1. **(c)**

Let $x=5, y=0 \Rightarrow f(5+0)=f(5).f(0)$

$\Rightarrow f(5)=f(5)f(0) \Rightarrow f(0)=1$

Therefore, $f'(5)=\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$

$=\lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}=\lim_{h\to 0}2\left[\frac{f(h)-1}{h}\right]\ \{\because f(5)=2\}$

$=2\lim_{h\to 0}\left[\frac{f(h)-f(0)}{h}\right]=2\times f'(0)=2\times 3=6$.

2. **(b)**

$\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$. We add and subtract $g(a)f(a)$ in numerator

$=\lim_{x\to a}\frac{g(x)f(a)-g(a)f(a)+g(a)f(a)-g(a)f(x)}{x-a}$

$=\lim_{x\to a}f(a)\left[\frac{g(x)-g(a)}{x-a}\right]-\lim_{x\to a}g(a)\left[\frac{f(x)-f(a)}{x-a}\right]$

$=f(a)\lim_{x\to a}\left[\frac{g(x)-g(a)}{x-a}\right]-g(a)\lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}\right]$

$=f(a)g'(a)-g(a)f'(a)$ [by using first principle formula]

= 3.4 - (-1)(-2) = 12 - 2 = 10

Trick : $\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$

Using L-Hospital's rule, Limit = $\lim_{x\to a}\frac{g'(x)f(a)-g(a)f'(x)}{1}$;

Limit = $g'(a)\ f(a)-g(a)f'(a)=(4)(3)-(-1)(-2)$ = 12 - 2 = 10.

3. **(a)**

$f(x)=\frac{2\sin x.\cos x.\cos 2x.\cos 4x.\cos 8x.\cos 16x}{2\sin x}=\frac{\sin 32x}{2^5\sin x}$

$\therefore\ f'(x)=\frac{1}{32}.\frac{32\cos 32x.\sin x-\cos x.\sin 32x}{\sin^2 x}$

$\therefore\ f'\left(\frac{\pi}{4}\right)=\frac{32.\frac{1}{\sqrt2}-\frac{1}{\sqrt2}.0}{32.\left(\frac{1}{\sqrt2}\right)^2}=\sqrt2$.

4. **(b)**

$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a[\cos\theta-\theta(-\sin\theta)-\cos\theta]}{a[-\sin\theta+\theta\cos\theta+\sin\theta]}=\frac{\theta\sin\theta}{\theta\cos\theta}=\tan\theta$.

5. **(d)**

Given $f(x)=\begin{vmatrix}x^3 & \sin x & \cos x\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$, 2nd and 3rd rows are constant, so only 1st row will take part in differentiation

$\therefore\ \frac{d^3}{dx^3}f(x)=\begin{vmatrix}\frac{d^3}{dx^3}x^3 & \frac{d^3}{dx^3}\sin x & \frac{d^3}{dx^3}\cos x\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$

We know that $\frac{d^n}{dx^n}x^n=n!, \frac{d^n}{dx^n}\sin x=\sin(x+\frac{n\pi}{2})$

and $\frac{d^n}{dx^n}\cos x=\cos(x+\frac{n\pi}{2})$

Using these results,

$\frac{d^3}{dx^3}f(x)=\begin{vmatrix}3! & \sin\left(x+\frac{3\pi}{2}\right) & \cos\left(x+\frac{3\pi}{2}\right)\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}$

$\frac{d^3}{dx^3}f(x)\Big|_{\text{at }x=0}=\begin{vmatrix}6 & -1 & 0\\ 6 & -1 & 0\\ 1 & p^2 & p^3\end{vmatrix}=0$ *i.e.*, independent of *p*.

