**1. (d)**

$$\lim_{x\to 0}\frac{\int_0^{x^2}(\tan^{-1}t)^2dt}{\int_0^{x^4}\sin\sqrt{t}\,dt} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 0}\frac{[(\tan^{-1}t)^2]_{t=x^2}\cdot\frac{d}{dx}(x^2)}{[\sin\sqrt{t}]_{t=x^4}\cdot\frac{d}{dx}(x^4)}$$

[Using L′ Hospital's Rule]

$$=\lim_{x\to 0}\frac{(\tan^{-1}x^2)^2.2x}{\sin x^2.4x^3}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{\tan^{-1}x^2}{x^2}\right)^2\Bigg/\left(\frac{\sin x^2}{x^2}\right)=\frac{1}{2}.$$

Hence (D) is the correct answer.

**2. (a)**

We have, $f(x)=\int_1^{x^3}\frac{dt}{1+t^4}$

$$\Rightarrow f'(x)=\left[\frac{1}{1+t^4}\right]_{t=x^3}\cdot\frac{d}{dx}(x^3)=\frac{1}{1+x^{12}}\ .\ 3x^2$$

$$\therefore f''(x)=3\left[\frac{(1+x^{12}).2x-x^2.12x^{11}}{(1+x^{12})^2}\right]$$

$$f'(x)=\frac{6x(1+x^{12}-6x^{12})}{(1+x^{12})^2}$$

$$f''(x)=\frac{6x(1-5x^{12})}{(1+x^{12})^2}$$

**3. (d)**

$$\frac{d}{dx}F(x)=\frac{e^{\sin x}}{x},\ x>0$$

On integrating both sides of above equation, we get

$$F(x)=\int\frac{e^{\sin x}}{x}dx \qquad \ldots(1)$$

Also, $\int_1^4\frac{3}{x}e^{\sin x^3}dx=\int_1^4\frac{3x^2}{x^3}e^{\sin x^3}dx$

= F (k) - F (1)

Let $x^3=z \quad\Rightarrow\quad 3x^2\,dx=dz$

$$\Rightarrow\int_1^{64}\frac{e^{\sin z}}{z}dz=F(k)-F(1)$$

Using equation (1), we get

$[F(z)]_1^{64}=F(k)-F(1)$

$\Rightarrow F(64)-F(1)=F(k)-F(1)$

$\Rightarrow k=64.$

Hence (D) is the correct answer.

**4. (a)**

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\ldots+\frac{1}{8n}\right]$$

$$=\lim_{n\to\infty}\left[\frac{n^2}{(n+0)^3}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\ldots+\frac{n^2}{(n+n)^3}\right]$$

$$=\lim_{n\to\infty}\sum_{r=0}^{n}\frac{n^2}{(n+r)^3}=\lim_{n\to\infty}\sum_{r=0}^{n}\frac{1}{n}\cdot\frac{1}{\left(1+\frac{r}{n}\right)^3}$$

$$=\int_0^1\frac{dx}{(1+x^3)}=\left[-\frac{1}{2(1+x)^2}\right]_0^1=-\frac{1}{2}\left(\frac{1}{4}-1\right)=\frac{3}{8}.$$

Hence (A) is the correct answer.

**5. (b)**

$\lim_{n\to\infty}S_n$

$$=\lim_{n\to\infty}\left[\frac{1}{\sqrt{4n^2-0}}+\frac{1}{\sqrt{4n^2-1^2}}+\frac{1}{\sqrt{4n^2-2^2}}+\ldots+\frac{1}{\sqrt{4n^2-(n-1)^2}}\right]$$

$$=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{(2n)^2-r^2}}$$

$$=\frac{1}{2}\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{1-\left(\frac{r}{2n}\right)^2}}\cdot\frac{1}{n}$$

$$=\frac{1}{2}\int_0^1\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx=\frac{1}{2}\cdot\left[2\sin^{-1}\frac{x}{2}\right]_0^1=\frac{\pi}{6}.$$

Hence (B) is the correct answer.

