1. (d) Given, position vectors of A, B and C are $7\mathbf{j}+10\mathbf{k}$, $-\mathbf{i}+6\mathbf{j}+6\mathbf{k}$ and $-4\mathbf{i}+9\mathbf{j}+6\mathbf{k}$ respectively.

$\therefore \quad |\overrightarrow{AB}| = |-\mathbf{i}-\mathbf{j}-4\mathbf{k}| = \sqrt{18}$

$|\overrightarrow{BC}| = |-3\mathbf{i}+3\mathbf{j}| = \sqrt{18}$

$|\overrightarrow{AC}| = |-4\mathbf{i}+2\mathbf{j}-4\mathbf{k}| = \sqrt{36}$

Clearly, $AB = BC$ and $(AC)^2 = (AB)^2 + (BC)^2$

Hence, triangle is right angled isosceles.

2. (b) Let $-2\mathbf{a}+3\mathbf{b}-\mathbf{c} = x\mathbf{p}+y\mathbf{q}+z\mathbf{r}$

$\Rightarrow -2\mathbf{a}+3\mathbf{b}-\mathbf{c}$

$= (2x+y-3z)\mathbf{a}+(-3x-2y+z)\mathbf{b}+(y+2z)\mathbf{c}$

$\therefore 2x+y-3z=-2, \quad -3x-2y+z=3$ and $y+2z=-1$

Solving these, we get $x=0, \quad y=-\frac{7}{5}, \quad z=\frac{1}{5}$

$\therefore \quad -2\mathbf{a}+3\mathbf{b}-\mathbf{c} = \frac{(-7\mathbf{q}+\mathbf{r})}{5}$.

Trick : Check alternates one by one

i.e., (a) $\mathbf{p}-4\mathbf{q} = -2\mathbf{a}+5\mathbf{b}-4\mathbf{c}$

(b) $\frac{-7\mathbf{q}+\mathbf{r}}{5} = -2\mathbf{a}+3\mathbf{b}-\mathbf{c}$.

3. (b) A, B, C, D, E are five co-planar points.

$\overrightarrow{DA}+\overrightarrow{DB}+\overrightarrow{DC}+\overrightarrow{AE}+\overrightarrow{BE}+\overrightarrow{CE}$

$= (\overrightarrow{DA}+\overrightarrow{AE})+(\overrightarrow{DB}+\overrightarrow{BE})+(\overrightarrow{DC}+\overrightarrow{CE})$

$= \overrightarrow{DE}+\overrightarrow{DE}+\overrightarrow{DE} = 3\overrightarrow{DE}$.

4. (a) Resultant vector

$= (2\mathbf{i}+4\mathbf{j}-5\mathbf{k})+(\mathbf{i}+2\mathbf{j}+3\mathbf{k}) = 3\mathbf{i}+6\mathbf{j}-2\mathbf{k}$

Unit vector $= \frac{3\mathbf{i}+6\mathbf{j}-2\mathbf{k}}{\sqrt{9+36+4}} = \frac{1}{7}(3\mathbf{i}+6\mathbf{j}-2\mathbf{k})$.

5. (b) Let the B divide AC in ratio $\lambda : 1$, then

$5\mathbf{i}-2\mathbf{k} = \frac{\lambda(11\mathbf{i}+3\mathbf{j}+7\mathbf{k})+\mathbf{i}-2\mathbf{j}-8\mathbf{k}}{\lambda+1}$

$\Rightarrow 3\lambda-2=0 \Rightarrow \lambda = \frac{2}{3}$ *i.e.*, ratio = 2 : 3.

6. (a) $|\mathbf{a}+\mathbf{b}| > |\mathbf{a}-\mathbf{b}|$

Squaring both sides, we get

$a^2+b^2+2\mathbf{a}.\mathbf{b} > a^2+b^2-2\mathbf{a}.\mathbf{b}$

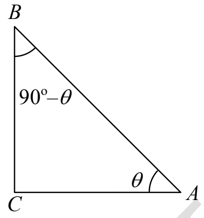
$\Rightarrow 4\mathbf{a}.\mathbf{b} > 0 \Rightarrow \cos\theta > 0$. Hence $\theta < 90^\circ$, (acute).

7. (d) Obviously $\mathbf{a}, \mathbf{b}$ are unit vectors.

8. (c) It is obvious.