6. **(b)**

$x=\int_0^y\frac{1}{\sqrt{1+4t^2}}dt \Rightarrow \frac{dx}{dy}=\frac{1}{\sqrt{1+4y^2}} \Rightarrow \frac{dy}{dx}=\sqrt{1+4y^2}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{4y}{\sqrt{1+4y^2}}\frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2}=\frac{4y}{\sqrt{1+4y^2}}.\sqrt{1+4y^2}=4y$

7. **(d)**

Acceleration in *x*-direction = $\frac{d^2x}{dt^2}=-2c$ and acceleration in *y*-direction = $\frac{d^2y}{dt^2}=2b$

Resultant acceleration is = $\sqrt{(-2c)^2+(2b)^2}=2\sqrt{b^2+c^2}$

8. **(c)**

$\because$ Surface area $s=4\pi r^2$ and $\frac{dr}{dt}=2$

$\therefore\ \frac{ds}{dt}=4\pi\times 2r\frac{dr}{dt}=8\pi r\times 2=16\pi r \Rightarrow \frac{ds}{dt}\propto r$.

9. **(d)**

$y^3+3x^2=12y$

$\Rightarrow 3y^2.\frac{dy}{dx}+6x=12.\frac{dy}{dx} \Rightarrow \frac{dy}{dx}(3y^2-12)+6x=0$

$\Rightarrow \frac{dy}{dx}=\frac{6x}{12-3y^2} \Rightarrow \frac{dx}{dy}=\frac{12-3y^2}{6x}$

Tangent is parallel to *y*-axis, $\frac{dx}{dy}=0 \Rightarrow 12-3y^2=0$ or $y=\pm 2$. Then $x=\pm\frac{4}{\sqrt3}$, for $y=2$

$y=-2$ does not satisfy the equation of the curve,

$\therefore$ The point is $\left(\pm\frac{4}{\sqrt3},2\right)$

10. **(b)**

$f(x)=\left(\frac{1}{x}\right)^x \Rightarrow f'(x)=\left(\frac{1}{x}\right)^x\left(\log\frac{1}{x}-1\right)$

$f'(x)=0 \Rightarrow \log\frac{1}{x}=1=\log e \Rightarrow \frac{1}{x}=e \Rightarrow x=\frac{1}{e}$.

Therefore, maximum value of function is $e^{1/e}$.

11. **(c)**

Let $x+y=4$ or $y=4-x$

$\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$ or $f(x)=\frac{4}{xy}=\frac{4}{x(4-x)}$

$f(x)=\frac{4}{4x-x^2}$, $f'(x)=\frac{-4}{(4x-x^2)^2}.(4-2x)$

Put $f'(x)=0 \Rightarrow 4-2x=0 \Rightarrow x=2$ and $y=2$

$\therefore$ min. $\left(\frac{1}{x}+\frac{1}{y}\right)=\frac{1}{2}+\frac{1}{2}=1$.

12. (c)

$2x+2y=100 \Rightarrow x+y=50$ .....(i)

Let area of rectangle is A, $\therefore A=xy \Rightarrow y=\frac{A}{x}$

From (i), $x+\frac{A}{x}=50 \Rightarrow A=50x-x^2 \Rightarrow \frac{dA}{dx}=50-2x$

for maximum area $\frac{dA}{dx}=0$

$\therefore 50-2x=0 \Rightarrow x=25$ and $y=25$

$\therefore$ adjacent sides are 25 *cm* and 25 *cm*.

13. (b)

$f(x)=x\sin x+\cos x+\cos^2 x$ $\therefore$

$f'(x)=\sin x+x\cos x-\sin x-2\cos x\sin x = \cos x(x-2\sin x)$

Hence $x\to 0$ to π, then $f'(x)\le 0$, *i.e.,* $f(x)$ is decreasing function.

14. (b)

$f(x)=\sin^4 x+\cos^4 x = (\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$

$=1-\frac{4\sin^2 x\cos^2 x}{2}=1-\frac{\sin^2 2x}{2}=1-\frac{1}{4}(2\sin^2 2x)$

$=1-\left(\frac{1-\cos 4x}{4}\right)=\frac{3}{4}+\frac{1}{4}\cos 4x$

Hence function $f(x)$ is increasing when $f'(x)>0$

$f'(x)=-\sin 4x>0 \Rightarrow \sin 4x<0$

Hence $\pi<4x<\frac{3\pi}{2}$ or $\frac{\pi}{4}<x<\frac{3\pi}{8}$.

