1. (b) Here $f(0) = 0$

Since $-1 \le \sin\frac{1}{x} \le 1 \Rightarrow -|x| \le x\sin\frac{1}{x} \le |x|$

We know that $\lim_{x\to 0}|x| = 0$ and $\lim_{x\to 0} -|x| = 0$

In this way $\lim_{x\to 0} f(x) = 0$.

2. (b) $\lim_{n\to\infty}\frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}$.

3. (b) $\lim_{x\to a}\frac{\sqrt{3x-a}-\sqrt{x+a}}{x-a}$

$= \lim_{x\to a}\frac{\sqrt{3x-a}-\sqrt{x+a}}{(x-a)} \times \frac{\sqrt{3x-a}+\sqrt{x+a}}{\sqrt{3x-a}+\sqrt{x+a}}$

$= \frac{2}{2\sqrt{2a}} = \frac{1}{\sqrt{2a}}$

Aliter : Apply L-Hospital's rule

$\lim_{x\to a}\frac{\sqrt{3x-a}-\sqrt{x+a}}{x-a} = \lim_{x\to a}\frac{3}{2\sqrt{3x-a}} - \frac{1}{2\sqrt{x+a}}$

$= \frac{3}{2\sqrt{2a}} - \frac{1}{2\sqrt{2a}} = \frac{1}{\sqrt{2a}}$.

4. (b) $\lim_{x\to 2}\frac{x^n - 2^n}{x-2} = n.2^{n-1} \Rightarrow n.2^{n-1} = 80 \Rightarrow n = 5$.

5. (a) $\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)\times(\sqrt{x}+1)}{(x-1)(2x+3)\times(\sqrt{x}+1)} = \frac{-1}{5.2} = \frac{-1}{10}$.

6. (a) $\lim_{x\to 0}\frac{\log\cos x}{x} = \lim_{x\to 0}\frac{\log\left[1-2\sin^2\frac{x}{2}\right]}{x}$

$= \lim_{x\to 0}\frac{-\left[2\sin^2\frac{x}{2} + \left(\frac{2\sin^2\frac{x}{2}}{2}\right)^2 + ...\right]}{x} = 0$

Aliter : Apply L-Hospital's rule,

$\lim_{x\to 0}\frac{\log\cos x}{x} = \lim_{x\to 0}\frac{-\tan x}{1} = 0$.

7. (b) Applying L-Hospital's rule,

$\lim_{x\to 9}\frac{\frac{1}{2\sqrt{f(x)}}\cdot f'(x)}{\frac{1}{2\sqrt{x}}} = \frac{\frac{f'(9)}{\sqrt{f(9)}}}{\frac{1}{\sqrt{9}}} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$

8. (a) $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{2\sqrt{x}}$.

Aliter : Apply L-Hospital rule,

$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{1}{2\sqrt{x+h}} = \frac{1}{2\sqrt{x}}$.

9. (c) $\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx} = \lim_{x\to 0}\left\{\frac{2\sin^2\frac{mx}{2}}{2\sin^2\frac{nx}{2}}\right\}$

$= \lim_{x\to 0}\left[\left\{\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right\}^2 \frac{m^2x^2}{4}.\frac{1}{\left\{\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right\}^2}.\frac{4}{n^2x^2}\right]$

$= \frac{m^2}{n^2}\times 1 = \frac{m^2}{n^2}$.

Aliter : Apply L-Hospital's rule,

$\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx} = \lim_{x\to 0}\frac{m\sin mx}{n\sin nx} = \lim_{x\to 0}\frac{m^2\cos mx}{n^2\cos nx} = \frac{m^2}{n^2}$.

10. (c) $\lim_{x\to 0}\frac{2\times 9\sin^2 3x}{(3x)^2} = 18$

11. (a) $\lim_{\alpha\to\pi/4}\frac{\sin\alpha-\cos\alpha}{\alpha-\pi/4}$

$= \lim_{\alpha\to\pi/4}\left\{\frac{\sqrt{2}\left(\sin\alpha.\frac{1}{\sqrt{2}} - \cos\alpha.\frac{1}{\sqrt{2}}\right)}{\left(\alpha-\frac{\pi}{4}\right)}\right\}$

$= \sqrt{2}\lim_{\alpha\to\pi/4}\frac{\sin\left(\alpha-\frac{\pi}{4}\right)}{\left(\alpha-\frac{\pi}{4}\right)} = \sqrt{2}\times 1 = \sqrt{2}$.