9. (c) We have $\overrightarrow{AB}.\overrightarrow{AC}+\overrightarrow{BC}.\overrightarrow{BA}+\overrightarrow{CA}.\overrightarrow{CB}$

$(AB)(AC)\cos\theta+(BC)(BA)\cos(90^\circ-\theta)+0$



$= AB(AC\cos\theta+BC\sin\theta) = AB\left(\frac{(AC)^2}{AB}+\frac{(BC)^2}{AB}\right)$

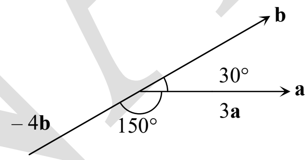
$= AC^2+BC^2 = AB^2 = p^2$.

10. (c) $|\mathbf{x}-\mathbf{y}|^2 = (\mathbf{x}-\mathbf{y}).(\mathbf{x}-\mathbf{y}) = 1+1-2|\mathbf{x}||\mathbf{y}|\cos\pi$

$= 2-2\cos\pi, \quad \therefore |\mathbf{x}-\mathbf{y}|^2 = 4$

So, $\frac{1}{2}|\mathbf{x}-\mathbf{y}| = 1$, $[\because |\mathbf{x}|^2 = |\mathbf{y}|^2 = 1, |\mathbf{x}| = |\mathbf{y}| = 1]$.

11. (a) It is obvious from figure.



12. (b) $14(\mathbf{a}\times\mathbf{b})+15(\mathbf{b}\times\mathbf{a}) = \mathbf{b}\times\mathbf{a}$.

13. (b) If angle between $\mathbf{b}$ and $\mathbf{c}$ is α and $|\mathbf{b}\times\mathbf{c}| = \sqrt{15}$

$|\mathbf{b}|\ |\mathbf{c}|\ \sin\alpha = \sqrt{15}$

$\sin\alpha = \frac{\sqrt{15}}{4}$; $\therefore \cos\alpha = \frac{1}{4}$

$\mathbf{b}-2\mathbf{c} = \lambda\mathbf{a} \Rightarrow |\mathbf{b}-2\mathbf{c}|^2 = \lambda^2|\mathbf{a}|^2$

$|\mathbf{b}|^2+4|\mathbf{c}|^2-4\,\mathbf{b}.\mathbf{c} = \lambda^2|\mathbf{a}|^2$

$16+4-4\{|\mathbf{b}||\mathbf{c}|\cos\alpha\} = \lambda^2$

$16+4-4\times4\times1\times\frac{1}{4} = \lambda^2 \Rightarrow \lambda^2 = 16 \Rightarrow \lambda = \pm 4$.

14. (b) $\Delta = \sqrt{\Delta_x^2+\Delta_y^2+\Delta_z^2}$

where $\Delta_x = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}$ and so on.

Alter : Form two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$

$$\Delta = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}| = \frac{1}{2}\left|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}\right|$$

$$= \frac{1}{2}\left|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1\end{vmatrix}\right| = \frac{1}{2}|8\mathbf{i}+4\mathbf{j}+4\mathbf{k}|$$

$$= \frac{1}{2}\sqrt{64+16+16} = \frac{\sqrt{96}}{2} = 2\sqrt{6}.$$

15. (c) Required area $= \left|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 3 \\ 2 & 1 & -4\end{vmatrix}\right|$

$= |5\mathbf{i}+10\mathbf{j}+5\mathbf{k}| = \sqrt{150} = 5\sqrt{6}$.

16. (d) $(\mathbf{a}\times\mathbf{j}).(2\mathbf{j}-3\mathbf{k}) = \mathbf{a}.\{\mathbf{j}\times(2\mathbf{j}-3\mathbf{k})\}$

$= \mathbf{a}.\{-3(\mathbf{j}\times\mathbf{k})\} = -3(\mathbf{a}.\mathbf{i}) = -12.$

17. (b) Let vector be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$.

$\because a\mathbf{i}+b\mathbf{j}+c\mathbf{k}, \mathbf{i}+\mathbf{j}, \mathbf{j}+\mathbf{k}$ are coplanar.

$$\therefore \begin{vmatrix} a & b & c \\ 1 & 1 & 0 \\ 0 & 1 & 1\end{vmatrix} = 0 \Rightarrow a-b+c=0$$

Also, since $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k}) \,||\, (2\mathbf{i}-2\mathbf{j}-4\mathbf{k})$

$\therefore (a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\times(2\mathbf{i}-2\mathbf{j}-4\mathbf{k}) = 0$

i.e., $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & b & c \\ 2 & -2 & -4\end{vmatrix} = 0$

$\Rightarrow \mathbf{i}(-4b+2c) - \mathbf{j}(-4a-2c) + \mathbf{k}(-2a-2b) = 0$

$\Rightarrow -4b+2c=0,\ 4a+2c=0,\ 2a+2b=0$

$\Rightarrow \frac{c}{2}=\frac{b}{1},\ \frac{c}{2}=\frac{a}{-1},\ \frac{a}{-1}=\frac{b}{1}$

i.e., $\frac{a}{-1}=\frac{b}{1}=\frac{c}{2}$ or $\frac{a}{1}=\frac{b}{-1}=\frac{c}{-2}$

$\therefore$ Required vector is $\mathbf{i}-\mathbf{j}-2\mathbf{k}$.

