1. (a)

Re-writing the given equation,

$$2xy^2dx + ye^x dx = e^x dy \Rightarrow 2x\,dx + \frac{ye^x dx - e^x dy}{y^2} = 0$$

$$\Rightarrow d(x^2) + d\left(\frac{e^x}{y}\right) = 0$$

Integrating, $x^2 + \frac{e^x}{y} = c$

$\therefore\ yx^2 + e^x = cy$

2. (a)

Comparing given equation with $Mdx + Ndy = 0$,

We get, $M = x^2 - 4xy - 2y^2$, $N = y^2 - 4xy - 2x^2$

$$\frac{\partial M}{\partial y} = -4x - 4y$$

$$\frac{\partial N}{\partial x} = -4y - 4x$$

$$\therefore\ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

So the given differential equation is exact.

Integrating *m w.r.t. x,* treating *y* as constant,

$\int Mdx = \int (x^2 - 4xy - 2y^2)dx = \frac{x^3}{3} - 2x^2y - 2y^2x$ Integrating *N w.r.t. y,* treating *x* as constant,

$$\int Ndy = \int (y^2 - 4xy - 2x^2)dy = \frac{y^3}{3} - 2xy^2 - 2x^2y = \frac{y^3}{3};$$

(omitting $-2xy^2 - 2x^2y$ which already occur in $\int Mdx$)

$\therefore$ Solution of the given equation is $\frac{x^3}{3} - 2x^2y - 2xy^2 + \frac{y^3}{3} = \lambda$

$$\Rightarrow x^3 + y^3 - 6xy(x+y) = 3\lambda$$

$\therefore\ x^3 + y^3 - 6xy(x+y) = c \qquad (3\lambda = c)$

3. (c)

(a), (b), (d) do not fulfill the criteria of a linear differential equation but (c) do.

$\frac{dy}{dx} + \frac{y}{x} = \log x$ is a linear differential equation.

4. (b)

We have $(x - e^{\tan^{-1}y})\frac{dy}{dx} = -(1+y^2) \Rightarrow \frac{dx}{dy} = -\left(\frac{x - e^{\tan^{-1}y}}{1+y^2}\right)$

$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$ ....(i)

This is a linear differential equation of the form

$$\frac{dx}{dy} + R(y).x = S(y)$$

$$R = \frac{1}{1+y^2}, \quad S = \frac{e^{\tan^{-1}y}}{1+y^2}$$

Integrating factor $= e^{\int Rdy} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}y}$

Multiplying (i) by I.F. and integrating,

$$xe^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2}\cdot e^{\tan^{-1}y}dy = \int \frac{(e^{\tan^{-1}y})^2dy}{1+y^2} = \frac{(e^{\tan^{-1}y})^2}{2} + \frac{k}{2}$$

$\therefore \phi 0 \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

5. (b)

We have $\frac{dx}{x+1} = dt$

Integrating, $\int_0^{99} \frac{dx}{x+1} = \int_0^t dt \Rightarrow [\ln(x+1)]_0^{99} = t$

$\therefore\ t = \ln 100 = \log_e(10)^2 = 2\log_e 10$

6. (c)

We have $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \Rightarrow v + x\frac{dv}{dx} = v - \cos^2 v$

$\Rightarrow x\frac{dv}{dx} = -\cos^2 v \Rightarrow \sec^2 v dv = -\frac{dx}{x} \Rightarrow \tan v = -\ln x + c$

$\Rightarrow \tan(y/x) = -\ln x + c$

For $x = 1$, $y = \pi/4$

$\Rightarrow \tan \pi/4 = -\ln 1 + c \Rightarrow 1 = 0 + c$

$\therefore c = 1$

$\therefore\ \tan(y/x) = 1 - \ln x$

$$\Rightarrow y/x = \tan^{-1}(1 - \ln x) = \tan^{-1}(\ln e - \ln x) = \tan^{-1}\left[\ln\left(\frac{e}{x}\right)\right]$$

$$\therefore\ y = x\tan^{-1}\left[\ln\left(\frac{e}{x}\right)\right]$$

7. (d)