Aliter : Apply L-Hospital's rule,

$\lim_{\alpha\to\pi/4}\frac{\sin\alpha-\cos\alpha}{\alpha-(\pi/4)} = \lim_{\alpha\to\pi/4}\frac{\cos\alpha+\sin\alpha}{1} = \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} = \sqrt{2}$.

12. (c) $\lim_{x\to a}\frac{x^2-a^2}{x-a} = \lim_{x\to a}(x+a) = 2a$.

13. (a) Apply the L-Hospital's rule, $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$.

14. (b) $\lim_{x\to 0}\frac{\cos ax - \cos bx}{x^2}$

$= \lim_{x\to 0}\frac{2\sin\left(\frac{a+b}{2}\right)x.\sin\left(\frac{b-a}{2}\right)x}{\left(\frac{a+b}{2}\right)x.\frac{2}{a+b}.\frac{2}{b-a}.\left(\frac{b-a}{2}\right)x} = \frac{b^2-a^2}{2}$

Aliter : Apply L-Hospital's rule,

$\lim_{x\to 0}\frac{\cos ax-\cos bx}{x^2} = \lim_{x\to 0}\frac{-a\sin ax + b\sin bx}{2x}$

$$= \lim_{x\to 0} \frac{-a^2\cos ax + b^2\cos bx}{2} = \frac{b^2 - a^2}{2}.$$

15. (c) $\lim_{x\to 0} \frac{x[{}^5C_1 + {}^5C_2x + {}^5C_3x^2 + {}^5C_4x^3 + {}^5C_5x^4]}{x[{}^3C_1 + {}^3C_2x + {}^3C_3x^2]} = \frac{5}{3}.$

Aliter : Apply L-Hospital's rule.

16. (a) $\lim_{x\to a} \frac{x^9 + a^9}{x + a} = 9 \Rightarrow \frac{2a^9}{2a} = 9 \Rightarrow a^8 = 9 \Rightarrow \quad a = 9^{1/8}$

17. (a) $\lim_{x\to 0+} \frac{x}{1 + e^{-1/x}} = 0$ as $e^{-1/x} \to 0$ when $x \to 0^+$

18. (c) $\lim_{x\to 1-}[x] = \lim_{h\to 0}[1-h] = \lim_{h\to 0} 0 = 0$

and $\lim_{x\to 1+}[x] = \lim_{h\to 0}[1+h] = \lim_{h\to 0} 1 = 1$

Hence limit does not exist.

19. (d) $\lim_{x\to 0} \frac{2\sin 4x\cos 2x}{2\sin x\cos 4x} = \lim_{x\to 0} 4\left(\frac{\sin 4x}{4x}\right)\left(\frac{x}{\sin x}\right)\frac{\cos 2x}{\cos 4x} = 4.$

Aliter : $\lim_{x\to 0} \frac{\frac{2\sin 2x}{2x} + \frac{6\sin 6x}{6x}}{\frac{5\sin 5x}{5x} - \frac{3\sin 3x}{3x}} = \frac{2+6}{5-3} = 4.$

20. (d) $\lim_{x\to\infty} x(a^{1/x} - 1) = \lim_{x\to\infty}\left[\frac{a^{1/x} - 1}{1/x}\right]$

$$= \lim_{x\to\infty} \frac{[e^{\log_e a^{1/x}} - 1]}{1/x} = \log_e a = -\log_e \frac{1}{a}.$$

21. (c) $\lim_{n\to\infty}\left[\frac{n(n+1)(2n+1)}{6n^3}\right] = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6} = \frac{1}{3}.$

Note : Students should remember that

$\lim_{n\to\infty} \frac{\Sigma n}{n^2} = \frac{1}{2}, \ \lim_{n\to\infty} \frac{\Sigma n^2}{n^3} = \frac{1}{3}$ and $\lim_{n\to\infty} \frac{\Sigma n^3}{n^4} = \frac{1}{4}.$

22. (d) $\lim_{x\to\alpha} \frac{\sin x - \sin\alpha}{x - \alpha}$

$\lim_{x\to\alpha} \frac{\cos x}{1} = \cos\alpha$, (Apply L-Hospital's rule)

23. (a) $\lim_{x\to 3}\left\{\frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}\right\} = \lim_{x\to 3} \frac{(x-3)\left\{\sqrt{x-2} + \sqrt{4-x}\right\}}{2(x-3)} = 1.$

Aliter : Apply *L*-Hospital's rule.