18. (c) Since $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanar vectors.

$$\therefore [\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = 0 \Rightarrow \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 1 & \alpha & 0\end{vmatrix} = 0$$

$\Rightarrow 1[0-\alpha] - 1[0-1] - 1[2\alpha-3] = 0$

$\Rightarrow -3\alpha+4=0 \Rightarrow \alpha = \frac{4}{3}$.

19. (c) $\mathbf{a}\times[\mathbf{a}\times\{\mathbf{a}\times(\mathbf{a}\times\mathbf{b})\}] = \mathbf{a}\times[\mathbf{a}\times\{\mathbf{a}\times ab\,\hat{\mathbf{n}}\}]$

$= \mathbf{a}\times[\mathbf{a}\times a^2\mathbf{b}] = \mathbf{a}\times a^3 b\,\hat{\mathbf{n}} = |\mathbf{a}|^4\ \mathbf{b}.$

20. (d) Required distance $= \left|\frac{d-\mathbf{a}.\mathbf{n}}{|\mathbf{n}|}\right| = \left|\frac{5-(2\mathbf{i}-2\mathbf{j}+3\mathbf{k}).(\mathbf{i}+5\mathbf{j}+\mathbf{k})}{\sqrt{1+25+1}}\right|$

$$= \left|\frac{5-(2-10+3)}{\sqrt{27}}\right| = \frac{10}{3\sqrt{3}}.$$

21. (b) $|\mathbf{a}-\mathbf{b}|^2+|\mathbf{b}-\mathbf{c}|^2+|\mathbf{c}-\mathbf{a}|^2$

$= 2(\mathbf{a}^2+\mathbf{b}^2+\mathbf{c}^2) - 2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})$

$= 2\times 3 - 2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})$

$= 6 - \{(\mathbf{a}+\mathbf{b}+\mathbf{c})^2-\mathbf{a}^2-\mathbf{b}^2-\mathbf{c}^2\} = 9 - |\mathbf{a}+\mathbf{b}+\mathbf{c}|^2 \le 9.$

22. (b) $\mathbf{u} = 2\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i}-3\mathbf{j}+2\mathbf{k}$

Let vector $\mathbf{c} = c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$, then $\mathbf{c}.\mathbf{u}=0$

$\Rightarrow 2c_1+2c_2-c_3=0$ .....(i)

and $\mathbf{c}.\mathbf{v}=0 \Rightarrow 6c_1-3c_2+2c_3=0$ .....(ii)

Solving equation (i) and (ii) by cross multiplication

$\frac{c_1}{4-3} = \frac{c_2}{-6-4} = \frac{c_3}{-6-12} = \lambda$, (say)

$\Rightarrow \frac{c_1}{1} = \frac{c_2}{-10} = \frac{c_3}{-18} = \lambda$

$\Rightarrow c_1 = \lambda,\ c_2 = -10\lambda$ and $c_3 = -18\lambda$

Thus $\mathbf{c} = \lambda(\mathbf{i}-10\mathbf{j}-18\mathbf{k})$

$|\mathbf{c}| = \lambda\sqrt{1+100+324} = \lambda\sqrt{425}$

Hence required unit vector is, $\frac{\mathbf{c}}{|\mathbf{c}|}$

$= \frac{\lambda(\mathbf{i}-10\mathbf{j}-18\mathbf{k})}{\lambda\sqrt{425}} = \frac{1}{\sqrt{425}}(\mathbf{i}-10\mathbf{j}-18\mathbf{k})$

$= \frac{1}{5\sqrt{17}}(\mathbf{i}-10\mathbf{j}-18\mathbf{k}) = \frac{1}{\sqrt{17}}\left(\frac{1}{5}\mathbf{i}-2\mathbf{j}-\frac{18}{5}\mathbf{k}\right)$

Aliter : Required vector is $\frac{\mathbf{u}\times\mathbf{v}}{|\mathbf{u}\times\mathbf{v}|} = \frac{\mathbf{i}-10\mathbf{j}-18\mathbf{k}}{\sqrt{425}}$.

23. (c) It is given that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ are the position vectors of vertices of a quadrilateral *ABCD* respectively.

Let *E, F, G* and *H* are the middle points of sides *AB, BC, CD* and *DA* respectively.

