1. (a)

Break integral at x = 1, 2, 3

2. (b)

Put $e^x - 1 = t^2$

$\Rightarrow e^x\,dx = 2t\,dt$

$\Rightarrow I = \int_0^2 \frac{t.2t\,dt}{t^2+4}$

3. (c)

$\int_{n-1}^{n} e^{\{x\}}dx \;= n - (n-1)\int_0^1 e^{\{x\}}dx$

$= \int_0^1 e^x dx \;=\; e - 1$

4. (c)

$A = \int_1^{\sin\theta} \frac{t\,dt}{1+t^2}$

$B = \int_1^{\operatorname{cosec}\theta} \frac{dt}{t(1+t^2)}$ Put $t = \frac{1}{z}$, $dt = -\frac{1}{z^2}\,dz$

$B = \int_1^{\sin\theta} \frac{-\frac{1}{z^2}dz}{\frac{1}{z}\left(1+\frac{1}{z^2}\right)}$

$B = \int_1^{\sin\theta} \frac{-z}{(z^2+1)}dz$

$B = -\int_1^{\sin\theta} \frac{z}{(z^2+1)}dz \qquad z \Rightarrow t$

$B = -\int_1^{\sin\theta} \frac{t}{t^2+1}dt = -A$

$B = -A \qquad A + B = 0$

Now $\Delta = \begin{vmatrix} A & A^2 & -A \\ 1 & A^2 & -1 \\ 1 & 2A^2 & -1 \end{vmatrix} = 0$

5. (d)

$\int_0^{21} [x]^3 dx \;=\; \sum_{r=0}^{20} \int_r^{r+1} [x]^3 dx$

$\sum_{r=0}^{20} \int_r^{r+1} r^3 dx$

$= \sum_{1}^{20} r^3 = \left(\frac{20.21}{2}\right)^2 = 44100$

6. (d)

Let $I = \int_e^{e^4} \sqrt{\ell n(x)}\,dx.$

Put $\ell n\, x = t^2 \quad \therefore \phi 0 \;\; \delta\xi = 2\tau \;\; e^{t^2}$ dt

$= \int_1^2 2t^2 e^{t^2}\,dt = (te^{t^2})_1^2 - \int_1^2 e^{t^2}dt$

$= 2e^4 - e - a$

7. (a)

$\int_0^x [t]dt \;=\; \int_0^{[x]} [t]dt \;+\; \int_{[x]}^{[x]+\{x\}} [t]dt \;=\; \int_0^{[x]} t\,dt$

$\int_{[x]}^{[x]+\{x\}} [t]dt \;=\; \int_0^{[x]} t\,dt \;-\; \int_0^{[x]} [t]dt \;=\; \int_0^{[x]} \{t\}\,dt$

$\{x\}\,[x] = [x]\,.\,\frac{1}{2}$ i.e. $\{x\} = \frac{1}{2}$

thus $3 < x = n + \frac{1}{2} < 15$ i.e. $3 - \frac{1}{2} < n < 15 - \frac{1}{2}$

$\therefore$ n can take 12 values.

No. of solutions is 12.

8. (a)

$I = \int_{2n}^{2n+1/2} (\sin x)\left\{\frac{x}{2}\right\}dx$

$= 2n \int_0^2 (\sin \pi x)\,\frac{x}{2}\,dx + \int_0^{1/2} (\sin \pi x)\,\frac{x}{2}\,dx$

$= \frac{-2n\pi + 1}{\pi^2}$

9. (a)

$I_1 = \int_0^{\frac{\pi}{2}} \cos\theta\; f(\sin\theta + \cos 2\theta)\,d\theta$

$I_2 = \int_0^{\frac{\pi}{2}} 2\sin\theta\cos\theta\; f(\sin\theta + \cos^2\theta)\,d\theta$

Let $\sin\theta = t$, then $I_1 = \int_0^1 f(t + 1 - t^2)\,dt.$

and $I_2 = 2\int_0^1 t\,f(t+1-t^2)\,dt$

replace t by 1 - t then $I_1 = I_2$

10. (a)