24. (b) $\lim_{x\to 0} \frac{x\cos x - \sin x}{x^2\sin x} = \lim_{x\to 0} \frac{-\sin x}{2\sin x + x\cos x}$

(By L-Hospital's rule)

$= \lim_{x\to 0} \frac{-\cos x}{3\cos x - x\sin x} = -\frac{1}{3}$, (Again by L-Hospital's rule)

25. (c) $\lim_{x\to 0} \frac{ax + bx^2 + cx^3}{x} = \lim_{x\to 0} \frac{a + bx + cx^2}{1} = a.$

26. (c) Multiply function by $\frac{(1+x)^{1/2} + (1-x)^{1/2}}{(1+x)^{1/2} + (1-x)^{1/2}}$ and solve.

Aliter : Apply L-Hospital's rule,

$$\lim_{x\to 0} \frac{(1+x)^{1/2} - (1-x)^{1/2}}{x} = \lim_{x\to 0} \frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{1-x}} = 1.$$

27. (b) $\lim_{x\to 1} \frac{1 - x^{-1/3}}{(1 - x^{-1/3})(1 + x^{-1/3})} = \frac{1}{2}.$

Aliter : Apply L-Hospital's rule.

28. (a) $\lim_{x\to 0} \frac{(1 + nx + {}^nC_2x^2 + \ldots\text{higher powers of } x \text{ to } x^n) - 1}{x} = n.$

Aliter : Apply L-Hospital's rule.

29. (c) $\lim_{\theta\to 0} \frac{2\sin^2(\theta/2)}{\theta^2} = \frac{1}{2}.$

Aliter : Apply L-Hospital's rule.

30. (a) $\lim_{\theta\to 0} \frac{5\cos\theta - \frac{2\sin\theta}{\theta}}{3 + \frac{\tan\theta}{\theta}} = \frac{5-2}{3+1} = \frac{3}{4}.$

31. (c) Apply formula of $\sin C - \sin D$,

i.e., $\lim_{x\to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = \lim_{x\to 0} \frac{2\cos 2.\sin x}{x}$

$$= 2\cos 2.\lim_{x\to 0} \frac{\sin x}{x} = 2\cos 2$$

You may also apply L-Hospital rule.

32. (c) $\lim_{x\to\infty} \frac{\left(2+\frac{1}{x}\right)^{40}\left(4-\frac{1}{x}\right)^5}{\left(2+\frac{3}{x}\right)^{45}} = \frac{2^{40}.4^5}{2^{45}} = 2^5 = 32.$

33. (b) Let $\tan^{-1} 2x = \theta \Rightarrow x = \frac{1}{2}\tan\theta$ and as $x \to 0, \theta \to 0$

$$\Rightarrow \lim_{x\to 0} \frac{x}{\tan^{-1} 2x} = \lim_{\theta\to 0} \frac{\frac{1}{2}\tan\theta}{\theta} = \frac{1}{2}.$$

34. (d) $\pi - 2x = \theta \Rightarrow x = \frac{\pi}{2} - \frac{\theta}{2}$ and as $x \to (\pi/2), \theta \to 0$

Now solve yourself.

35. (b) $\frac{m}{n}$ (formula).

36. (a) $\lim_{x\to 0}\frac{4\sin^3 x}{x^3}=4.$

37. (c) $\lim_{x\to\pi/2}\{(1-\sin x)\tan x\}=\lim_{x\to\pi/2}\frac{\sin x-\sin^2 x}{\cos x}$

Apply L-Hospital's rule, we get

$\lim_{x\to\pi/2}\frac{\cos x-\sin 2x}{-\sin x}=0$.

38. (a) It is a fundamental concept.

39. (b) Given limit $=\lim_{n\to\infty}(4^n+5^n)^{1/n}$

$$=\lim_{n\to\infty}5\left[\left\{1+\left(\frac{4}{5}\right)^n\right\}^{(5/4)^n}\right]^{(1/n).(4/5)^n}=5.e^0=5\ .$$

$$\left(\because\left(\frac{4}{5}\right)^n\to 0\text{ as }n\to\infty\right)$$

40. (a) $\lim_{x\to\infty}\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}$

$$=\lim_{x\to\infty}\frac{ax}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}$$

$$=\lim_{x\to\infty}\frac{a}{\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}}=\frac{a}{2a}=\frac{1}{2}.$$

41. (a) $\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}=\lim_{x\to 0}\frac{e^x[e^{\tan x-x}-1]}{\tan x-x}$

$$=\lim_{x\to 0}e^x.\lim_{x\to 0}\frac{e^{\tan x-x}-1}{\tan x-x}=e^0\times 1=1.$$