15. (a)

Given that equation of curve $y=x^3=f(x)$

So $f(2)=8$ and $f(-2)=-8$

Now $f'(x)=3x^2 \Rightarrow f'(x)=\frac{f(2)-f(-2)}{2-(-2)}$

$\Rightarrow \frac{8-(-8)}{4}=3x^2; \therefore x=\pm\frac{2}{\sqrt{3}}$.

16. (a)

Slope of C_1 is sinx and for x > 0 slope of C_2 is $\frac{\sqrt{3}}{2}$. Thus for $x=\frac{\sqrt{3}}{2} \Rightarrow x=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

Hence point of contact is $\left(\frac{\pi}{3},\frac{1}{2}\right)$ or $\left(\frac{2\pi}{3},\frac{3}{2}\right)$

For $\left(\frac{\pi}{3},\frac{1}{3}\right)$ we get $a=\frac{1}{2}-\frac{\pi}{2\sqrt{3}}$

For $\left(\frac{2\pi}{3},\frac{3}{2}\right)$ we get $a=\frac{3}{2}-\frac{\pi}{\sqrt{3}}$

17. (c)

$x\sin\alpha\cos y+x\cos\alpha\sin y=\sin y$

$x\sin\alpha\cot y+x\cos\alpha=1$

$\cot y=\frac{1-x\cos\alpha}{x\sin\alpha}$

$\tan y=\frac{x\sin\alpha}{1-x\cos\alpha}$

$\sec^2 y\,.\,\frac{dy}{dx}$

$=\frac{\sin\alpha(1-x\cos\alpha)-x\sin\alpha(-\cos\alpha)}{(1-x\cos\alpha)^2}$ $\sec^2 y\,.\,\frac{dy}{dx}$

$=\frac{\sin\alpha-x\sin\alpha\cos\alpha+x\sin\alpha\cos\alpha}{(1-x\cos\alpha)^2}$

$\sec^2 y\frac{dy}{dx}=\frac{\sin\alpha}{(1-x\cos\alpha)^2}$

$m=\sin\alpha$ $n=-\cos\alpha$

$m^2+n^2=1$

18. (a)

$y=\tan^{-1}\sqrt{\frac{(\sin x/2-\cos x/2)^2}{(\sin x/2+\cos x/2)^2}}$

$=\tan^{-1}\left|\frac{\sin x/2-\cos x/2}{\sin x/2+\cos x/2}\right|$

$x=\pi/6$, $\sin x/2-\cos x/2<0$

$\therefore \left|\frac{\sin x/2-\cos x/2}{\sin x/2+\cos x/2}\right|=-\frac{\sin x/2-\cos x/2}{\sin x/2+\cos x/2}$

$=\frac{1-\tan x/2}{1+\tan x/2}$

$=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$y=\frac{\pi}{4}-\frac{x}{2} \Rightarrow \frac{dy}{dx}=-\frac{1}{2}$

19. (d)

Let $f(x)=ax^2+bx+c$

$f(1)=f(-1) \Rightarrow a+b+c=a-b+c \Rightarrow 2b=0$

$\Rightarrow b=0$

$\therefore f(x)=ax^2+c$ $\therefore f'(x)=2ax$

$\therefore f'(a)=2a^2$, $f'(b)=2ab$, $f'(c)=2ac$

Since a, b, c are in A.P.,

$\therefore f'(a), f'(b), f'(c)$ are also in A.P.

20. (a)

$\because \frac{dy}{dx}=0 \Rightarrow y+x^2y=x+c$

$\frac{dy}{dx}+x^2\frac{dy}{dx}+y(2x)=1$; $\therefore xy=½$

21. (c)

Here, $\frac{dy}{dx}=0$ at $\left(x=\frac{\pi}{3}, y=\frac{3\sqrt{3}}{2}\right)$

$\Rightarrow$ Tangent at $x = \frac{\pi}{3}$ is parallel to x-axis

$\Rightarrow$ Equation of tangent is, $y = \frac{3\sqrt{3}}{2}$

Also equation of normal is, $x = \frac{\pi}{3}$

Area of quadrilateral $= \frac{\pi}{3} \cdot \frac{3\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{2}$ sq. unit.