Tangent at $P(x, y)$ to the curve $y = f(x)$ may be expressed as

$$Y - y = \frac{dy}{dx}(X - x)$$

$$\therefore\ Q = \left(x - y\frac{dx}{dy}, 0\right)$$

As per question, $OQ \propto y$

$$\Rightarrow x - y\frac{dx}{dy} \propto y \Rightarrow x - y\frac{dx}{dy} = by \Rightarrow \frac{x}{y} - \frac{dx}{dy} = b$$

$$\therefore\ \frac{dx}{dy} = \frac{x}{y} - b$$

Let $\frac{x}{y} = v \Rightarrow x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy} \Rightarrow \frac{x}{y} - b = v + y\frac{dv}{dy}$

$$\Rightarrow v - b = v + y\frac{dv}{dy} \Rightarrow -b = y\frac{dv}{dy} \Rightarrow -b\frac{dy}{y} = dv$$

Integrating, $\int dv = -b\int \frac{dy}{y} \Rightarrow v = -b\ln y + a$

$\Rightarrow \frac{x}{y} = a - b\ln y$ (*a*, an arbitrary constant)

$\therefore\ x = y(a - b\ln y)$

8. (d)

We have $\frac{d^2y}{dx^2}=\frac{\ln x}{x^2} \Rightarrow d\left(\frac{dy}{dx}\right)=\frac{\ln x}{x^2}dx$

Integrating,

$$\frac{dy}{dx}=\int \ln x\, d\left(-\frac{1}{x}\right)=-\frac{\ln x}{x}+\int \frac{1}{x^2}dx=-\frac{\ln x}{x}-\frac{1}{x}+c$$

$$\Rightarrow \frac{dy}{dx}=-\frac{1+\ln x}{x}+c$$

When $x=1$, $\frac{dy}{dx}=-1$

$\therefore -1=-1+c \Rightarrow c=0$

$$\therefore \frac{dy}{dx}=-\frac{1+\ln x}{x} \Rightarrow dy=-\frac{1+\ln x}{x}dx$$

$$\Rightarrow -\int dy=+\int \frac{dx}{x}+\int \ln x.\frac{1}{x}dx \Rightarrow -y=\ln x+\frac{1}{2}(\ln x)^2+\lambda$$

$y=0$ when $x=1$

$$\therefore\ 0=0+0^2+\lambda \Rightarrow \lambda=0 \Rightarrow -y=\ln x+\frac{1}{2}(\ln x)^2$$

$$\therefore\ y=-\frac{1}{2}(\ln x)^2-\ln x$$

9. (d)

For $\phi(x)=y,\ y'=1+y^2 \Rightarrow \frac{dy}{dx}=1+y^2 \Rightarrow \int \frac{dy}{1+y^2}=\int dx$

$\Rightarrow \tan^{-1}y=x+c$

$\therefore y=\tan(x+c)$

i.e., $\phi(x)=\tan(x+c)$

As $y(0)=0$, $0=\tan c$ and $y(\pi)=0 \Rightarrow 0=\tan(\pi+c)=\tan c$

$\therefore c=0$

$\therefore \phi(x)=y=\tan x$.

But $\tan x$ is not continuous in $(0, \pi)$

Since $\tan\frac{\pi}{2}$ is not defined.

Hence there exists not a function satisfying the given condition.

10. (a)

The given differential equation can be re–written as

$\left[\left(\frac{dy}{dx}\right)^2-1\right]^2=\left(\frac{d^2y}{dx^2}\right)^5$. Hence its order is 2 and degree is 5.

11. (b)

$$\frac{dy}{dx}=\left(\frac{dx}{dy}\right)^{-1}$$

$$\frac{d^2y}{dx^2}=-\left(\frac{dx}{dy}\right)^{-2}\frac{d}{dx}\left(\frac{dx}{dy}\right)=-\left(\frac{dy}{dx}\right)^2\left(\frac{d^2x}{dy^2}\right)\frac{dy}{dx}$$

$$=\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\frac{d^2x}{dy^2}=0$$

12. (a)

The equation of family is $y=Ax^2+Bx+C$, where A, B, C are arbitrary constants. Hence, $\frac{d^3y}{dx^3}=0$

13. (b)

Differentiating the given relation we have,

$$\frac{dy}{dx}=ckx^{k-1} \Rightarrow c=\frac{1}{k}x^{1-k}\frac{dy}{dx}.$$

Putting this value in the given equation we have

$$y=\frac{1}{k}x^{1-k}\frac{dy}{dx}x^k=\frac{1}{k}x\frac{dy}{dx}.$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we get

$$y=-\frac{1}{k}x\frac{dx}{dy} \Rightarrow ky\,dy+x\,dx=0 \Rightarrow ky^2+x^2=\text{constant}$$

14. (c)

Dividing the given equation by $z(\log z)^2$, we get

$$\frac{1}{z(\log z)^2}\frac{dz}{dx}+\frac{1}{\log z}\frac{1}{x}=\frac{1}{x^2}. \quad \ldots (1)$$

Writing $\frac{1}{\log z}=u$, we have $\frac{du}{dx}=-(\log z)^{-2}\frac{1}{z}\frac{dz}{dx}$.