42. (b) Put $\cos^{-1}x=y$. So if $x\to -1,\ y\to\pi$

$$\therefore\ \lim_{x\to -1}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}=\lim_{y\to\pi}\frac{\sqrt{\pi}-\sqrt{y}}{\sqrt{1+\cos y}}$$

$$=\lim_{y\to\pi}\frac{\sqrt{\pi}-\sqrt{y}}{\sqrt{2}\cos(y/2)}=\lim_{y\to\pi}\frac{\sqrt{\pi}-\sqrt{y}}{\sqrt{2}\sin\left(\frac{\pi}{2}-\frac{y}{2}\right)}\frac{\left(\frac{\pi}{2}-\frac{y}{2}\right)}{\left(\frac{\pi}{2}-\frac{y}{2}\right)}$$

$$=\lim_{y\to\pi}\frac{1}{\frac{\sqrt{2}}{2}(\sqrt{\pi}+\sqrt{y})}.\frac{1}{\frac{\sin\left(\frac{\pi}{2}-\frac{y}{2}\right)}{\left(\frac{\pi}{2}-\frac{y}{2}\right)}}=\frac{1}{\sqrt{2\pi}}.$$

43. (c) We have $f(x)+g(x)+h(x)=\frac{x^2-4x+17-4x-2}{x^2+x-12}$

$$=\frac{x^2-8x+15}{x^2+x-12}=\frac{(x-3)(x-5)}{(x-3)(x+4)}$$

$$\therefore\ \lim_{x\to 3}[f(x)+g(x)+h(x)]=\lim_{x\to 3}\frac{(x-3)\ (x-5)}{(x-3)\ (x+4)}=-\frac{2}{7}.$$

44. (d) Let $y=\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x}$

$$\Rightarrow \log y=\lim_{x\to 0}\frac{2}{x}\log\left(\frac{a^x+b^x+c^x}{3}\right)$$

$$=2\lim_{x\to 0}\frac{\log(a^x+b^x+c^x)-\log 3}{x}$$

Now applying L-Hospital's rule, we have

$\log y=\log(abc)^{2/3}\Rightarrow y=(abc)^{2/3}$

45. (d) $\lim_{x\to\infty}\frac{(2x-3)\ (3x-4)}{(4x-5)\ (5x-6)}$

$$=\lim_{x\to\infty}\frac{x^2\left(2-\frac{3}{x}\right)\left(3-\frac{4}{x}\right)}{x^2\left(4-\frac{5}{x}\right)\left(5-\frac{6}{x}\right)}=\frac{6}{20}=\frac{3}{10}.$$

46. (d) $\lim_{x\to 2}f(x)=\lim_{x\to 2}\frac{\sin(e^{x-2}-1)}{\log(x-1)}$

$$=\lim_{t\to 0}\frac{\sin(e^t-1)}{\log(1+t)},\ \{\text{Putting } x=2+t\}$$

$$=\lim_{t\to 0}\frac{\sin(e^t-1)}{e^t-1}.\frac{e^t-1}{t}.\frac{t}{\log(1+t)}$$

$$=\lim_{t\to 0}\frac{\sin(e^t-1)}{e^t-1}.\left(\frac{1}{1!}+\frac{t}{2!}+...\right)\times\left[\frac{1}{\left(1-\frac{1}{2}t+\frac{1}{3}t^2-...\right)}\right]$$

$=1\ .\ 1\ .\ 1=1,\ \ (\because \text{As } t\to 0,\ e^t-1\to 0).$

47. (a) $\lim_{x\to 0^-}f(x)=\lim_{h\to 0}f(0-h)=\lim_{h\to 0}f(0-h)=0$

and $\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)=\lim_{h\to 0}-(0+h)=0$

$\therefore\ \lim_{x\to 0}f(x)=0,\ \left(\because \lim_{x\to 0^-}f(x)=\lim_{x\to 0^+}f(x)\right).$

48. (a) Let $y = \lim_{x\to\infty}\left(1+\frac{1}{mx}\right)^x = \lim_{x\to\infty}\left(1+\frac{1}{mx}\right)^{mx\cdot\frac{1}{m}}$

$\Rightarrow y = e^{1/m}, \quad \left(\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right)$.

49. (d) Let $y = \lim_{x\to 3}\frac{x^3 - x^2 - 18}{x-3}$, $\left(\frac{0}{0}\text{form}\right)$

Applying L-Hospital's rule, we get

$y = \lim_{x\to 3} 3x^2 - 2x = (27-6) = 21$.