Hence, (C) is the correct answer.

22. (a)

Equation $3ax^2 + 2bx + c$

$f(x) = 3a\frac{x^3}{3} + 2\frac{bx^2}{2} + cx$

$f(0) = 0$

& $f(1) = a + b + c = 0$

$\Rightarrow f(0) = f(1) \Rightarrow (0, 1)$ are roots of f(x)

having atleast one root of f'(x)

23. (a)

$h(x) = f(x) + f(-x) \Rightarrow h'(x) = f'(x) - f'(-x)$

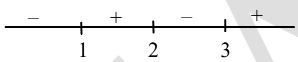
$\Rightarrow$ If h(x) has extreme point $\Rightarrow h'(x) = 0$

$\Rightarrow f'(x) = f'(-x)$

$\Rightarrow$ f '(x)is even function.

24. (c)

$f'(x) = x.(e^x - 1)(x-1)(x-2)^3(x-3)^5$



so at x = 2

25. (b)

Let $y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$\Rightarrow y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right\} = \sin^2 \cot^{-1}\left(\tan\frac{\theta}{2}\right)$

$\Rightarrow y = \sin^2\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos^2\frac{\theta}{2} = \frac{1}{2}(1+\cos\theta) = \frac{1}{2}(1+x)$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

26. (d)

$y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots \text{ to } \infty}}} \Rightarrow y = \sqrt{x+y} \Rightarrow y^2 = x + y$

$\Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx}(2y-1) = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y-1}$

27. (b)

$y = x^{x^{x\ldots\ldots\infty}} \Rightarrow y = x^y \Rightarrow \log_e y = y\log_e x$

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{y}{x} + \log_e x\frac{dy}{dx} \Rightarrow \left(\frac{1}{y} - \log_e x\right)\frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow x(1 - y\log_e x)\frac{dy}{dx} = y^2$

28. (a)

$y = x^2 + \frac{1}{y} \Rightarrow y^2 = x^2y + 1 \Rightarrow 2y\frac{dy}{dx} = y.2x + x^2\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2xy}{2y - x^2}$

29. (c)

Differentiating the given equation *w.r.t. x*, $2y\frac{dy}{dx} = 4$ at point

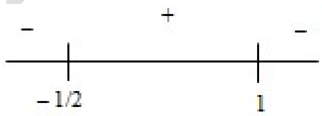
(2, 4) $\frac{dy}{dx} = \frac{1}{2}$

$$P = \frac{y_1 - x_1\left(\frac{dy}{dx}\right)}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} = \frac{4 - 2\left(\frac{1}{2}\right)}{\sqrt{1 + \frac{1}{4}}} = \frac{6}{\sqrt{5}}.$$

30. (a)

$f'(x) = e^{x(1-x)} + x.e^{x(1-x)}.(1-2x)$

$= e^{x(1-x)}\{1 + x(1-2x)\} = e^{x(1-x)}.(-2x^2 + x + 1)$ Now by the sign-scheme for $-2x^2 + x + 1$



$f'(x) \geq 0$, if $x \in \left[-\frac{1}{2}, 1\right]$, because $e^{x(1-x)}$ is always positive.

So, $f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$.

31. (b)

For $xy = -2$, $\frac{dy}{dx} = \frac{2}{x^2}$ which is always positive.

Thus $\frac{-a}{b} > 0$ or signs of 'a' and 'b' must be opposite.

32. (a)

Clearly A, B $\in \left(0, \frac{\pi}{3}\right)$. For y = sec x, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are positive in $\left(0, \frac{\pi}{3}\right)$. Hence

$\sec A + \sec B \geq 2\sec\left(\frac{A+B}{2}\right) = 2\sec\frac{\pi}{6} = \frac{4}{\sqrt{3}}$

33. (d)

$f'(x) = 3x^2 + 2ax + b = 3(x - x_1)(x - x_2)$ where $x_1 < x_2$.

clearly $f'(x) < 0\ \forall\ x \in (x_1, x_2)$ and $f'(x) > 0\ \forall\ x \in (-\infty, x_1) \cup$

(x_2, ∞). Thus $x = x_1$ is the point of local maxima and $x = x_2$ is the point of local minima.

