 2 + 3 + ...n)^2$

$$= \left(\frac{n(n+1)}{2}\right)^2$$

$= \Sigma n^3$

21. (a)

$$aC_0 + \frac{a^2}{2}C_1 + \frac{a^3}{3}C_2 + ... + \frac{a^{n+1}}{n+1}C_n$$

$$= \frac{(1+a)^{n+1} - 1}{n+1}$$

Put a = 2 and n = 10

22. (c)

Conversely $(x+y)^n = {}^nC_0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + ... + {}^nC_nx^0y^n$

$(x+2a)^5 =$

${}^5C_0x^5 + {}^5C_1x^4(2a)^1 + {}^5C_2x^3(2a)^2 + {}^5C_3x^2(2a)^3 + {}^5C_4x^1(2a)^4 + {}^5C_5x^0(2a)^5$

$= x^5 + 10x^4a + 40x^3a^2 + 80x^2a^3 + 80xa^4 + 32a^5$.

23. (c)

$(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a^1 + {}^nC_2x^{n-2}a^2 + ... + {}^nC_nx^{n-n}.a^n =$

$(x^n + {}^nC_2x^{n-2}a^2 + ..) + ({}^nC_1x^{n-1}a^1 + {}^nC_3x^{n-3}a^3 + ...) = A + B$

.....(i)

Similarly, $(x-a)^n = A - B$ .....(ii)

From (i) and (ii), we get $4AB = (x+a)^{2n} - (x-a)^{2n}$

Trick: Put $n=1$ in $(x+a)^n$. Then, $x+a = A+B$.

Comparing both sides $A = x, B = a$.

Option (c) L.H.S. $4AB = 4xa$, R.H.S. $(x+a)^2 - (x-a)^2 = 4ax$

. *i.e.,* L.H.S. = R.H.S

24. (c)

$\because \frac{T_2}{T_3} = \frac{2}{n-2+1}.\frac{b}{a} = \frac{2}{n-1}\left(\frac{b}{a}\right)$ and

$\frac{T_3}{T_4} = \frac{3}{n+3-3+1}.\left(\frac{b}{a}\right) = \frac{3}{n+1}\left(\frac{b}{a}\right)$

$\because \frac{T_2}{T_3} = \frac{T_3}{T_4}$ (given) ;

$\therefore \frac{2}{n-1}\left(\frac{b}{a}\right) = \frac{3}{n+1}\left(\frac{b}{a}\right) \Rightarrow 2n+2 = 3n-3 \Rightarrow n = 5$

25. (b)

For coefficient of x^{-7},

$(11-r)(1) + (-2).r = -7 \Rightarrow 11 - r - 2r = -7 \Rightarrow r = 6$;

$T_7 = {}^{11}C_6(a)^5\left(-\frac{1}{b}\right)^6 = \frac{462a^5}{b^6}$

26. (b)

$A =$ coefficient of x^n in $(1+x)^{2n} = {}^{2n}C_n = \frac{(2n)!}{n!n!} =$

$\frac{2.(2n-1)!}{(n-1)!n!}$ .....(i)

$B =$ coefficient of x^n in $(1+x)^{2n-1} = {}^{2n-1}C_n = \frac{(2n-1)!}{n!(n-1)!}$ .....(ii)

By (i) and (ii) we get, $A = 2B$

27. (a)

Since $f = R - [R]$, $R = f + [R]$

$[5\sqrt{5}+11]^{2n+1} = f + [R]$, where $[R]$ is integer

Now let $f' = [5\sqrt{5}-11]^{2n+1}, 0 < f' < 1$

$f + [R] - f' = [5\sqrt{5}+11]^{2n+1} - [5\sqrt{5}-11]^{2n+1} =$

$2\left[{}^{2n+1}C_1(5\sqrt{5})^{2n}(11)^1 + {}^{2n+1}C_3(5\sqrt{5})^{2n-2}(11)^3 + ...\right]$

$= 2.$(Integer) $= 2K$ $(K \in N)$ = Even integer

Hence $f - f'$ = even integer - $[R]$, but $-1 < f - f' < 1$.

Therefore, $f - f' = 0$ $\therefore f = f'$

Hence R.f = $R.f' = (5\sqrt{5}+11)^{2n+1}(5\sqrt{5}-11)^{2n+1} = 4^{2n+1}$.