50. (a) $\lim_{x\to\infty}\frac{2x^2+3x+4}{3x^2+3x+4} = \lim_{x\to\infty}\frac{2+\frac{3}{x}+\frac{4}{x^2}}{3+\frac{3}{x}+\frac{4}{x^2}} = \frac{2}{3}$.

51. (d) Here $f(2) = 0$

$\lim_{x\to 2-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} |2-h-2| = 0$

$\lim_{x\to 2+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0} |2+h-2| = 0$

Hence it is continuous at $x = 2$.

52. (b) Since limit of a function is $a+b$ as $x\to 0$, therefore to be continuous at a function, its value must be

$a+b$ at $x=0 \Rightarrow f(0) = a+b$.

53. (c) $f(0) = 0$

$\lim_{x\to 0-} f(x) = \lim_{h\to 0} e^{-1/h} = 0$ and $\lim_{x\to 0+} f(x) = \lim_{h\to 0} e^{1/h} = \infty$

Hence function is discontinuous at $x = 0$.

54. (c) $f(x) = \left\{\frac{x^2-4x+3}{x^2-1}\right\}$, for $x \ne 1$

$= 2$, for $x = 1$

$f(1) = 2,\ f(1+) = \lim_{x\to 1+}\frac{x^2-4x+3}{x^2-1} = \lim_{x\to 1+}\frac{(x-3)}{(x+1)} = -1$

$f(1-) = \lim_{x\to 1-}\frac{x^2-4x+3}{x^2-1} = -1 \Rightarrow f(1) \ne f(1-)$

Hence the function is discontinuous at $x = 1$.

55. (c) $f(0+) = f(0-) = 2$ and $f(0) = 2$

Hence $f(x)$ is continuous at $x = 0$.

56. (c) $\lim_{x\to 0^+} f(x) = x^2 \sin\frac{1}{x}$, but $-1 \le \sin\frac{1}{x} \le 1$ and $x \to 0$

Therefore, $\lim_{x\to 0^+} f(x) = 0 = \lim_{x\to 0^-} f(x) = f(0)$

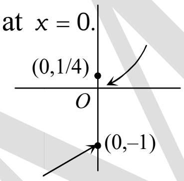
Hence $f(x)$ is continuous at $x = 0$.

57. (b) $\lim_{x\to 0-} f(x) = \lim_{x\to 0}\left[(1+2x)^{1/2x}\right]^2 = e^2$.

58. (d) $\lim_{x\to 0+} f(x) = \lim_{h\to 0} 2^{1/h} = \infty$

$\lim_{x\to 0-} f(x) = \lim_{h\to 0} 2^{-1/h} = \lim_{h\to 0}\frac{1}{2^{1/h}} = 0$.

59. (c) Clearly from curve drawn of the given function $f(x)$ is discontinuous at $x = 0$.



60. (a) It is obvious.

61. (b) $\lim_{x\to 1} f(x) = \lim_{x\to 1} x+1 = 2 = k$.

62. (b) (i) When $0 \le x < 1$

$f(x)$ doesn't exist as $[x] = 0$ here.

(ii) Also $\lim_{x\to 1+} f(x)$ and $\lim_{x\to 1-} f(x)$ does not exist.

Hence $f(x)$ is discontinuous at all integers and also in

(0, 1).

63. (b) $\lim_{x\to 0} f(x) = \frac{\sin^2 ax}{(ax)^2}a^2 = a^2$ and $f(0) = 1$.

Hence $f(x)$ is discontinuous at $x = 0$, when $a \ne 0$.

64. (b) $\lim_{x\to 0-} f(x) = 0$

$f(0) = 0,\ \lim_{x\to 0+} f(x) = -4$

$f(x)$ discontinuous at $x = 0$.

and $\lim_{x\to 1-} f(x) = 1$ and $\lim_{x\to 1+} f(x) = 1,\ f(1) = 1$

Hence $f(x)$ is continuous at $x = 1$.

Also $\lim_{x\to 2-} f(x) = 4(2)^2 - 3.2 = 10$

$f(2) = 10$ and $\lim_{x\to 2+} f(x) = 3(2)+4 = 10$

Hence $f(x)$ is continuous at $x = 2$.

65. (c) $\lim_{x\to 1+} f(x) = 0$ and $\lim_{x\to 1-} f(x) = 1+1 = 2$.

Hence $f(x)$ is discontinuous at $x = 1$.

66. (d) $\lim_{x\to 1-} f(x) = -2$ and $f(-1) = -2$.

67. (b) Obviously $\lim_{x\to b} f(x) = f(b) = 0$.