Thus bigger root of $f'(x) = 0$ must be negative. Hence $a^2 - 3b > 0$, $a > 0$, $b > 0$

34. (c)

In this case PA must be normal to the given curve. For $x^2 = 4y$, $\frac{dy}{dx} = \frac{x}{2}$. Thus equation of normal at $P(x_1,y_1)$ is ;

$\left(y - \frac{x_1^2}{4}\right) = -\frac{2}{x_1}(x - x_1)$.

It must pass through (0, 4)

$\Rightarrow \left(4 - \frac{x_1^2}{4}\right) = \frac{2}{x_1}\, x_1 = 2$

$\Rightarrow x_1 = \pm\sqrt{8}$. Apart from this y-axis is also a normal to the curve passing through A(0, 4) and corresponding $x_1 = 0$.

If $x_1 = 0 \Rightarrow P(0, 0) \Rightarrow PA = 4$

If $x_1 = \pm\sqrt{8} \Rightarrow P(\pm\sqrt{8}, 2)$

$\Rightarrow PA = \sqrt{8+4} = \sqrt{12}$

Thus $p \equiv (\pm\sqrt{8}, 2)$

35. (c)

$f''(x) > 0\ \forall\ x \in R \Rightarrow f'(x)$ is increasing $\forall\ x \in R$.

$g'(x) = -f'(2 - x) + f'(4 + x)$. If $g'(x) > 0$

$\Rightarrow f'(4 + x) > f'(2 - x)$

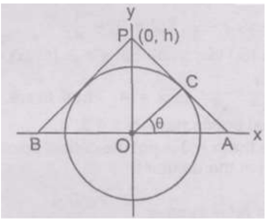
$\Rightarrow 4 + x > 2 - x$

$\Rightarrow x > -1$

36. (d)

Let $\angle COA = \theta \Rightarrow OA = OC.\sec\theta = 4\sec\theta$ Also $\angle OPC = \theta$

$\Rightarrow OP = OC.\text{cosec}\theta = 4\text{cosec}\theta$



Now $\Delta_{PAB} = OA.OP = \frac{32}{\sin 2\theta}$

For Δ_{PAB} to be minimum $\sin 2\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\Rightarrow P = (0, 4\sqrt{2})$

37. (d)

Solving $y^2 = 4ax$ and $x^2 = 4by$ we get $x = 0$ or $x^3 = 64ab^2$. Slope of the curves at the common points are $\frac{2a}{y}$ and $\frac{x}{2b}$ respectively. If these parabola intersect orthogonally then $\frac{2a}{y}.\frac{x}{2b} = -1$

$\Rightarrow ax + by = 0 \Rightarrow ax + \frac{x^2}{4} = 0$

$\Rightarrow x = -4a$ (as $x \neq 0$)

$\Rightarrow -x^3 = 64a^3$

$\Rightarrow 64ab^2 + 64a^3 = 0$

$\Rightarrow a^2 + b^2 = 0$

which is not possible.

38. (c)

Equation of tangent $\frac{x}{5}\cos\phi + \frac{y}{4}\sin\phi = 1$

Length of AB = $\sqrt{\frac{25}{\cos^2\phi} + \frac{16}{\sin^2\phi}} = \sqrt{t}$

$t = \frac{25}{\cos^2\phi} + \frac{16}{\sin^2\phi} = 25\sec^2\phi + 16\text{cosec}^2\phi$

