1. **(c)**

$x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow (x - 1)^3 = -8$

$\Rightarrow \frac{x-1}{-2} = (1)^{1/3} \quad \Rightarrow 1, \omega, \omega^2$

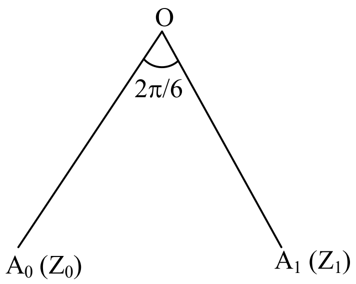
$\alpha = -1, \beta = 1 - 2\omega, \gamma = 1 - 2\omega^2$

$\therefore E = \frac{-2}{-2\omega} + \frac{-2}{-2\omega^2} + \frac{-2\omega^2}{-2}$

$= \frac{1}{\omega} + \frac{1}{\omega} + \frac{1}{\omega} = \frac{3}{\omega} = 3\omega^2.$

2. **(c)**

Let the vertices be $z_0, z_1, \ldots\ldots z_5$ w.r.t. centre O as origin



$|z_0| = 1, A_0A_1 = |z_1 - z_0| = |z_0 e^{i\theta} - z_0|$

$\therefore A_0A_1 = |z_0|\,|\cos\theta + i\sin\theta - 1|$

$= 1 . \sqrt{(\cos\theta - 1)^2 + \sin^2\theta} =$

$\sqrt{2(1 - \cos\theta)}$.

$\therefore A_0A_1 = \sqrt{2.2\sin^2\frac{\theta}{2}} = 2\sin\frac{\theta}{2},$

where $\theta = \frac{2\pi}{6} = \frac{\pi}{3}$

Replacing θ by 2θ and 4θ, we get

$A_0A_2 = 2\sin\frac{2\theta}{2} = 2\sin\theta,$

$A_0A_4 = 2\sin\frac{4\theta}{2} = 2\sin 2\theta$

$\therefore (A_0A_1)(A_0A_2)(A_0A_4) = 8\sin\frac{\pi}{6}\sin\frac{\pi}{3}\sin\frac{2\pi}{3}$

$= 8.\frac{1}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = 3.$

3. **(b)**

We have $\log_{0.3}|z - 1| > \log_{0.3}|z - i|$

$\Rightarrow |z - 1| < |z - i| (\because 0 < 0.3 < 1)$

$\Rightarrow |x + iy - 1| < |x + iy - i|$

$\Rightarrow (x - 1)^2 + y^2 < x^2 + (y - 1)^2$

$\Rightarrow x^2 + y^2 - 2x + 1 < x^2 + y^2 - 2y + 1$

$\Rightarrow -2(x - y) < 0 \Rightarrow x - y > 0$.

4. **(c)**

Equation of normals of circle are

$(2 - i)z = (2 + i)z \quad \ldots(1)$

and $(2 + i)z + (i - 2)\bar{z} - 4i = 0 \quad \ldots(2)$

replace $\bar{z}$ form (2) with the help of (1)

$(2 + i)z + \frac{(i-2)(2-i)}{(2+i)}z - 4i = 0$

$\Rightarrow (2 + i)z + \frac{(4i-3)(2-i)}{5}z - 4i = 0$

$\Rightarrow (2 + i)z + \frac{(11i-2)}{5}z = 4i$

$\Rightarrow (16i + 8)z = 20i$

$\Rightarrow z = \frac{20i}{16i+8} = \frac{5i(2-4i)}{(2+4i)(2-4i)}$

centre $z = \left(1 + \frac{i}{2}\right)$

(Intersection point of two normals) equation of line in standard form

$= \frac{iz}{1+i} + \frac{\bar{z}}{1+i} + 1 = 0$

$(1 + i)z + (1 - i)\bar{z} + 2 = 0 \qquad$ radius of circle

$= \frac{\left|\frac{(1+i)(2+i)}{2} + \frac{(1-i)(2-i)}{2} + 2\right|}{|1+i| + |1-i|} = \frac{3}{2\sqrt{2}}.$

5. **(d)**

We have $I_m(Z^2) = 4$

$I_m[(x^2 - y^2) + 2ixy] = 4$ (putting $Z = x + iy$)