28. (b)

Coefficient of

$$x^r = \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)...\left(-\frac{1}{2}-r+1\right)}{r!}(-2)^r$$

$$= \frac{1.3.5...(2r-1).(-1)^r.(-1)^r.2^r}{2^r\, r!} = \frac{1.3.5...(2r-1)}{r!} = \frac{(2r)!}{r!r!2^r}$$

29. (c)

Coefficient of x^{25} in $(1+x+x^2+x^3+x^4)^{-1}$

= Coefficient of x^{25} in $\left[\frac{1(1-x^5)}{1-x}\right]^{-1}$

= Coefficient of x^{25} in $(1-x^5)^{-1}.(1-x)$

= Coefficient of x^{25} in $[\,(1-x^5)^{-1} - x(1-x^5)^{-1}\,]$

$= [1 + (x^5)^1 + (x^5)^2 + ...] - x[1 + (x^5)^1 + (x^5)^2] + ...] =$

Coefficient of x^{25} in $[1 + x^5 + x^{10} + x^{15} + ...]$

- Coefficient of x^{24} in $[1 + x^5 + x^{10} + x^{15} + ...]$ $= 1 - 0 = 1$.

30. (c)

We have $(1 + 3x + 6x^2 + 10x^3 + ...)^{1/3}$

$= [(1-x)^{-3}]^{1/3}$; $[\because (1-x)^{-3} = 1 + 3x + 6x^2 + ...\infty]$

$\Rightarrow (1-x)^{-1} = 1 + x + x^2 + ...\infty$

31. (a)

General term = $\frac{10!}{\alpha!\beta!\gamma!} 2^{\alpha/2} 3^{\beta/3} 5^{\gamma/6}$

α = 0, 2, 4, 6,8, 10

β = 0, 3, 6 γ = 0, 6

Hence possible sets = (4, 6, 0), (4, 0, 6), (10, 0, 0)

Hence there are 3 rational terms

Sum = $\frac{10!}{4!\,6!} 2^2 3^2 + \frac{10!}{4!\,6!} 2^2 5 + \frac{10!}{10!} 2^5 = 12632$

32. (b)

General term of $\left(x - \frac{3}{x}\right)^{11}$

$T_{r+1} = {}^{11}C_r\, x^{11-r}\left(-\frac{3}{x}\right)^r = {}^{11}C_r\, x^{11-2r}(-3)^r$

$11 - 2r \neq 0$, -2 so $11 - 2r = -1$ is only possible case $r = 6$

$\Rightarrow (-3)^6\, {}^{11}C_6$

33. (b)

$\sum\limits_{0\le i<j<k<\ell\le n}\sum\sum\sum\sum n = n \sum\limits_{0\le i<j<k<\ell\le n}\sum\sum\sum\sum$

$\Downarrow$

Selection of 4 terms (from n + 1 terms)

$= n.\, {}^{n+1}C_4$

34. (c)

As $b_1, b_2, ...b_n$ are n^{th} roots of unity

$\Rightarrow b_1, b_2, ...b_n$ are in G.P.

where $b_1 = 1$, $b_2 = e^{i2\pi/n}$, $b_3 = e^{i4\pi/n}$....

$b_n = e^{\frac{2i(n-1)\pi}{n}}$

Clearly b_2 is common ratio and $b_n = b_1(b_2)^{n-1}$

given expression = $^nC_1 . b_1 + {^nC_2} . b_2 + … + {^nC_n} . b_n$

$= b_1 (^nC_1 + {^nC_2}b_2 + {^nC_2}\, b_2^2 + …. + {^nC_n}\, b_2^{n-1})$

$= \frac{b_1}{b_2}((1+b_2)^n - 1)$

35. (c)

Number of terms are infinite

36. (c)

The co-efficients of x^{r-1}, x^r, x^{r+1} in the expansion of $(1 + x)^n$ are respectively $^nC_{r-1}$, ^nC_r, $^nC_{r+1}$, which are in A.P.

$\therefore 2 \cdot {^nC_r} = {^nC_{r-1}} + {^nC_{r+1}}$

or $2 \bullet \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!} + \frac{n!}{(r+1)!(n-r-1)!}$

or $\frac{2}{r(r-1)!(n-r)(n-r-1)!}$

$= \frac{1}{(r-1)!(n-r+1)(n-r)(n-r-1)!} + \frac{1}{(r+1)r(r-1)!(n-r-1)!}$

or $\frac{2}{r(n-r)} = \frac{1}{(n-r+1)(n-r)} + \frac{1}{r(r+1)}$

or $2(n-r+1)(r+1) = r(r+1) + (n-r+1)(n-r)$

or $n^2 - n(4r+1) + 4r^2 - 2 = 0.$

37. (a)

$x - x^2 + x[x] = x - x(x - [x]) = x - x\{x\} = x(1 - \{x\})$

$x = (2+\sqrt{3})^n = {^nC_0}\, 2^n + {^nC_1}\, 2^{n-1}\sqrt{3} + {^nC_2}\, 2^{n-2}(\sqrt{3})^2 + …$

Let $x_1 = (2-\sqrt{3})^n = {^nC_0}\, 2^n - {^nC_1}\, 2^{n-1}\sqrt{3} + {^nC_2}\, 2^{n-2}(\sqrt{3})^2 + …$

$x + x_1 = 2(^nC_0\, 2^n + {^nC_2}\, 2^{n-2} \cdot (\sqrt{3})^2 + …)$

= Even integer.