So (1) can be written as

$$-\frac{du}{dx}+\frac{u}{x}=\frac{1}{x^2} \Rightarrow \frac{du}{dx}-\frac{u}{x}=\frac{-1}{x^2}$$

which is the required form with $P(x)=\frac{-1}{x}$ and $Q(x)=\frac{-1}{x^2}$

15. (b)

The given equation can be written as

$$x\,dx+y\,dy+\frac{ydx-xdy}{\sqrt{xy}(x+y)}=0$$

$$\text{or } \frac{1}{2}d(x^2+y^2)=\frac{xdy-ydx}{x^2\sqrt{\frac{y}{x}}\left(1+\frac{y}{x}\right)}=\frac{2}{\left(1+\frac{y}{x}\right)}d\left(\sqrt{\frac{y}{x}}\right)$$

$$\Rightarrow x^2+y^2=4\tan^{-1}\sqrt{\frac{y}{x}}+c$$

16. (b)

The general equation of all non-horizontal lines in xy-plane is $ax+by=1$, where $a\neq 0$.

$$\Rightarrow a\frac{dx}{dy}+b=0 \qquad \text{[Diff. w.r.to y]}$$

$\Rightarrow a\frac{d^2x}{dy^2}=0$ [Diff. w.r.to y]

$\Rightarrow \frac{d^2x}{dy^2}=0$

Hence, the required differential equations is $\frac{d^2x}{dy^2}=0$

17. (d)

We have, $y=(C_1+C_2)\sin(x+C_3)-C_4e^{x+C_5}$

$\Rightarrow y=C_6\sin(x+C_3)-C_4e^{C_5}.e^x$, where $C_6=C_1+C_2$

$\Rightarrow y=C_6\sin(x+C_3)-C_7e^x$, where $C_4e^{C_5}=C_7$ Clearly, the above relation contains three arbitrary constants. So, the order of the differential equation satisfying it is 3.

18. (c)

Clearly, y = 2x - 4 satisfies the given differential equation.

19. (c)

We have, $y^2=2c(x+\sqrt{c})$

$\Rightarrow 2yy_1=2c$

$\Rightarrow yy_1=c$

$\Rightarrow c=\frac{y}{y_1}$

Eliminating c from (i) and (ii) , we get.

$y^2=\frac{2y}{y_1}\left(x+\frac{\sqrt{y}}{y_1}\right)$

$\Rightarrow yy_1-2x=2\frac{\sqrt{y}}{y_1}$

$\Rightarrow (yy_1-2x)^2y_1=4y$

Clearly, it is a differential equation of order one and degree 3.

20. (b)

We have, $f''(x)=g''(x)$. On integration,

we get $f'(x)=g'(x)+C$ ……. (i)

Putting x=1, we get

$f'(1)=g(1)+C$

$\Rightarrow 4=2+C$

$\Rightarrow C=2$

$\therefore f'(x)=g'(x)+2$

Integrating w.r.t.x, we get f(x) =g(x) +2x+c_1 ……. (ii)

Putting x = 2, we get.

$f(2)=g(2)+4+c_1$

$\Rightarrow 9=3+4+c_1$

$\Rightarrow c_1=2$

$\therefore f(x)=g(x)+2x+2$

Putting x=4, we get f(4) -g(4) =10.

21. (c)

The given differential equation is $y^2(x^2+1)=2xy_1$

$\Rightarrow \frac{y_2}{y_1}=\frac{2x}{x^2+1}$

Integrating both sides, we get.

