1. (a)

$$\left(\frac{dy}{dx}\right)=\left(\frac{dx}{dy}\right)^{-1}$$ for a differentiable coefficient

or $\frac{d^2y}{dx^2}=-1\left(\frac{dx}{dy}\right)^{-2}\frac{d}{dy}\left(\frac{dx}{dy}\right)\frac{dy}{dx}=-\left(\frac{dx}{dy}\right)^{-2}\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)=-$

$\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3$ or $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\frac{d^2x}{dy^2}=0$

2. (b)

We have, $y^2 = 4a(x + a)$... (1)

On differentiating w.r.t. x, we get

$2y\frac{dy}{dx}=4a \Rightarrow a=\frac{y}{2}\frac{dy}{dx}$.

On substituting the value of a in equation (1), we get

$y^2=2y\frac{dy}{dx}\left[x+\frac{y}{2}\frac{dy}{dx}\right]$

$\Rightarrow y=2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2$

$\Rightarrow y\left[1-\left(\frac{dy}{dx}\right)^2\right]=2x\,.\,\frac{dy}{dx}$,

which is the required differential equation. The degree of the differential equation is 2.Hence (B) is the correct answer

3. (c)

We can write $y = A\cos(x + B) - Ce^x$

where $A = c_1 + c_2$, $B = c_3$ and $C = c_4\, e^{c_5}$

$\frac{dy}{dx} = -A\sin(x + B) - Ce^x$

$\Rightarrow \frac{d^2y}{dx^2} = -A\cos(x + B) - Ce^x$

$\Rightarrow \frac{d^2y}{dx^2} + y = -2Ce^x$

$\Rightarrow \frac{d^3y}{dx^3}+\frac{dy}{dx} = -2Ce^x = \frac{d^2y}{dx^2}+y$

$\Rightarrow \frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$

Which is a differential equation of degree 1.

Hence (C) is the correct answer

4. (c)

We have, $y - x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$

$\Rightarrow y\,dx - x\,dy = ay^2\,dx + ady$

$\Rightarrow y(1 - ay)\,dx = (x + a)dy \Rightarrow \frac{dx}{x+a}-\frac{dy}{y(1-ay)}=0$

Integrating, we get

$\log(x + a) - \log y + \log(1 - ay) = \log c$

$\log\frac{(a+x)(1-ay)}{y}=\log c \Rightarrow (x + a)$

$(1 - ay) = cy$.

Since the curve passes through $\left(a,-\frac{1}{a}\right)$,

$2a \times (1 + 1) = -\frac{c}{a} \Rightarrow c = -4a^2$.

So, the equation of curve is

$(x + a)(1 - ay) = -4a^2y$.

Hence (C) is the correct answer

5. (b)

We have, $x\,dy=\left(y+\frac{x f(y/x)}{f'(y/x)}\right)dx$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}+\frac{f(y/x)}{f'(y/x)}$ which is homogeneous.

Put $y = Vx \Rightarrow \frac{dy}{dx}=V+x\frac{dV}{dx}$,

We obtain

$V+x\frac{dV}{dx}=V+\frac{f(V)}{f'(V)}\,dV$

$\Rightarrow \frac{f'(V)}{f(V)}dV=\frac{dx}{x}$

Integrating, we get

$\Rightarrow \log f(V) = \log cx \quad \Rightarrow f\left(\frac{y}{x}\right)=cx$.

Hence (B) is the correct answer

6. (a)

Given, $\frac{dy}{dx}=\frac{x-y}{x+y}$ This is a homogeneous equation

Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

Given equation becomes

$v+x\frac{dv}{dx}=\frac{1-v}{1+v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1-v}{1+v}-v$