68. (b) $\lim_{x\to a-} f(x) = -1, \lim_{x\to a+} f(x) = 1, f(a) = 1$.

69. (a) $\lim_{x\to 2-} f(x) = 3, \lim_{x\to 2+} f(x) = 3$ and $f(2) = 3$.

70. (a) $\lim_{x\to \pi/2-} f(x) = \frac{\pi}{2}, \lim_{x\to \pi/2+} f(x) = \frac{-\pi}{2}$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$.

71. (a) $\lim_{x\to 0-} f(x) = \lim_{x\to 0-}\left(\frac{2\sin^2 2x}{(2x)^2}\right)4 = 8$

$\lim_{x\to 0+} f(x) = \lim_{x\to 0+}\sqrt{16+\sqrt{x}+4} = 8$. Hence $a = 8$.

72. (b) $\lim_{x\to 0-} f(x) = 1+1 = 2, \lim_{x\to 0+} f(x) = 0, f(0) = 2$.

73. (b) $\lim_{x\to 1-} f(x) = 1, \lim_{x\to 1+} f(x) = 6$.

74. (d) $\lim_{x\to 4-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0}\frac{4-h-4}{|4-h-4|} + a$

$= \lim_{h\to 0} -\frac{h}{h} + a = a-1$.

$= \lim_{x\to 4+} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0}\frac{4+h-4}{|4+h-4|} + b = b+1$

and $f(4) = a+b$

Since $f(x)$ is continuous at $x = 4$

Therefore $\lim_{x\to 4-} f(x) = f(4) = \lim_{x\to 4+} f(x)$

$\Rightarrow a-1 = a+b = b+1 \Rightarrow b = -1$ and $a = 1$.

75. (c) Given function is continuous at all point in $(-\infty, 6)$ and at $x = 1, x = 3$ function is continuous.

If function $f(x)$ is continuous at $x = 1$, then

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \Rightarrow 1 + \sin\frac{\pi}{2} = a+b$

$\therefore\ a+b = 2$ .....(i)

If at $x = 3$, function is continuous, then

$\lim_{x\to 3^-} f(3) = \lim_{x\to 3^+} f(x) \Rightarrow 3a+b = 6\tan\frac{3\pi}{12}$

$\therefore\ 3a+b = 6$ .....(ii)

From (i) and (ii), $a = 2, b = 0$.

76. (b) $f(\pi/2) = 3$. Since $f(x)$ is continuous at $x = \pi/2$

$\Rightarrow \lim_{x\to \pi/2}\left(\frac{k\cos x}{\pi - 2x}\right) = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$.

77. (a) For continuous $\lim_{x\to 2} f(x) = f(2) = k$

$\Rightarrow k = \lim_{x\to 2}\frac{x^3 + x^2 - 16x + 20}{(x-2)^2}$

$= \lim_{x\to 2}\frac{(x^2-4x+4)\ (x+5)}{(x-2)^2} = 7$.

78. (b) $f(a) = 0$

$\lim_{x\to a-} f(x) = \lim_{x\to a-}\left(\frac{x^2}{a} - a\right) = \lim_{h\to 0}\left\{\frac{(a-h)^2}{a} - a\right\} = 0$

and $\lim_{x\to a+} f(x) = \lim_{h\to 0}\left\{a - \frac{(a+h)^2}{a}\right\} = 0$

Hence it is continuous at $x = a$.

79. (b) $\lim_{x\to 2} f(x) = \lim_{x\to 2}(x+2)\ (x^2+4) = 32,\ f(2) = 16$.

80. (d) For any $x \neq 1, 2$ we find that $f(x)$ is the quotient of two polynomials and a polynomial is everywhere continuous. Therefore $f(x)$ is continuous for all $x \neq 1, 2$. Check continuity at $x = 1, 2$.

81. (b) $f(0+0) = \lim_{h\to 0} f(x) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0}(0+h)\frac{e^{1/0+h} - e^{-1/0+h}}{e^{1/0+h} + e^{-1/0+h}} = \lim_{h\to 0} h\frac{e^{1/h} - e^{-1/h}}{e^{1/h} + e^{-1/h}} = 0$

and $f(0-0) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} -h\frac{e^{-1/h} - e^{1/h}}{e^{-1/h} + e^{1/h}} = 0$

and $f(0) = 0$; $\therefore\ f(0+0) = f(0-0) = f(0)$

Hence f is continuous at $x = 0$.

At remaining points $f(x)$ is obviously continuous.

Thus it is everywhere continuous.