$2xy = 4$ or $xy = 2$ which is a rectangular hyperbola

6. **(b)**

Let $z = x + iy$

$z = \alpha + 3 + i\sqrt{(5 - \alpha^2)}$

$x + iy = \alpha + 3 + i\sqrt{(5 - \alpha^2)}$

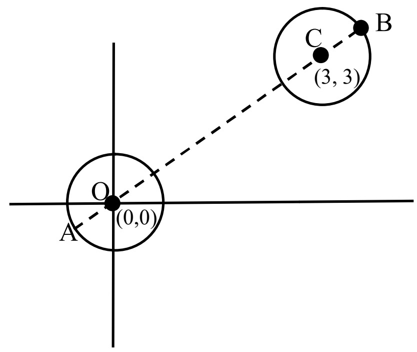
$x = \alpha + 3$

$y = \sqrt{(5 - \alpha^2)}$

$y = \sqrt{5 - (x-3)^2} \Rightarrow y^2 = 5 - x^2 - 9 + 6x$

$x^2 + y^2 - 6x + 4 = 0$ (focus is circle)

7. **(a)**



AB = Max distance

AB = AO + OC + BC

$= 2 + \sqrt{(3-0)^2 + (3-0)^2} + 1$

$= 3 + 3\sqrt{2}$

$= 3(1 + \sqrt{2})$

8. (a)

$1 + \omega + \omega^2 = 0$

$(-\omega^2)^3 - (-\omega)^3 = -\omega^6 + \omega^3 = -1 + 1 = 0$

9. (c)

Taking modulus both sides

$|1+i|\,|1+2i|\,|1+3i| \,.........\, |1+ni|\,|\alpha + i\beta|$

$\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{10}\ ....\ \sqrt{1+n^2} = \sqrt{\alpha^2+\beta^2}$

i.e. $2.5.10\ ....\ (1+n^2) = \alpha^2 + \beta^2$

10. (c)

$z = \dfrac{i(1-\sqrt{3}\,i)}{2} = -i\omega$

$z^{101} + i^{103} = (-i\omega)^{103} + i^{103} = -i\omega^2 - i = i\omega$

$(z^{101} + i^{103})^{105} = (i\omega)^{105} = i\ (w^3 = 1, i^4 = 1)$

Also, $z = -i\omega \neq i$, $z^2 = (-i\omega)^2 = -(\omega^2) = -\omega^2 \neq i$

$z^3 = (-i\omega)^3 = -i^3\omega^3 = i$

so, $(z^{101} + i^{103})^{105} = z^3$

11. (b)

Least value of $|z+1|$ is 0 and this happens when $z = -1$

$\Rightarrow |z+4| = |-1+4| = 3 \le 3|z+1|$

$= |(z+4) - 3| \le |z+4| + |-3|$

$\le 3 + 3 \le 6$

greatest value of $|z+1| = 6$

12. (a)

$\dfrac{1+7i}{(2-i)^2} = \dfrac{(1+7i)(3+4i)}{(3-4i)(3+4i)} = \dfrac{-25+25i}{25} = -1+i$

Let $z = x+iy = -1+i$, $\therefore\ x = r\cos\theta = -1$ and $y = r\sin\theta = 1$

$\therefore\ \theta = \dfrac{3\pi}{4}$ and $r = \sqrt{2}$, Thus $\dfrac{1+7i}{(2-i)^2} = \sqrt{2}\left[\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right]$

Alternative method: $\left|\dfrac{1+7i}{(2-i)^2}\right| = \left|\dfrac{1+7i}{3-4i}\right| = \sqrt{2}$ and *arg*

$\left(\dfrac{1+7i}{3-4i}\right) = \tan^{-1}7 - \tan^{-1}\left(\dfrac{-4}{3}\right) = \tan^{-1}7 + \tan^{-1}\dfrac{4}{3} = \dfrac{3\pi}{4}$

$\therefore\ \dfrac{1+7i}{(2-i)^2} = \sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$

13. (a)

$e^{e^{i\theta}} = e^{(\cos\theta + i\sin\theta)} = e^{\cos\theta}.e^{i\sin\theta} = e^{\cos\theta}[\cos(i\sin\theta) + i\sin(\sin\theta)]$

$\therefore$ Real part of $e^{e^{i\theta}}$ is $e^{\cos\theta}[\cos(\sin\theta)]$.