Clearly $x_1 \in (0, 1)\ \forall\ n \in N$.

$\Rightarrow [x] + \{x\} + x_1$ = Even integer

$\Rightarrow \{x\} + x_1$ = Integer

$\{x\} \in (0, 1)$, $x_1 \in (0, 1)$

$\Rightarrow \{x\} + x_1 \in (0, 2)$

$\Rightarrow \{x\} + x_1 = 1$

$\Rightarrow x_1 = 1 - \{x\}$

$\Rightarrow x(1 - \{x\}) = x \cdot x_1 = (2+\sqrt{3})^n(2-\sqrt{3})^n = 1.$

38. (c)

We have,

$(1-x)^n = {^nC_0} - {^nC_1}x + {^nC_2}x^2 - {^nC_3}x^3 + …. + (-1)^n\, {^nC_n}x^n$

and,

$(1 + x + x^2 + ….. + x^p)^n = (a_0 + a_1x + a_2x^2 + ….. + a_{np}x^{np})$

$\therefore (1-x)^n(1 + x + x^2 + ….. + x^p)^n$

$= (^nC_0 - {^nC_1}x + {^nC_2}x^2 - {^nC_3}x^3 + …. + (-1)^n\, {^nC_n}x^n)$

$\times (a_0 + a_1x + a_2x^2 + ….. + a_{np}x^{np})$ …(1)

Now, Coefficient of x^r on R.H.S. of (1)

$= a_r\, {^nC_0} - a_{r-1}\, {^nC_1} + a_{r-2}\, {^nC_2} - …. + a_0(-1)^r\, {^nC_r}$

LHS of (1) $= (1-x)^n(1 + x + x^2 + …. + x^p)$

$= (1-x)^n\left\{\frac{1-x^{p+1}}{1-x}\right\}^n$

$= (1-x^{p+1})^n$

Since r is not a multiple of (p + 1). Therefore, the expansion of $(1-x^{p+1})^n$ does not contain x^r in any term.

$\therefore$ Since r is not a multiple of (p + 1). Therefore, the expansion of $(1-x^{p+1})^n$ does not contain x^r in any term.

$\therefore$ Coefficient of x^r on LHS of (1) = 0

Hence,

$^nC_0\, a_r - {^nC_1}a_{r-1} + {^nC_2}\, a_{r-2} - ….. + (-1)^r\, {^nC_r}\, a_0 = 0.$

If r is not a multiple of (p + 1)

Hence (C) is correct answer.

39. (a)

We have, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)(C_r + C_s)$

$= \sum_{r=0}^{n}\sum_{s=0}^{n}(rC_r + rC_s + sC_r + sC_s)$

$= \sum_{r=0}^{n}\left[\sum_{s=0}^{n} rC_r + r\sum_{s=0}^{n} C_s + C_r\sum_{s=0}^{n} s + \sum_{s=0}^{n} sC_s\right]$

$= \sum_{r=0}^{n}\left[(n+1)r.C_r + r2^n + \frac{n(n+1)}{2}C_r + n.2^{n-1}\right]$

$= (n+1)(n \cdot 2^{n-1}) + 2^n\ \frac{n(n+1)}{2} + \frac{n(n+1)}{2}\, 2^n + n\, 2^{n-1}(n+1)$

$= n(n+1)2^n + n(n+1)\, 2^n$

$= 2n(n+1)2^n$ …(1)

Also, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)(C_r + C_s)$

$= \sum_{r=0}^{n} 4r\, C_r + 2\sum\sum_{0\le r<s\le n}(r+s)(C_r + C_s)$

$\therefore 2n(n+1)\, 2^n = 4n \cdot 2^{n-1} + 2\sum\sum_{0\le r<s\le n}(r+s)(C_r + C_s)$

$\Rightarrow \sum\sum_{0\le r<s\le n}(r+s)(C_r + C_s) = n^2 \cdot 2^n.$

Hence (A) is correct answer.

40. (b)

(r + 1)th term in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$ is given by

$T_{r+1} = {^{n-3}C_r}(x)^{n-3-r}\left(\frac{1}{x^2}\right)^r = {^{n-3}C_r}x^{n-3-3r}$

As x^{2k} occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$,

we must have n -3 - 3r = 2k for some non-negative interger r.