$\frac{dt}{d\phi} = 50\sec^2\phi.\tan\phi - 32\text{cosec}^2\phi\cot\phi = 0$

$\tan^4\phi = \frac{32}{50} \Rightarrow \tan^2\phi = \frac{4}{5}$

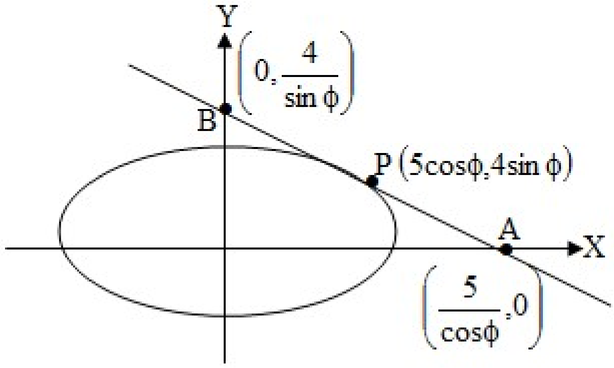
so $t = 25\sec^2\phi + 16\text{cosec}^2\phi$

$= 25.\frac{9}{5} + 16.\frac{9}{4} = 45 + 36$

$\therefore\ t = 81 \quad \sqrt{t} = \sqrt{81} = 9$

so minimum length intercepted = '9'

so 'C' is correct.



39. (c)

Area of trapezium ABCD

$S = \frac{1}{2}[6 + (6 + 2 \times 6\cos\theta)] \times 6\sin\theta$

$= 36(\sin\theta + \sin\theta\cos\theta)$

$= 36\sin\theta + 18\sin 2\theta$

$\frac{ds}{d\theta} = 36\cos\theta + 36\cos 2\theta = 0$

$= 36(\cos\theta + \cos 2\theta) = 0$

$\Rightarrow \cos\theta + 2\cos^2\theta - 1 = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$\cos\theta = -1$ or $\cos\theta = \frac{1}{2}$

$\theta = \pi$ or $\frac{\pi}{3}$ $\qquad \theta \neq \pi$

S_{max} at $\theta = \frac{\pi}{3}$

$$36\sin\frac{\pi}{3} + 18\sin\frac{2\pi}{3}$$

$$36 \text{ x } \frac{\sqrt{3}}{2} + 18.\frac{\sqrt{3}}{2} = 27\sqrt{3} \text{ sq.inch.}$$

So 'C' is correct

40. (a)

Differentiating partially w.r.t. x

$g'(x) = g(y)[g'(x - y)]$

Put y = x

$g'(x) = g(x) . g'(0) = a . g(x)$

$\Rightarrow g(x) = ae^x \qquad (\because g(0) = 1)$

Now $g'(x) = ae^x$, $g'(3) = ae^3$

and $g'(-3) = ae^{-3} = \frac{a^2}{b}$

41. (b)

$$f(x) = \sqrt{(x-1)+4-4\sqrt{x-1}} + \sqrt{(x-1)+9-6\sqrt{x-1}}$$

$$= \left|\sqrt{x-1}-2\right| + \left|\sqrt{x-1}-3\right|$$

at x = 1.5

$$f(x) = \left(2-\sqrt{x-1}\right) + \left(3-\sqrt{x-1}\right)$$

$$= 5 - 2\sqrt{x-1}$$

$$f'(x) = \frac{-2}{2\sqrt{x-1}}$$

$$\therefore f'(1.5) = \frac{-1}{\sqrt{\frac{3}{2}-1}} = -\sqrt{2}$$

42. (d)

$$f(x) = \frac{\ell n(\ell n(x))}{\ell n(x)}$$

$$f'(x) = \frac{\frac{1}{\ell nx}\times\frac{1}{x}\times\ell n(x) - \ell n(\ell n(x))\times 1/x}{[\ell n(x)]^2}$$

$$f'(x) = \frac{\frac{1}{e}-0}{1} = 1/e$$

43. (b)

$g(x) = f(e^x) . e^{f(x)}$

$g'(x) = f'(e^x).e^x . e^{f(x)} + f(e^x). e^{f(x)}.f'(x)$

$g'(0) = 2$

44. (b)