$\Rightarrow \frac{1+v}{2-(1+v)^2}dv=\frac{dx}{x}$

On integrating both sides

$\int\frac{1+v}{2-(1+v)^2}dv=\int\frac{dx}{x}$

Put $(1 + v)^2 = t \Rightarrow 2(1 + v)\,dv = dt$

$\Rightarrow \frac{1}{2}\int\frac{dt}{2-t}=\int\frac{dx}{x}$

$\Rightarrow -\frac{1}{2}\log(2 - t) = \log x + \log c$

$\Rightarrow\ -\frac{1}{2}\log[2-(1+v)^2] = \log xc$

$\Rightarrow -\frac{1}{2}\log[-v^2-2v+1] = \log xc$

$\Rightarrow \log\frac{1}{\sqrt{1-2v-v^2}} = \log xc$

$\Rightarrow x^2c^2(1-2v-v^2) = 1$

$\Rightarrow x^2c^2\left(1-\frac{2y}{x}-\frac{y^2}{x^2}\right) = 1 \left[\because v=\frac{y}{x}\right]$

$\Rightarrow \frac{x^2c^2(x^2-2yx-y^2)}{x^2} = 1$

$\Rightarrow y^2+2xy-x^2 = c$

7. (a)

We have, $y = \left(x+\sqrt{1+x^2}\right)^n$ ... (i)

Let $\frac{d^2y}{dx^2} = y_2$

and $\frac{dy}{dx} = y_1$

On differentiating Equation (i), we get

$\frac{dy}{dx} = n\left[x+\sqrt{1+x^2}\right]^{n-1}\left(1+\frac{x}{\sqrt{x^2+1}}\right)$

$= \frac{n\left[x+\sqrt{1+x^2}\right]^n}{\sqrt{1+x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{ny}{\sqrt{1+x^2}}$

$y_1^2(1+x^2) = n^2y^2$

Again differentiating, we get

$2y_1y_2(1+x^2)+2x\,y_1^2 = 2n^2yy_1$

Dividing by $2y_1$

$y_2(1+x^2)+xy_1 = n^2y$

$\Rightarrow \frac{d^2y}{dx^2}(1+x^2)+x\frac{dy}{dx} = n^2y$

8. (c)

We have, $x\,dx+y\,dy+\frac{x\,dy-y\,dx}{x^2+y^2} = 0$

$\Rightarrow \frac{1}{2}d(x^2+y^2)+d\tan^{-1}\left(\frac{y}{x}\right) = 0$

Integrating, $\frac{1}{2}(x^2+y^2)+\tan^{-1}\frac{y}{x} = \frac{c}{2}$

$\Rightarrow x^2+y^2+2\tan^{-1}\frac{y}{x} = c.$

$\therefore y = x\tan\left(\frac{c-x^2-y^2}{2}\right)$ is the required solution.

Hence (C) is the correct answer

9. (a)

We have,

$x\,dy - y\,dx = xy^3(1+\log x)\,dx$

$\Rightarrow -\left(\frac{ydx-xdy}{y^2}\right) = xy(1+\log x)\,dx$

$\Rightarrow -d\left(\frac{x}{y}\right) = xy(1+\log x)\,dx$

$\Rightarrow -\frac{x}{y}d\left(\frac{x}{y}\right) = x^2(1+\log x)\,dx$

Integrating, we get

$-\frac{\left(\frac{x}{y}\right)^2}{2} = (1+\log x)\frac{x^3}{3} - \int\frac{x^3}{3}\cdot\frac{1}{x}dx$

$\Rightarrow -\frac{x^2}{2y^2} = \frac{x^3}{3}(1+\log x) - \frac{x^3}{9} + \frac{C}{2}$

$\Rightarrow -\frac{x^2}{y^2} = \frac{2x^3}{3}\left(\frac{2}{3}+\log x\right)+C$

10. (b)

$\frac{d^2y}{dx^2} = e^{-2x}$

$\Rightarrow \frac{dy}{dx} = \frac{e^{-2x}}{2}+c$

$\Rightarrow y = \frac{e^{-2x}}{4}+cx+d$

11. (b)

We have , I. F. = sin x $\Rightarrow e^{\int P\,dx} = \sin x \Rightarrow \int P\,dx = \log\sin x$

$\Rightarrow P = \frac{d}{dx}(\log\sin x) = \cot x.$

Hence (B) is the correct answer

12. (c)

We have, $xy^4\,dx + y\,dx - x\,dy = 0$

$\Rightarrow x\,dx + \frac{y\,dx-x\,dy}{y^4} = 0$

$\Rightarrow x^3\,dx + \left(\frac{x}{y}\right)^2\cdot\frac{ydx-xdy}{y^2} = 0$

$\Rightarrow x^3\,dx + \left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right) = 0$

Integrating, we get $\frac{x^4}{4}+\frac{1}{3}\left(\frac{x}{y}\right)^3 = c$

$\Rightarrow 3x^4y^3+4x^3 = cy^3,$

Which is the required solution.