$\because \sin x < x \;\forall\; x \in (0, \infty)$

so, $\cos(\sin x) > \cos x$, so $I_1 > I_3$

and sin sinx > sinx

so $\int_0^{\pi/2} \sin(\sin x)dx \;>\; \int_0^{\pi/2} (\sin x)dx$

$\int_0^{\pi/2} \sin(\cos x)dx > \int_0^{\pi/2} (\cos x)dx \Rightarrow I_2 > I_3$

11. (c)

Put $x = \frac{1}{t}$

$$I = \int_{3}^{1/3} t\sin\left(t - \frac{1}{t}\right)\left(-\frac{1}{t^2}\right)dt$$

$$= \int_{3}^{1/3} \sin\left(\frac{1}{t} - t\right)\frac{1}{t}dt$$

$= - I \Rightarrow I = 0$

12. (b)

$$I = \int_0^{\pi/2} \frac{\cos^3 x \sin x}{\sin^4 x + \cos^4 x}dx$$

$$\Rightarrow I + I = \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}dx$$

put $\sin^2 x = t$

13. (c)

$f'(x) = x(x^2 - 3x + 2) = x (x - 1) (x - 2)$. The sign scheme for $f'(x)$ is as shown in figure.

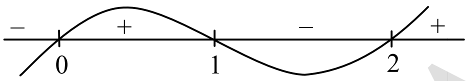
$\therefore$ $f'(x) \le 0$ in $1 \le x \le 2$ and $f'(x) \ge 0$ in $2 \le x \le 3$

$\therefore$ $f'(x)$ is decreasing in [1, 2] and increasing in [2, 3]

$$\therefore \text{ min. } f(x) = f(2) = \int_1^2 x(x^2 - 3x + 2)\,dx$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2 = -\frac{1}{4}$$

max. f(x) = the greatest among [f(1), f(3)]



f(1) =

$$\int_1^1 x(x^2 - 3x + 2)dx = 0$$

$$f(3) = \int_1^3 x(x^2 - 3x + 2)dx = 2$$

$\therefore$ max f(x) = 2, so the range = $\left[-\frac{1}{4}, 2\right]$

14. (c)

$$\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x \quad \{\text{given}\}$$

Differentiating both sides w.r.t. x using Newton Leibnitz formula, we have

$- \sin^2 x\, f(\sin x) \cos x = - \cos x$

$\Rightarrow \sin^2 x\, f(\sin x) \cos x = \cos x$ $\{\cos x \ne 0\}$

$$\Rightarrow f(\sin x) = \frac{1}{\sin^2 x}, \quad x \ne (2n+1)\frac{\pi}{2}$$

$$f(x) = \frac{1}{x^2}$$

$$\text{Now, } f\left(\frac{1}{\sqrt{3}}\right) = 3$$

15. (a)

Since, $f'(x) = x \sin \frac{1}{x}$

Now, at all points in $(0, \pi)$, $f'(x)$ has a definite finite value.

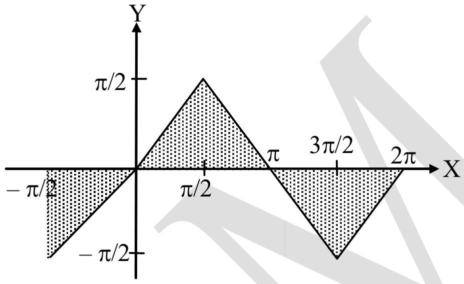
$\therefore$ f(x) is differentiable finitely in $(0, \pi)$

As a finitely differentiable function is also continuous

$\therefore$ f(x) is continuous in $(0, \pi)$

16. (b)

The graph of $f(x) = \sin^{-1}(\sin x)$ is shown as in figure.



$$\text{Therefore } \int_{-\pi/2}^{2\pi} \sin^{-1}(\sin x)\,dx = \int_{-\pi/2}^{0} \sin^{-1}(\sin x)\,dx + \int_{0}^{\pi} \sin^{-1}(\sin x)\,dx + \int_{\pi}^{2\pi} \sin^{-1}(\sin x)\,dx$$