$f(x) = [\tan^2 x] = 0$ for $-\pi/4 < x < \pi/4$. Thus $\lim_{x\to 0} f(x)$ exists and the value is 0. Moreover, it is continuous at x = 0. Being a constant function f is differentiable at x = 0 and $f'(0) = 0$

45. (a)

$$y_n = \sin\left(\frac{n\pi}{2}+x\right) + \cos\left(\frac{n\pi}{2}+x\right)$$

$y_{n+2} = - y_n$ similarly $y_{n+5} + y_{n+3} = 0$

and $y_{n+8} + y_{n+6} = 0$

46. (b)

$\because 2 < x < 3$

$\therefore [x] = 2$

then $f(x) = \sin\left\{\frac{2\pi}{3} - x^2\right\}$

$$f'(x) = \cos\left(\frac{2\pi}{3}-x^2\right)(-2x)$$

$$\therefore f'\left(\sqrt{\frac{\pi}{3}}\right) = \cos\left(\frac{2\pi}{3}-\frac{\pi}{3}\right)\left(-2\sqrt{\frac{\pi}{3}}\right)$$

$$= \frac{1}{2}.\left(-2\sqrt{\frac{\pi}{3}}\right)$$

$$= -\sqrt{\frac{\pi}{3}}.$$

47. (a)

$f(x)\ \sqrt{x-1}+\sqrt{x+24-10\sqrt{x-1}} \Rightarrow \sqrt{x-1}+\sqrt{(5-\sqrt{x-1})^2}$

$\sqrt{x-1}+|5-\sqrt{x-1}|, 1 < x < 26$

$\Rightarrow \sqrt{x-1}+(5-\sqrt{x-1}) \Rightarrow dy/dx = 0$

48. (b)

$$\frac{dy}{dx} = \frac{3x^2}{x^3} + \frac{3}{\sqrt{1-x^2}} + k(2x)$$

$$\Rightarrow \frac{dy}{dx}_{\left(x=\frac{1}{2}\right)} = 6 + \frac{6}{\sqrt{3}} + k$$

Now $\frac{dy}{dx}_{\left(x=\frac{1}{2}\right)} = 2\sqrt{3}$ given

$\therefore 6 + (6/\sqrt{3}) + k = 2\sqrt{3} \Rightarrow k = -6$

49. (b)

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{2^x}{1+2^{(x+1)}}\right)^2} \times \frac{(1+2^{x+1}).2^x\log 2 - 2^x.2^{x+1}\log 2}{(1+2^{x+1})^2}$$

$$\frac{dy}{dx}_{(x=0)} = \frac{1}{10}\log_e 2$$

50. (a)

$f(x) = \log_x(\log_e x) = \dfrac{\log_e(\log_e x)}{\log_e x}$

$f'(x) = \dfrac{(\log_e x)\left(\dfrac{1}{\log_e x}.\dfrac{1}{x}\right) - \dfrac{1}{x}.\log(\log x)}{(\log_e x)^2}$

$\Rightarrow f'(e) = \dfrac{1-\log 1}{e.(1)^2} = 1/e$

51. (c)

$f(x) = \text{Max.}\ \{x + 1, |x| + 1\}$

$= |x| + 1\ \forall\ x \in R$

which is non differentiable

at $x = 0$

52. (b)

$y = \tan^{-1}\left(\dfrac{2^x}{1+2^{x+1}}\right)$

$\dfrac{dy}{dx} = \dfrac{1}{1+\left(\dfrac{2^x}{1+2^{x+1}}\right)^2} \times \dfrac{(1+2^{x+1})2^x \log 2 - 2^x.2^{x+1}\log 2}{(1+2^{x+1})^2}$ r = 0

$= \dfrac{1}{1+\left(\dfrac{1}{3}\right)^2} . \dfrac{3.\log 2 - 1.2\log 2}{9}$

$= \dfrac{1}{10} . \dfrac{(\log 2^3 - \log 2^2)}{1} = \dfrac{1}{10}\log 2$

53. (a)

$\dfrac{dy}{dx} = f'\left(\dfrac{3x+4}{5x+6}\right)\left[\dfrac{(5x+6)3-(3x+4)5}{(5x+6)^2}\right]$