13. (a)

The component linear equations are ;

$p = x,\ p = e^x,\ p = 1/y$

If $\frac{dy}{dx} = x$ then $dy = x\,dx \Rightarrow y = \frac{x^2}{2} + c_1$

If $\frac{dy}{dx} = e^x$ then $dy = e^x dx \Rightarrow y = e^x + c_2$

If $\frac{dy}{dx} = \frac{1}{y}$ then $y\,dy = dx \Rightarrow \frac{y^2}{2} = x + c_3$

$\therefore$ The required solution is ;

$$\left(y - \frac{x^2}{2} + c\right)(y - e^x + c)\left(\frac{y^2}{2} - x + c\right) = 0$$

14. (b)

The given equation can be written as ;

$(xy^2p^2 - x^3) + 2(xy^2 - py^3) = 0 \Rightarrow x(y^2p^2 - x^2) + 2y^2(x - py) = 0 \Rightarrow (py - x)[x(py + x) - 2y^2] = 0$

If $py - x = 0$ then $y\,dy - x\,dx = 0 \Rightarrow y^2 - x^2 = c_1$

If $x\,yp + x^2 - 2y^2 = 0$ then $2y\frac{dy}{dx} - \frac{4y^2}{x} = -2x \Rightarrow \frac{dt}{dx} - \frac{4}{x}t = -2x$; where $t = y^2$

$\text{I.F.} = e^{-\int \frac{4}{x}dx} = e^{-4\ln x} = \frac{1}{x^4}$

Its solution is $t\left(\frac{1}{x^4}\right) = \int -2x \cdot \frac{1}{x^4}dx$

i.e. $\frac{t}{x^4} = \frac{1}{x^2} + c_2$ i.e., $y^2 = x^2 + c_2 x^4$

Hence the required solutions is ;

$(y^2 - x^2 - c)(y^2 - x^2 - cx^4) = 0$

15. (b)

Differentiating the given equation with respect to x, we get ;

$\frac{dy}{dx} = 2p + 2x\frac{dp}{dx} - 2p\frac{dp}{dx}$

or $2(x - p)\frac{dp}{dx} + p = 0$

or $\frac{dx}{dp} + \frac{2}{p}x = 2$

It is a linear equation in x and p.

$\text{I.F.} = e^{\int \frac{2}{p}dx} = e^{2\log p} = p^2$

$\therefore$ The solution is;

$xp^2 = \int p^2 \cdot 2\,dp = \frac{2}{3}p^3 + c$

Thus, the solution of the given equation is ;

$x = \frac{2}{3}p + cp^{-2}$ where p is parameter

16. (c)

The given equation can be written as;

$y = \frac{xp}{2} + \frac{ax}{2p}$

$\frac{dy}{dx} = \frac{p}{2} + \frac{x}{2}\cdot\frac{dp}{dx} + \frac{a}{2p} - \frac{ax}{2p^2}\cdot\frac{dp}{dx}$

$\Rightarrow p(p^2 - a) = x(p^2 - a)\cdot\frac{dp}{dx}$

$\Rightarrow \frac{dp}{dx} = \frac{p}{x} \Rightarrow p = cx.$

(The equation $p^2 - a = 0$ given us singular solution in which we are not interested).

The substitute p in (i) we get the required solution;

$2y = cx^2 + \frac{a}{c}$

17. (a)

Let x denote the population at a time t in years.

then $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx$

when k is a constant of proportionality.

Solving $\frac{dx}{dt} = kx$, we get

$\int \frac{dx}{x} = \int k\,dt \Rightarrow \log x = kt + c \Rightarrow x = e^{kt + c}$

$\Rightarrow x = x_0 e^{kt}$

Where x_0 is the population at time t = 0.

Since it doubles in 50 years, at t = 50, we must have $x = 2x_0$.