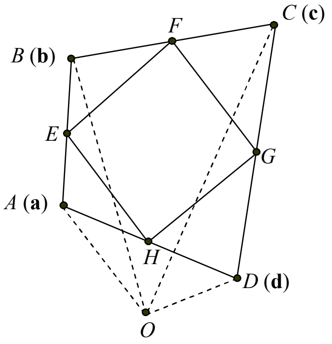
The position vectors of these points will be

$\overrightarrow{OE} = \frac{1}{2}(\mathbf{a}+\mathbf{b}),\quad \overrightarrow{OF} = \frac{1}{2}(\mathbf{b}+\mathbf{c})$,

$\overrightarrow{OG} = \frac{1}{2}(\mathbf{c}+\mathbf{d}),\quad \overrightarrow{OH} = \frac{1}{2}(\mathbf{a}+\mathbf{d})$

Then $\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE} = \left(\frac{\mathbf{c}-\mathbf{a}}{2}\right)$

and $\overrightarrow{FG} = \frac{1}{2}(\mathbf{d} - \mathbf{b}), \overrightarrow{GH} = \frac{1}{2}(\mathbf{a} - \mathbf{c}), \overrightarrow{GH} = \frac{1}{2}(\mathbf{b} - \mathbf{d})$



It is clear that $\overrightarrow{EF}$ is parallel to $\overrightarrow{GH}$ and $\overrightarrow{FG}$ is parallel to $\overrightarrow{HE}$. Thus $EFGH$ is a parallelogram.

$\therefore \quad \overrightarrow{EF} \times \overrightarrow{FG} = \frac{1}{4}\{(\mathbf{c} - \mathbf{a}) \times (\mathbf{d} - \mathbf{b})\}$

$= \frac{1}{4}(\mathbf{c} \times \mathbf{d} - \mathbf{c} \times \mathbf{b} - \mathbf{a} \times \mathbf{d} + \mathbf{a} \times \mathbf{b})$

$= \frac{1}{4}(\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{d} + \mathbf{d} \times \mathbf{a})$

$\therefore$ Area of parallelogram $EFGH$ is,

$A = |\overrightarrow{EF} \times \overrightarrow{FG}| = \frac{1}{4}|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{d} + \mathbf{d} \times \mathbf{a}|$.

24. (c) We have $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{c})$
$= ((\mathbf{a} \times \mathbf{b}).\mathbf{c})\mathbf{b} - ((\mathbf{a} \times \mathbf{b}).\mathbf{b})\mathbf{c} = [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]\mathbf{b}$

$(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a}) = ((\mathbf{b} \times \mathbf{c}).\mathbf{a})\mathbf{c} - ((\mathbf{b} \times \mathbf{c}).\mathbf{c})\mathbf{a} = [\mathbf{b}\,\mathbf{c}\,\mathbf{a}]\mathbf{c}$

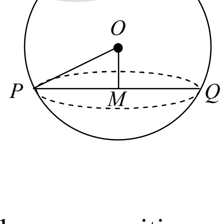
$(\mathbf{c} \times \mathbf{a}) \times (\mathbf{a} \times \mathbf{b}) = ((\mathbf{c} \times \mathbf{a}).\mathbf{b})\mathbf{a} - ((\mathbf{c} \times \mathbf{a}).\mathbf{a})\mathbf{b} = [\mathbf{c}\,\mathbf{a}\,\mathbf{b}]\mathbf{a}$

$\therefore\ [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{c})(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})(\mathbf{c} \times \mathbf{a}) \times (\mathbf{a} \times \mathbf{b})]$

$= [[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]\mathbf{a}\,[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]\mathbf{b}\,[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]\mathbf{c}] = [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^3[\mathbf{a}\,\mathbf{b}\,\mathbf{c}] = [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^4$.

25. (d) The centre of the sphere $|\mathbf{r}| = 5$ is at the origin and radius $= 5$. Let M be the foot of perpendicular from O to the given plane. Then OM = length of perpendicular from O to the given plane $= \frac{|\overrightarrow{OM}.(\mathbf{i} + \mathbf{j} + \mathbf{k}) - 3\sqrt{3}|}{|\mathbf{i} + \mathbf{j} + \mathbf{k}|}$

$= \frac{3\sqrt{3}}{\sqrt{1^2 + 1^2 + 1^2}} = 3$



Let P be any position of circle, then P lies on plane as well as on sphere.

$\therefore\ OP$ = radius of sphere = 5

In ΔOPM, we have $OP^2 = OM^2 + PM^2$

$\Rightarrow PM = \sqrt{5^2 - 3^2} = 4$.