14. (a)

$\log_{1/3}|z+1| > \log_{1/3}|z-1|$

$\Rightarrow |z+1| < |z-1| \Rightarrow x^2+1+2x+y^2 < x^2+1-2x+y^2$

$\Rightarrow x < 0 \Rightarrow \text{Re}(z) < 0.$

15. (b)

Projection of z_1 on $z_2 = \dfrac{z_1 o z_2}{|z_2|} = \dfrac{a_1a_2 + b_1b_2}{\sqrt{a_2^2 + b_2^2}} = \dfrac{1}{\sqrt{10}}$.

16. (d)

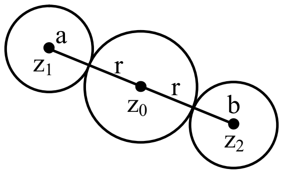
If z_1 be the new complex number then $|z_1| = |z| + \sqrt{2} = 2\sqrt{2}$.

Also $\dfrac{z_1}{z} = \dfrac{|z_1|}{|z|}e^{i3\pi/2} \Rightarrow z_1 = z.\ 2\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)$

$= 2(1+i)(0-i) = -2i + 2 = 2(1-i)$

17. (b)

Centre of the circle be z_0 and radius r. Then its equation is $|z - z_0| = r$ circle (1) touches the circle $|z - z_1| = a$ externally.



Distance between centres = sum of radii

$|z_0 - z_1| = a + r$ …(1)

$|z_0 - z_2| = b + r$ …(2)

(1) - (2)

$|z_0 - z_1| - |z_0 - z_2| = a - b$

$\therefore z_0$ lies on the curve $|z - z_1| - |z - z_2| = a - b$ which is equation of a hyperbola.

18. (b)

$2\left|z - \dfrac{1}{2}\right| = |z-1|$ $\qquad \dfrac{|z-1|}{|z-1/2|} = 2$

So locus of z is a circle

19. (a)

Let $\dfrac{a}{|z_2 - z_3|} = \dfrac{b}{|z_3 - z_1|} = \dfrac{c}{|z_1 - z_2|} = \lambda$ (say)

$\Rightarrow a = |z_2 - z_3|$, $b = \lambda|z_3 - z_1|$, $c = \lambda|z_1 - z_2|$

$\Rightarrow a^2 = \lambda^2|z_2 - z_3|^2 = \lambda^2(z_2 - z_3)(\bar{z}_2 - \bar{z}_3)$

$\therefore\ \dfrac{a^2}{(z_2 - z_3)} = \lambda^2(\bar{z}_2 - \bar{z}_3)$

Similarly, $\dfrac{a^3}{(z_3 - z_1)} = \lambda^2(\bar{z}_3 - \bar{z}_1)$ and $\dfrac{c^2}{(z_1 - z_2)}$

$= \lambda^2(\bar{z}_1 - \bar{z}_2)$

$\therefore\ \dfrac{a^2}{z_2 - z_3} + \dfrac{b^2}{z_3 - z_1} + |\dfrac{c^2}{z_1 - z_2} = 0.$

20. (c)

Sum $= i^2[1 - (i^2)^{2n+1}/1 - i^2]$

$= -1/2[1 - (-1)^{2n+1}]\ (\because (-1)^{odd} = -1) = -1/2\ [2]$

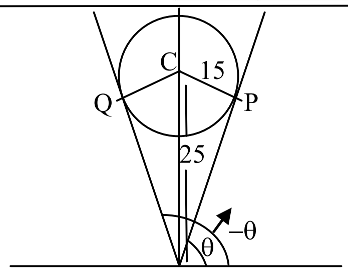
$\Rightarrow -1$

21. (b)

$2^{300}\left(e^{i\frac{\pi}{3}}\right)^{300} = a + ib$

$2^{300}(\cos(100\pi) + i\sin(100\pi)) = a + ib$; $a = 2^{300}$, $b = 0$

22. (c)



min arg (z) = $\theta = \cos^{-1}\left(\frac{15}{25}\right) = \cos^{-1}\left(\frac{3}{5}\right)$

max arg (z) = $\pi - \theta = \pi - \cos^{-1}\left(\frac{3}{5}\right)$

| max arg z - min arg z | = $|\pi - 2\theta|$

$= \left|\pi - 2\cos^{-1}\left(\frac{3}{5}\right)\right|$

23. (a)

In an equilateral triangle the circumcentre and the centroid are the same point. So,