Again, $Lf'(0) = \lim_{h\to 0}\frac{f(0-h) - f(0)}{-h}$

$= \lim_{h\to 0}\frac{h.\frac{e^{-1/h} - e^{1/h}}{e^{-1/h} + e^{1/h}} - 0}{-h} = -1$

$Rf'(0) = \lim_{h\to 0}\frac{f(0+h) - f(0)}{h} = \lim_{h\to 0}\frac{h\frac{e^{1/h} - e^{-1/h}}{e^{1/h} + e^{-1/h}}}{h} = 1$

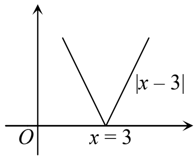
$\because\ L\ f'(0) \neq R\,f'(0)$

$\therefore f$ is not differentiable at $x = 0$.

82. (d) $\lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0} |3-h-3| = 0$

$\lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0} |3+h-3| = 0$

$\because \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$



Hence *f* is continuous at $x = 3$

Now $Lf'(3) = \lim_{h\to 0} \frac{f(3-h)-f(3)}{-h}$

$= \lim_{h\to 0} \frac{|3-h-3|-0}{-h} = \lim_{h\to 0} \frac{h}{-h} = -1$

$Rf'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{|3+h-3|-0}{h} = 1$

$\because Lf'(3) \neq Rf'(3)$. Hence f is not differentiable at $x = 3$.

Trick : Can be seen by graph it is continuous but tangent is not defined at $x = 3$.

83. (a,c,d) $x \le x^2 \Rightarrow x(1-x) \le 0 \Rightarrow x(x-1) \ge 0$

$\Rightarrow x \le 0$ or $x \ge 1$; $\therefore h(x) = \begin{cases} x & : \quad x \le 0 \\ x^2 & : \quad 0 < x < 1 \\ x & : \quad x \ge 1 \end{cases}$

$h(x)$ is continuous for every x but not differentiable at $x = 0$ and 1. Also

$h'(x) = \begin{cases} 1 & x < 0 \\ \text{not exists} & x = 0 \\ 2x & 0 < x < 1 \\ \text{not exists} & x = 1 \\ 1 & x > 1 \end{cases}$

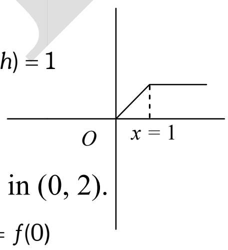
$\therefore h'(x) = 1$ for all $x > 1$.

84. (d) It is obvious.

85. (a) $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 1, & 1 < x \le 2 \end{cases}$

$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} (1-h) = 1$

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = 1$



Hence function is continuous in (0, 2).

Now $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} (0+h) = 0 = f(0)$

$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} (2-h) = 1 = f(2)$

Hence function is continuous in [0, 2]

Clearly, from graph it is not differentiable at $x = 1$.

86. (a) Since this function is continuous at $x = 0$

Now for differentiability

$f(x) = |x| = |0| = 0$ and $f(0+h) = f(h) = |h|$

$\therefore \lim_{h\to 0-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0-} \frac{|h|}{h} = -1$

and $\lim_{h\to 0+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0+} \frac{|h|}{h} = 1$.

Therefore it is continuous and non-differentiable.

87. (c) $\lim_{h\to 0^-} 1+(2-h) = 3$, $\lim_{h\to 0^+} 5-(2+h) = 3$, $f(2) = 3$

Hence, *f* is continuous at $x = 2$

Now $Rf'(x) = \lim_{h\to 0} \frac{5-(2+h)-3}{h} = -1$

$Lf'(x) = \lim_{h\to 0} \frac{1+(2-h)-3}{-h} = 1$

$\because Rf'(x) \neq Lf'(x)$; $\therefore$ *f* is not differentiable at $x = 2$.

88. (a) $f'(k-0) = \lim_{h\to 0} \frac{[k-h]\sin \pi(k-h) - [k]\sin \pi k}{-h}$

$= \lim_{h\to 0} \frac{(-1)^{k-1}(k-1)\sin \pi h - k \times 0}{-h}$

$= \lim_{h\to 0} \frac{(-1)^{k-1}(k-1)\sin \pi h}{-h} = (-1)^k.(k-1)\pi$.

89. (d) $Rf'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$= \lim_{h\to 0} \frac{2(2+h)-1-(4-1)}{h} = \lim_{h\to 0} \frac{4+2h-1-3}{h} = 2$

and $Lf'(2) = \lim_{h\to 0} \frac{f(2-h)-f(2)}{-h} = \lim_{h\to 0} \frac{2-h+1-3}{-h} = 1$.