**6. (b)**

$$\int_{-100\pi}^{100\pi}(\sin^4x+\cos^4x)dx=2\int_0^{100\pi}(\sin^4x+\cos^4x)dx$$

($\because$ integrand is given)

$$=2\int_0^{\frac{\pi}{2}(200)}(\sin^4x+\cos^4x)dx=2\ .\ 200$$

$$\int_0^{\pi/2}(\sin^4x+\cos^4x)\,dx$$

$$\left[\because \sin^4x+\cos^4x\text{ is a periodic function of period }\frac{\pi}{2}\right]$$

$$=400\left[\int_0^{\pi/2}\sin^4x+\int_0^{\pi/2}\cos^4\left(\frac{\pi}{2}-x\right)dx\right]$$

$$=800\int_0^{\pi/2}\sin^4x\,dx=800\ .\ \frac{3.1}{4.2}\cdot\frac{\pi}{2}=150\,\pi.$$

Hence (B) is the correct answer.

**7. (b)**

$I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x(1+\tan^2 x)dx$

$= \int_0^{\pi/4} \tan^n x \sec^2 x \, dx$

$= \int_0^1 t^n \, dt$, where $t = \tan x$

$\therefore \quad I_n + I_{n+2} = \frac{1}{n+1}$

$\Rightarrow \lim_{n\to\infty} n\,[I_n + I_{n+2}]$

$= \lim_{n\to\infty} n \cdot \frac{1}{n+1} = \lim_{n\to\infty} \frac{n}{n+1}$

$= \lim_{n\to\infty} \frac{n}{n\left(1+\frac{1}{n}\right)} = 1.$

Hence (B) is the correct answer.

**8. (d)**

Let $I = \int_0^{\pi} \frac{x^3 \cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2} dx$

$= \int_0^{\pi} \frac{(\pi - x)^3 \cos^4(\pi - x)\sin^2(\pi - x)}{\pi^2 - 3\pi(\pi - x) + 3(\pi - x)^2} dx$

$= \int_0^{\pi} \frac{(\pi^3 - 3\pi^2 x + 3\pi x^2 - x^3)\cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2} dx$

$= \int_0^{\pi} \frac{(\pi^3 - 3\pi^2 x + 3\pi x^2)\cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2} dx - I$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(\pi^2 - 3\pi x + 3x^2)\cos^4 x \sin^2 x}{(\pi^2 - 3\pi x + 3x^2)} dx$

$\therefore \quad I = \pi \cdot \int_0^{\pi/2} \cos^4 x \sin^2 x \, dx = 2\pi \cdot \frac{3.1.1}{6.4.2} \cdot \frac{\pi}{2}$

$\therefore = \frac{\pi^2}{32}.$

Hence (D) is the correct answer.

**9. (d)**

Let $y = \lim_{n\to\infty} \left[\left(1+\frac{1^2}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\ldots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$

$\Rightarrow \log y =$

$\lim_{n\to\infty} \frac{1}{n}\left[\log\left(1+\frac{1^2}{n^2}\right) + \log\left(1+\frac{2^2}{n^2}\right) + \ldots \log\left(1+\frac{n^2}{n^2}\right)\right]$

$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \log\left(1+\frac{r^2}{n^2}\right) = \int_0^1 \log\,(1+x^2)\,dx$

$= [x \log(1+x^2)]_0^1 - \int_0^1 x \cdot \frac{2x}{1+x^2} dx$

$= \log 2 - 2\int_0^1 \left(1 - \frac{1}{1+x^2}\right) dx$

$= \log 2 - 2\,[x - \tan^{-1} x]_0^1 = \log 2 + \frac{\pi}{2} - 2.$

$\therefore \; y = e^{\log 2 + \frac{\pi}{2} - 2} = e^{\log 2} \cdot e^{\frac{\pi - 4}{2}} = 2e^{\frac{\pi-4}{2}}.$