$z_0 = \frac{z_1 + z_2 + z_3}{3} \Rightarrow z_1 + z_2 + z_3 = 3z_0$ ..... (i)

To shift the origin at z_0, we have to replace z_1, z_2, z_3 and z_0 by $z_1 + z_0, z_2 + z_0, z_3 + z_0$ and $0 + z_0$ then equation (i) becomes $(z_1 + z_0) + (z_2 + z_0) + (z_3 + z_0) = 3(0 + z_0) \Rightarrow$ $z_1 + z_2 + z_3 = 0$

On squaring $z_1^2 + z_2^2 + z_3^2 + 2(z_1z_2 + z_2z_3 + z_3z_1) = 0$

But triangle with vertices z_1, z_2 and z_3 is equilateral, then

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$ .....(iii)

From (ii) and (iii) we get, $3(z_1^2 + z_2^2 + z_3^2) = 0$. Therefore,

$z_1^2 + z_2^2 + z_3^2 = 0.$

24. (d)

$z = \frac{\alpha + \beta t}{\gamma + \delta t} \Rightarrow (\gamma + \delta t)z = \alpha + \beta t$

$\Rightarrow (\delta z - \beta)t = \alpha - \gamma z$

$\Rightarrow \quad t = \frac{\alpha - \gamma z}{\delta z - \beta}$ $\quad [\because \alpha\delta - \beta\gamma \neq 0]$

As t is real, $\frac{\alpha - \gamma z}{\delta z - \beta} = \frac{\bar{\alpha} - \bar{\gamma}\bar{z}}{\bar{\delta}\bar{z} - \bar{\beta}}$

$\Rightarrow \quad (\alpha - \gamma z)(\bar{\delta}\bar{z} - \bar{\beta}) = (\bar{\alpha} - \bar{\gamma}\bar{z})(\delta z - \beta)$

$\Rightarrow \quad (\bar{\gamma}\delta - \gamma\bar{\delta})z\bar{z} + (\gamma\bar{\beta} - \bar{\alpha}\delta)z + (\alpha\bar{\delta} - \beta\bar{\gamma})\bar{z}$

$= (\alpha\bar{\beta} - \bar{\alpha}\beta)$....(1)

Since $\frac{\gamma}{\delta}$ is real, $\frac{\gamma}{\delta} = \frac{\bar{\gamma}}{\bar{\delta}}$ or $\gamma\bar{\delta} - \delta\bar{\gamma} = 0$

Therefore (1) can be written as $\bar{a}z + a\bar{z} = c$ ...(2)

where $a = i(\alpha\bar{\delta} - \beta\bar{\gamma})$ and $c = i(\bar{\alpha}\beta - \alpha\bar{\beta})$

Note that $a \neq 0$ for if $a = 0$ then

$a\bar{\delta} - \beta\bar{\gamma} = 0 \Rightarrow \frac{\alpha}{\beta} = \frac{\bar{\gamma}}{\bar{\delta}} = \frac{\gamma}{\delta}$ $\quad [\because \frac{\gamma}{\delta}$ is real]

$\Rightarrow \quad \alpha\delta - \beta\gamma = 0,$

which is against hypothesis.

Also, note that $c = i(\bar{\alpha}\beta - \alpha\bar{\beta})$ is a purely real number.

Thus, $z = \frac{\alpha + \beta t}{\gamma + \delta t}$ represents a straight line.

25. (c)

Let $z = z_1/z_2$, then $z + 1/z = 1 \Rightarrow z^2 - z + 1 = 0$

$\Rightarrow z = \frac{1 \pm \sqrt{3}i}{2} \qquad \Rightarrow \frac{z_1}{z_2} = \frac{1 \pm \sqrt{3}i}{2}$

If z_1 and z_2 are represented by A and B respectively and O be the origin, then

$\frac{OA}{OB} = \frac{|z_1|}{|z_2|} = \left|\frac{1 \pm \sqrt{3}i}{2}\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\Rightarrow \quad OA = OB$

Also, $\frac{AB}{OB} = \frac{|z_2 - z_1|}{|z_2|} = \left|1 - \frac{z_1}{z_2}\right|$

$= \left|1 - \left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right)\right| = \left|\frac{1}{2} \mp \frac{\sqrt{3}}{2}i\right|$

$= \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\Rightarrow \quad AB = OB$

Thus, OA = OB = AB.

$\therefore$ ΔOAB is an equilateral triangle.