$\log y_1=\log(x^2+1)+\log C \Rightarrow y_1=C(x^2+1)$ ……. (i)

It is given that $y_1=3$ at $x=0$

Putting x=0, $y_1=3$ in (i) , we get $C=3$

Substituting the value of C in (i), we obtain $y_1=3(x^2+1)$

Integrating both sides w.r.t to x, we get

$y=x^3+3x+C_2$

This passes through the point (0,1). Therefore, C_2=1

Hence, the required equation of the curve is

$y=x^3+3x+1$

22. (d)

Solving the homogeneous equation, by using y = vx, we find the solution x + y = c(x^2+y^2) y(-1) = 1 ⇒ x + y = 0 which is a straight line.

23. (b)

$y\frac{dy}{dx}=(k-x)$

y dy = (k - x) dx

Integrate both side

$\int y\,dy=\int (k-x)\,dx$

$\frac{y^2}{2}=-\frac{(k-x)^2}{2}+c$

$\frac{(k-x)^2}{2}+\frac{y^2}{2}=c$ family of circle center is (k, 0)

24. (a)

We have $\frac{y_3}{y_2}=8 \Rightarrow \ell n\, y_2=8x+C_1$

Putting x = 0, we have $C_1=\log y_2(0)=\log 1=0$

$\therefore \log y_2=8x$ or $y_2=e^{8x}$

i.e. $y_1=\frac{e^{8x}}{8}+C_2$

Again, putting x = 0 , we have $C_2=-\frac{1}{8}$

So, $y_1=\frac{1}{8}(e^{8x}-1)\Rightarrow y=\frac{1}{8}\left(\frac{e^{8x}}{8}-x\right)+C_3$

Putting x = 0, we have $C_3=\frac{1}{8}-\frac{1}{64}=\frac{7}{64}$

Thus $y = \frac{1}{8}\left(\frac{e^{8x}}{8} - x + \frac{7}{8}\right)$

25. (a)

Differentiating the equation twice w.r.t. x, we have

$2a^2 x - 2b^2yy' = 0$, $a^2 - b^2(y'^2 + yy'') = 0$

Eliminating a^2 and b^2 we have the differential equation

$\frac{y''}{y'} + \frac{y'}{y} = \frac{1}{x}$

26. (b)

Solving the equations of the asymptotes the centre is x = 1 and y = 0, since $e = \sqrt{2}$ the equation of the family of the hyperbola is

$\frac{(x-1)^2}{a^2} - \frac{(y)^2}{a^2} = 1$

$\Rightarrow 2(x - 1) - 2y\frac{dy}{dx} = 0$

$\Rightarrow (x - 1) = yy'$ is differential equation

27. (a)

The parametric form of the given equation is x = t, $y = t^2$. The equation of any tangent at t is $2xt = y + t^2$. Differentiating, we get $2t = y_1$. Putting this value in the equation of tangent, we have $2x\, y_1/2 = y + (y_1/2)^2 \Rightarrow 4xy_1 = 4y + y_1^2$

The order of this equation is one.

28. (b)

[K is constant of proportionality] Let the curve be y = f(x). The equation of tangent at any point (x, y) is given by Y - y = f'(x) (X -x). So the portion of the axis of x which is cut off between the origin and the tangent at any point is obtained by putting Y = 0. Therefore,

$x - \frac{y}{f'(x)} = Ky \Rightarrow x - y\frac{dx}{dy} = Ky \Rightarrow \frac{dx}{dy} - \frac{x}{y} = -K$

which is a linear equation in x, so its integrating factor is

$e^{-\int (1/y)dy} = y^{-1}$. Therefore, multiplying by y^{-1}, we have

$\frac{d}{dy}(xy^{-1}) = -Ky^{-1} \Rightarrow xy^{-1} = -K \log y + C$

$\Rightarrow x = y(C - K \log y)$

where C is arbitrary constant

29. (d)

put $\frac{x}{y} = t \Rightarrow y = \frac{1}{t}x$ or $\frac{dy}{dx} = \frac{1}{t} + x\left(\frac{-1}{t^2}\right)\frac{dt}{dx}$

30. (c)

Since f(x) and g (x) are solution of given differential equation

$a f''(x) + x^2 f'(x) + f(x) = e^x$ ….(i)

& $ag''(x) + x^2 g'(x) + g(x) = e^x$ …….(ii)

(i) - (ii) given

$a[f''(x) - g''(x)] + x^2[f'(x) - g'(x)]$

$+ [f'(x) - g'(x)] = 0$