**10. (d)**

We know that $I_{2n} = \int_0^{\pi/2} \sin^{2n} x \, dx$

$= \frac{2n-1}{2n} \times \frac{2n-3}{2n-2} \times \ldots \times \frac{1}{2} \times \frac{\pi}{2},$

$I_{2n+1} = \int_0^{\pi/2} \sin^{2n+1} x \, dx = \frac{2n}{2n+1} \times \frac{2n-2}{2n-1} \times \ldots \times \frac{2}{3}$ and

Also, $I_{2m+1} = \frac{2m}{2m+1} I_{2m-1}.$

For all $x \in (0, \pi/2)$, $\sin^{2m-1} x > \sin^{2m} x > \sin^{2m+1} x$

Integrating from 0 to $\pi/2$, we get $I_{2m-1} \ge I_{2m} \ge I_{2m+1}$

Whence $\frac{I_{2m-1}}{I_{2m+1}} \ge \frac{I_{2m}}{I_{2m+1}} \ge 1$ ... (i)

Also $\frac{I_{2m-1}}{I_{2m+1}} = \frac{2m+1}{2m}.$

Hence $\lim_{m\to\infty} \frac{I_{2m-1}}{I_{2m+1}} = \lim_{m\to\infty} \frac{2m+1}{2m} = 1.$

From (i) and using sandwich theorem we have

$\lim_{m\to\infty} \frac{I_{2m}}{I_{2m+1}} = 1.$

**11. (d)**

$\lim_{n\to\infty} S_n$

$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{\sqrt{4-0}} + \frac{1}{\sqrt{4-1/n^2}} + \frac{1}{\sqrt{4-4/n^2}} \ldots + \frac{1}{\sqrt{4-\left(\frac{n-1}{n}\right)^2}}\right]$

$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{n-1} \frac{1}{\sqrt{4-(r/n)^2}} = \int_0^1 \frac{dx}{\sqrt{4-x^2}}$

$= \sin^{-1} \frac{x}{2}\Big|_0^1 = \frac{\pi}{6}.$

**12. (c)**

Let P =

$\lim_{n\to\infty} \left\{\frac{\sqrt{n}}{(3+4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{2}(3\sqrt{2}+4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{3}(3\sqrt{3}+4\sqrt{n})^2} + \ldots + \frac{1}{49n}\right\}$

$= \lim_{n\to\infty}$

$$\left\{\frac{\sqrt{n}}{\sqrt{1}(3\sqrt{1}+4\sqrt{n})^2}+\frac{\sqrt{n}}{\sqrt{2}(3\sqrt{2}+4\sqrt{n})^2}+\frac{\sqrt{n}}{\sqrt{3}(3\sqrt{3}+4\sqrt{n})^2}+\ldots+\frac{\sqrt{n}}{\sqrt{n}(3\sqrt{n}+4\sqrt{n})^2}\right\}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n\sqrt{\left(\frac{r}{n}\right)}\left\{3\sqrt{\left(\frac{r}{n}\right)}+4\right\}^2}$$

$$= \int_0^1\frac{dx}{\sqrt{x}(3\sqrt{x}+4)^2}$$

Put $3\sqrt{x}+4=t$

$$\therefore \frac{3}{2\sqrt{x}}dx = dt \Rightarrow \frac{dx}{\sqrt{x}}\ \frac{2}{3}dt$$

when $x=0 \Rightarrow t=4$

$x=1 \Rightarrow t=7$

$$\therefore P = \frac{2}{3}\int_4^7\frac{dt}{t^2} = \frac{2}{3}\left(-\frac{1}{t}\right)\Big|_4^7$$

$$= -\frac{2}{3}\left\{\frac{1}{7}-\frac{1}{4}\right\}$$

$$= \frac{2}{3}\left\{\frac{1}{4}-\frac{1}{7}\right\} = \frac{2}{3}\cdot\frac{3}{38} = \frac{1}{14}$$

Hence $P = \frac{1}{14}$.