Hence $2x_0 = x_0 e^{50k} \Rightarrow 50k = \log 2$

$\Rightarrow k = \frac{\log 2}{50}$ so that $x = x_0 e^{\frac{\log 2}{50}t}$

To find t, when it triples, $x = 3x_0 \Rightarrow 3x_0 = x_0 e^{\frac{\log 2}{50}t} \Rightarrow \log 3 = \frac{\log 2}{50}t$

$\Rightarrow t = \frac{50\log 3}{\log 2} = 79$ years

18. (b)

Given that, $\frac{dy}{dx} = \frac{2x - y}{x + 2y}$ ... (i)

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{2 - v}{1 + 2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{2 - v - v(1 + 2v)}{1 + 2v}$

$\Rightarrow \int \frac{1 + 2v}{2(1 - v - v^2)}dv = \int \frac{1}{x}dx$

$\Rightarrow \log k - \frac{1}{2}\log(1 - v - v^2) = \log x$

$\Rightarrow 2\log k - \log(1 - v - v^2) = 2\log x$

$\Rightarrow \log c = \log [x^2 (1 - v - v^2)]$

$\Rightarrow c = x^2\left(1-\frac{y}{x}-\frac{y^2}{x^2}\right)$

$\Rightarrow x^2 - xy - y^2 = c$

19. (a)

The given equation is $(y + 3)\, dy = (x + 2)\, dx$

$\Rightarrow \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + c$

Since, it passes through (2, 2)

$2 + 6 = 2 + 4 + c \Rightarrow c = 2\ \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + 2 \Rightarrow y^2 +$

$6y = x^2 + 4x + 4$

$\Rightarrow x^2 + 4x - y^2 - 6y + 4 = 0$

20. (c)

$\frac{dt}{dx} - t\,\frac{g'(x)}{g(x)} = -\frac{t^2}{g(x)}$

$\Rightarrow -\frac{1}{t^2}\frac{dt}{dx} + \frac{1}{t}\frac{g'(x)}{g(x)} = \frac{1}{g(x)}$ ... (1)

Let $z = \frac{1}{t} \Rightarrow -\frac{1}{t^2}\frac{dt}{dx} = \frac{dz}{dx}$ From (i)

$\frac{dz}{dx} + \frac{g'(x)}{g(x)} z = \frac{1}{g(x)}$

On comparing with $\frac{dz}{dx} + Pz = Q$, we get

$P = \frac{g'(x)}{g(x)}, Q = \frac{1}{g(x)}$

$IF = e^{\int \frac{g'(x)}{g(x)}dx} = e^{\log [g(x)]} = g(x)$

6tThus complete solution is

$z \,.\, g(x) = \int g(x)\,.\,\frac{1}{g(x)}\,dx + c \Rightarrow \frac{1}{t}g(x) = x + c \Rightarrow \frac{g(x)}{x+c} =$

t

21. (a)

The given equation is

$t = 1 + (ty)\left(\frac{dy}{dt}\right) + \frac{(ty)^2}{2!}\left(\frac{dy}{dt}\right)^2 + ... \infty \Rightarrow t = e^{ty\left(\frac{dy}{dt}\right)}$

$\Rightarrow \log t = ty\frac{dy}{dt}$

$\Rightarrow y\, dy = \frac{\log t}{t}\, dt$

On integration

$\frac{y^2}{2} = \frac{(\log t)^2}{2} + k$

$\Rightarrow y = \pm\sqrt{(\log t)^2 + 2k}$

$\Rightarrow y = \pm\sqrt{(\log t)^2 + c}$

22. (b)

Given $(1 + y^2) + (x - e^{\tan^{-1} y})\left(\frac{dy}{dx}\right) = 0$

This can be rewritten as

$(1 + y^2)\frac{dx}{dy} + x = e^{\tan^{-1} y}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1} y}}{(1+y^2)}$

It is a linear equation, comparing with the standard equation

$\frac{dx}{dy} + Px = Q \Rightarrow P = \frac{1}{1+y^2}, Q = \frac{e^{\tan^{-1} y}}{1+y^2} = e^{\int p\, dy}$

$= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1} y}$

Thus solution is

$x(e^{\tan^{-1} y}) = \int \frac{e^{\tan^{-1} y}}{1+y^2}\, e^{\tan^{-1} y}\, dy$

Put $e^{\tan^{-1} y} = t \Rightarrow \frac{e^{\tan^{-1} y}}{1+y^2}\, dy = dt$

$x(e^{\tan^{-1} y}) = \int t\, dt = \int \frac{t^2}{2} + c = \frac{e^{2\tan^{-1} y}}{2} + c$

$\Rightarrow x\, e^{\tan^{-1} y} = \frac{e^{2\tan^{-1} y}}{2} + c$

$\Rightarrow 2x\, e^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

23. (b)

The given equation can be written as;

$y = xp + p - p^2$; where $p = \frac{dy}{dx}$ ... (i)

differentiating both sides w.r.t. x, we get

$p = p + \frac{xdp}{dx} + \frac{dp}{dx} - 2p\,\frac{dp}{dx}$.

$\frac{dp}{dx}(x + 1 - 2p) = 0$

$\therefore$ either $\frac{dp}{dx} = 0$, i.e., $p = c$ ...(ii)

or $x + 1 - 2p = 0$, i.e., $p = \frac{1}{2}(x + 1)$ ... (iii)

Eliminating p between (i) and (ii) we get.

$y = cx + c - c^2$ as the complete solution and eliminating p between (i) and (iii)

$y = \frac{1}{2}(x + 1)\, x + \frac{1}{2}(x + 1) - \frac{1}{4}(x + 1)^2$

i.e., $4y = (x + 1)^2$ as the singular solution.

Hence (B) is the correct answer

24. (b)

The equation of the normal at (x, y) is :

$(X - x) + (Y - y)\frac{dy}{dx} = 0$

$\Rightarrow \dfrac{X}{x + y\dfrac{dy}{dx}} + \dfrac{Y}{\dfrac{(x + y\, dy/dx)}{dy/dx}} = 1$

$\Rightarrow OA = x + y\dfrac{dy}{dx},\ OB = \dfrac{\left(x + y\dfrac{dy}{dx}\right)}{\dfrac{dy}{dx}}$

Also, $\dfrac{1}{OA} + \dfrac{1}{OB} = 1$

$\Rightarrow 1 + \dfrac{dy}{dx} = x + y\dfrac{dy}{dx} \Rightarrow (y - 1)\dfrac{dy}{dx} + (x - 1) = 0$

Integrating, we get

$(y - 1)^2 + (x - 1)^2 = c$

Since the curve passes through (5, 4), c = 25.

Hence, the curve is $(x - 1)^2 + (y - 1)^2 = 25$

25. (b)

Integrate $dy/dx = -e^{-2x}/2 + c$

again Integrate $y = e^{-2x}/4 + cx + d$

26. (a)

$dy/dx = \ell nx \Rightarrow$ order = 1, degree = 1

27. (c)

$\int (by + k)\, dy = \int (ax + x)\, dx$

$by^2/2 + ky = ax^2/2 + hx + c$ represent parabola when $b = 0, a \neq 0$, $b \neq 0, a = 0$

28. (c)

$\left(1 + 3\dfrac{dy}{dx}\right)^2 = 4\left(\dfrac{d^3y}{dx^3}\right)^3$ order = 3, degree = 3

29. (a)

equation of circle is $x(x - a) + y^2 + \lambda y = 0$

$\Rightarrow x^2 + y^2 - ax + \lambda y = 0$ …(1)

deferential

$2x + 2y\, y' - a + \lambda y' = 0$ …(2)

from (1) and (2) $\dfrac{x^2 + y^2 - ax}{y} = \dfrac{2x + 2y\, y' - a}{y'}$

$\Rightarrow (x^2 + y^2 - ax)\, y' = (2x + 2y\, y' - a)\, y$

$\Rightarrow (x^2 - y^2 - ax)\, y' = (2x - a)y$

30. (b)

given equation is linear

$P = 1/x, \theta = x^2$

Solution is $y(x) = \int (x^2) x\, dx$

$xy = \dfrac{x^4}{4} + C$ …(1)

I.F. $= e^{\int \frac{1}{x} dx} = e^{\ell nx} = x$

Passes through (1, 1)

$1 = 1/4 + C \Rightarrow C = 3/4$

put C in (1)