Thus $f'(2)$ does not exist.

90. (c) $\because f$ is continuous at $x = 0$, $\therefore$ $f(0^-) = f(0^+) = f(0) = -1$

Also $Lf'(0) = Rf'(0)$

$\Rightarrow \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$\Rightarrow \lim_{h\to 0} \left(\frac{e^{-2h}-1+1}{-h} \right) = \lim_{h\to 0} \left(\frac{ah + \frac{bh^2}{2} - 1 + 1}{h} \right)$

$\Rightarrow \lim_{h\to 0} \left(\frac{-2e^{-2h}}{-1} \right) = \lim_{h\to 0} \left(a + \frac{bh}{2} \right)$

$\Rightarrow 2 = a + 0 \Rightarrow a = 2, b$ any number.

91. (d) $\lim_{x\to 0} f(x) = x^2 \sin\left(\frac{1}{x}\right)$, but $-1 \le \sin\left(\frac{1}{x}\right) \le 1$ and $x \to 0$

$\therefore \lim_{x\to 0^+} f(x) = 0 = \lim_{x\to 0^-} f(x) = f(0)$

Therefore $f(x)$ is continuous at $x=0$. Also, the function $f(x)=x^2\sin\frac{1}{x}$ is differentiable because

$Rf'(x)=\lim_{h\to 0}\frac{h^2\sin\frac{1}{h}-0}{h}=0$, $Lf'(x)=\lim_{h\to 0}\frac{h^2\sin(1/-h)}{-h}=0$.

92. (d) $Lf'(1)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$

$=\lim_{h\to 0}\frac{m(1-h)^2-m}{-h}=\lim_{h\to 0}\frac{m[1+h^2-2h-1]}{-h}$

$=\lim_{h\to 0}m(2-h)=2m$ and $Rf'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$=\lim_{h\to 0}\frac{2(1+h)-m}{h}$. For differentiability, $Lf'(1)=Rf'(1)$.

But for any value of m, $Rf'(1)=Lf'(1)$ not possible.

93. (c) $(gof)(x)=g[f(x)]=g[1-\cos x]=e^{1-\cos x}$, for $x\le 0$

$(gof)'(x)=e^{1-\cos x}.\sin x$, for $x\le 0$

$(gof)'(0)=0$.

94. (a) $f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$; As function is differentiable so it is continous as it is given that $\lim_{h\to 0}\frac{f(1+h)}{h}=5$ and hence $f(1)=0$. Hence $f'(1)=\lim_{h\to 0}\frac{f(1+h)}{h}=5$.

95. (d) $\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right|\le\lim_{x\to y}|x-y|$ or $|f'(x)|\le 0$

$\Rightarrow f'(x)=0\Rightarrow f(x)$ is constant, As $f(0)=0$

$\therefore f(1)=0$.

96. (b) $f(1)=-3$; $f'(x)\ge 9$ for all $x\in(1,5)$; $\therefore f(5)\ge 36$.

97. (c) $f(x)=1+\sin(3x)g(x)$

$f'(x)=3\cos 3x\,g(x)+\sin 3x\,g'(x)=f(x)\cos 3x$.

98. (d) If $f(x)=\begin{cases}x+2, & -1<x<3\\ 5, & x=3\\ 8-x, & x>3\end{cases}$ and $f(3)=5$

L.H.D $=\lim_{x\to 3^-}\frac{f(x)-f(3)}{x-3}=\lim_{h\to 0}\frac{f(3-h)-f(3)}{-h}$

$=\lim_{h\to 0}\frac{(3-h+2)-5}{-h}=\lim_{h\to 0}\frac{-h}{-h}=1$

R.H.D $=\lim_{x\to 3^+}\frac{f(x)-f(3)}{x-3}=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$

$=\lim_{h\to 0}\frac{8-(3+h)-5}{h}=\lim_{h\to 0}\frac{-h}{h}=-1$

L.H.D $\ne$ R.H.D $f(x)$ is not differentiable.

99. (c) $f(x)$ possesses derivative at $x=0$, so it is both continuous and differentiable at $x=0$. Now $f(0+0)=0$, $f(0-0)=b, f(0)=b$, $\therefore b=0$

Also $Rf'(0)=0, Lf'(0)=0, \forall a\in R$

$\therefore f'(0)=0$ if $b=0$.

100. (c) Since the function is defined for $x\ge 0$ i.e. not defined for $x<0$. Hence the function neither continuous nor differentiable at $x=0$.