$\Rightarrow 3(1 + r) = n - 2k$

$\Rightarrow n - 2k$ is a multiple of 3

41. (b)

$S = {}^{39}C_{39} + {}^{39}C_{38} + \dots\dots + {}^{39}C_{20}$

${}^{39}C_0 + {}^{39}C_1 + \dots + {}^{39}C_{19}) + ({}^{39}C_{20} + \dots + {}^{39}C_{39}) = 2^{39}$

$S + S = 2^{39}$

$\Rightarrow S = 2^{38}$

42. (b)

$T_5 + T_6 = 0$

$${}^nC_4\, a^{n-4}(-b)^4 + {}^nC_5\, a^{n-5}(-b)^5 = 0 \because \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$$

43. (a)

Let G = $(5\sqrt{5}-11)^{2n+1}$, then $0 < G < 1$,

Now, R - G = $(5\sqrt{5}+11)^{2n+1}$ - $(5\sqrt{5}-11)^{2n+1}$

$$= 2\left[{}^{2n+1}C_1\left(5\sqrt{5}\right)^{2n}(11) + \dots\right]$$

= an even integer

$\Rightarrow F - G = 0$

$\Rightarrow F = G$

$RF = RG = \left(5\sqrt{5}+11\right)^{2n+1}\left(5\sqrt{5}-11\right)^{2n+1}$

$= 4^{2n+1}$

44. (b)

$5^2 \equiv 12$ (divided by 13)

$5^9 \equiv 1$ (divided by 13)

$5^{96} \equiv 1$ (divided by 13)

$5^{99} \equiv 8$ (divided by 13)

45. (a)

Putting x = 1,

$1 = a_0 + a_1 + a_2 + a_3 + \dots\dots\dots a_{2n}$ ---- (i)

Putting x = -1

$3^n = a_0 - a_1 + a_2 - a_3 + \dots\dots\dots a_{2n}$ ---- (ii)

Adding (i) & (ii);

$$\frac{3^n+1}{2} = a_0 + a_2 + a_4 + \dots\dots a_{2n}$$

46. (b)

$$\left(4^{1/3} + \frac{1}{6^{1/4}}\right)^{20} \quad T_{r+1} = {}^{20}C_r \,.\, 4^{20 - r/3}\left(\frac{1}{3}\right)^{r/4}$$

The number of rational term = 2

when r = 12, 20

middle term $T_{11} = {}^{20}C_{10}\,(4^{1/3})^{10}\left(\frac{1}{6}\right)^{10/4}$

is irrational.

47. (c)

Coefficient of x^5 in the expansion of $(x^2 - x - 2)^5$ is

$$\sum \frac{5!}{n_1!.n_2!n_3!}(1)^{n_1}(-1)^{n_2}(-2)^{n_3}.$$

where $n_1 + n_2 + n_3 = 5$ and $n_2 + 2n_3 = 5$. The possible value of n_1, n_2 and n_3 are shown in margin

n_1	n_2	n_3
1	3	1
2	1	2
0	5	0

$\therefore$ The coefficient of x^5

$$= \frac{5!}{1!3!1!}(1)^1(-1)^3(-2)^1 + \frac{5!}{2!1!2!}(1)^2(-1)^1(-2)^2 +$$

$$\frac{5!}{0!5!0!}(1)^0(-1)^5(-2)^0 = 40 - 120 - 1 = -81$$

48. (d)

(1 + x) (1 + x + x2) (1+ x + x2 + x3) ....

(1 + x + x2 +....xn)

= a0 + a1x + a2x2 +.....+amxm

substituting x = 1

$$1 \times (2)(3)(4)\dots\dots(n) = \sum_{r=0}^{m} a_r = n!$$

49. (b)

$3^{37} = 3^{4\cdot 9}\cdot 3 = 3\cdot(81)^9 = 3(80+1)^9$

$= 3\,({}^9C_0\,(80)^9 + {}^9C_1\,(80)^8 + \dots\dots + {}^9C_9)$

Then remainder is 3

$\Rightarrow \alpha = 3$

and $4^{101} = 4^{100}\cdot 4$

4^{100} is divisible by 101

Remainder is 4, $\alpha^2\beta = 3^2\cdot 4 = 36$

$\beta\alpha^2 = 3\cdot 4^2 = 48$

equation $x^2 - 84x + 1728 = 0$.

50. (c)

$E = \frac{1-x}{1-x^2}$;

$E = (1-x)\left(1 - x^{2^{m+1}}\right)^{-1}$

$E = (1-x)\ \{1 + x^{2^{m+1}} + (x^{2^{m+1}})\dots\dots\}$

Coeffi. $x^{2^{m+1}} = 1$ ....(i)