**13. (c)**

Let $P = \lim_{n\to\infty}\frac{(n!)^{1/n}}{n}$

$$= \lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{1/n} = \lim_{n\to\infty}\left(\frac{1.2.3\ldots n}{n.n.n\ldots n}\right)^{1/n}$$

$$= \lim_{n\to\infty}\left(\left(\frac{1}{n}\right)\left(\frac{2}{n}\right)\left(\frac{3}{n}\right)\ldots\left(\frac{n}{n}\right)\right)^{1/n}$$

$$= \lim_{n\to\infty}\left(\prod_{r=1}^{n}\left(\frac{r}{n}\right)\right)^{1/n}$$

Taking logarithm then

$$\ln P = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{r}{n}\right)$$

$$= \int_0^1 \ln x\,dx$$

$$= (\ln x\,.\,x - x)\Big|_0^1$$

$= -1 - 0$

$= -1$

Hence $P = e^{-1} = 1/e$.

**14. (b)**

Let $I = \int_0^{\pi/2}\sin^9 x\cos^4 x\,dx$

$$= \frac{(8.6.4.2)(3.1)}{(13.11.9.7.5.3.1)}\ .\ 1 \text{ (from walli's formula)}$$

$$= \frac{128}{15015}.$$

**15. (c)**

$$\because \int_0^1 x^{r+2}dx = \frac{1}{r+3}$$

$$\therefore \lim_{n\to\infty}\sum_{r=0}^{n}{}^nC_r\,.\,\frac{1}{n^r}\,.\int_0^1 x^{r+2}dx$$

$$= \int_0^1\left(x^2.\lim_{n\to\infty}\sum_{r=0}^{n}{}^nC_r.\left(\frac{x}{n}\right)^r\right)dx$$

$$= \int_0^1\left\{x^2.\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n\right\}dx$$

$$= \int_0^1 x^2e^x dx$$

$= \{x^2(e^x) - (2x)(e^x) + 2e^x\}_0^1$ (By successive method)

$= (e-2e+2e)-(0-0+2)$

$= e - 2.$

**16. (c)**

For $0 \le x \le 1 \to e^0 \le e^{x^2} \le e^1$

$$\Rightarrow e^0(1-0) \le \int_0^1 e^{x^2}dx \le e^1(1-0)$$

$$\Rightarrow 1 \le \int_0^1 e^{x^2}dx \le e$$

**17. (a)**

Put $\log x = t \Rightarrow x = e^t$

$$\Rightarrow I_1 = \int_1^2\frac{e^t}{t}dt \xrightarrow{(P-1)} \int_1^2\frac{e^x}{x}dx = I_2$$

**18. (c)**

$$\int_{-2}^{2}\left\{p\ell n\left(\frac{1+x}{1-x}\right)+q\ell n\left(\frac{1-x}{1+x}\right)^{-2}+\gamma\right\}dx$$

$$= p\int_{-2}^{2}\ell n\left(\frac{1+x}{1-x}\right)dx - 2q\int_{-2}^{2}\ell n\left(\frac{1-x}{1+x}\right)dx + \int_{-2}^{2}\gamma\,dx$$

$= p \times 0 - 2q \times 0 + 4\gamma = 4\gamma$

**19. (b)**

$$I_{m,n} = \int_0^1 x^m (\log x)^n dx$$

$$= \left[(\log x)^n . \frac{x^{m+1}}{m+1}\right]_0^1 - \int_0^1 n(\log x)^{n-1} .$$

$$\frac{1}{x} . \frac{x^{m+1}}{m+1} dx$$

$$= 0 - \frac{n}{m+1} \int_0^1 x^m .(\log x)^{n-1} dx$$

$$= - \frac{n}{m+1} I_{m,n-1}$$

**20. (a)**

$\because$ f(x) is odd function

$$\therefore \int_{-\pi/4}^{+\pi/4} f(x)dx = 0.$$