= Area of shaded region

$$= -\left(\frac{1}{2}\times\frac{\pi}{2}\times\frac{\pi}{2}\right) + \left(\frac{1}{2}\times\pi\times\frac{\pi}{2}\right) - \left(\frac{1}{2}\times\pi\times\frac{\pi}{2}\right)$$

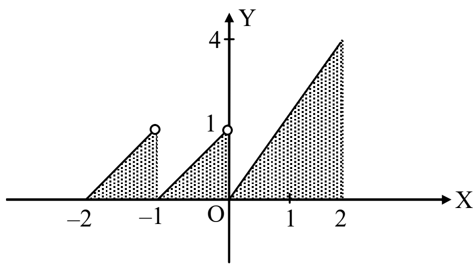
$$= -\frac{\pi^2}{8}.$$

17. (c)

$$\int_{-2}^{2} \max\{x + |x|, x - [x]\}\,dx$$

$$= \int_{-2}^{0} \max\{0, x - [x]\}\,dx + \int_{0}^{2} \max\{2x, x - [x]\}dx$$

the graph of $f(x) = \max\{x + |x|, x - [x]\}$ is shown as in figure



$$\text{therefore } \int_{-2}^{2} \max\{x + |x|, x - [x]\}\,dx = \text{Area of shaded region}$$

$$= 2\left(\frac{1}{2}\times 1\times 1\right) + \frac{1}{2}\times 2\times 4$$

$= 1 + 4$

$= 5.$

18. (c)

Let $I = \int_0^{\pi/4} \frac{(\sin x + \cos x)}{(9 + 16\sin 2x)} dx$

Put sin x - cos x = t then (cos x + sin x) dx = d t and $(\sin x - \cos x)^2 = t^2$

$\Rightarrow 1 - \sin 2x = t^2 \Rightarrow \sin 2x = 1 - t^2$

When $\left.\begin{matrix} x = 0 \Rightarrow t = -1 \\ x = \pi/4 \Rightarrow t = 0 \end{matrix}\right\}$

Then $I = \int_{-1}^{0} \frac{dt}{9 + 16(1 - t^2)}$

$= \int_{-1}^{0} \frac{dt}{25 - 16t^2} = \frac{1}{16} \int_{-1}^{0} \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$

$= \frac{1}{16} \cdot \frac{1}{2 \cdot \frac{5}{4}} \left\{ \ln \left| \frac{\frac{5}{4} + t}{\frac{5}{4} - t} \right| \right\}_{-1}^{0}$

$= \frac{1}{40} \left\{ \ln 1 - \ln \frac{1}{9} \right\}$

$= \frac{1}{40} \{0 + \ln 9\} = \frac{1}{40}\ 2 \ln 3$

$= \frac{1}{20} \ln 3.$

19. (a)

Let $I = \int_{\pi/4}^{\pi/3} x \operatorname{cosec}^2 x\, dx$

Integrating by parts taking x as first and $\operatorname{cosec}^2 x$ as second function, we have

$I = x(-\cot x) \Big|_{\pi/4}^{\pi/3} + \int_{\pi/4}^{\pi/3} \cot x\, dx$

$= -\left(\frac{\pi}{3} \cdot \frac{1}{\sqrt{3}} - \frac{\pi}{4} \right) + \ln |\sin x| \Big|_{\pi/4}^{\pi/3}$

$= -\frac{\pi}{3\sqrt{3}} + \frac{\pi}{4} + \left(\ln \frac{\sqrt{3}}{2} - \ln \frac{1}{\sqrt{2}} \right)$

$= \frac{\pi}{12\sqrt{3}} (3\sqrt{3} - 4) + \frac{1}{2} \ln \left(\frac{3}{2} \right).$

20. (b)

$f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$

$\therefore f'(x) = 2x \left[e^{-(x^2+1)^2} - e^{-(x^2)^2} \right]$

$= 2x\, e^{-(x^4+1+2x^2)} [1 - e^{2x^2+1}].$

Now $f'(x) > 0$ for $x \in (-\infty, 0)$.