$\tan\left(\dfrac{3x+4}{5x+6}\right)^2 . \left[\dfrac{-2}{(5x+6)^2}\right]$

54. (c)

$\because g(x) = f^{1}(x) \Rightarrow f(g(x)) = x$ differentiable

$\Rightarrow f'(g(x)) . g'(x) = 1$

$\Rightarrow g'(x) = \dfrac{1}{f'(g(x))}$ ....(1)

Now from curve

$f'(x) = 1 + \sec^2 x$

$f'(g(x)) = 1 + \sec^2 g(x) = 2 + \tan^2 g(x)$ ...(2)

Now from curve

$f(g(x)) = g(x) + \tan g(x)$

$x = g(x) + \tan g(x) \Rightarrow \tan g(x) = x - g(x)$ ...(3)

from (1), (2), (3)

$g'(x) = \dfrac{1}{2+[x-g(x)]^2}$

55. (a)

$y = (1 + x)(1 + x^2)(1 + x^4)....(1 + x^{2^n})$

Multiplying numerator and denominator by (1-x)

$\Rightarrow y = \dfrac{(1-x)(1+x)(1+x^2)....(1+x^{2^n})}{(1-x)}$

$\Rightarrow y = \left(\dfrac{1-x^{2^{n+1}}}{(1-x)}\right)$

$\dfrac{dy}{dx} = \dfrac{(1-x).\{-2^{n+1}.x^{2^{n+1}-1}\}-(1-x^{2^{n+1}})(-1)}{(1-x)^2}$

So $\left.\dfrac{dy}{dx}\right|_{x=0} = \dfrac{-2^{n+1}.0.1+1-0}{1^2} = 1$

56. (a)

$y = \cos^2\dfrac{\pi x}{180} = -\dfrac{\pi}{180}\sin\dfrac{2\pi x}{180} = -\dfrac{\pi}{180}\sin 2x^0$

57. (d)

$by^2 = (x + a)^3$

differentiating both sides

$2by\dfrac{dy}{dx} = 3(x + a)^2.1$

$\dfrac{dy}{dx} = \dfrac{3}{2}\dfrac{(x+a)^2}{by}$

length of subnormal $= SN = y\dfrac{dy}{dx} = \dfrac{3}{2}\dfrac{(x+a)^2}{b}$

and length of subtangent $= ST = y\dfrac{dx}{dy} = \dfrac{2by^2}{3(x+a)^2}$

$\dfrac{p}{q} = \dfrac{(ST)^2}{(SN)}$ (given)

$= \dfrac{(2by^2)^2.2b}{\{3(x+a)^2\}^2.3(x+a)^2}$

$= \dfrac{8b}{27}\dfrac{\{(x+a)^3\}^2}{(x+a)^6}$ {using $by^2 = (x + a)^3$

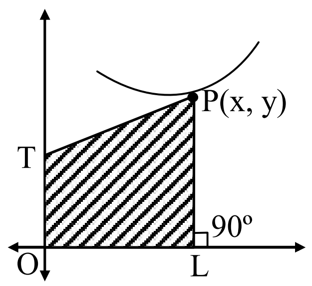
$= \dfrac{8b}{27}$

$\dfrac{p}{q} = \dfrac{8b}{27}$

58. (a)

length of intercept on y-axis by any tangent at

$P(x, y) = OT = y - x\dfrac{dy}{dx}$



Area of trapezium OLPTO $= \dfrac{1}{2}(PL + OT)\,OL$

$= \frac{1}{2}\left(y + y - x\frac{dy}{dx}\right)x$

$= \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x$

Now Area of trapezium OLPTO = $\frac{1}{2}x^2$

$\frac{1}{2}\left(2y - x\frac{dy}{dx}\right) = \pm\frac{1}{2}x^2$

$2y - x\frac{dy}{dx} = \pm x$

$\frac{dy}{dx} - \frac{2y}{x} = \pm 1$

Solve diff. equation

$y = \pm x + cx^2$

59. (a)

TM is sub-tangent where T $\equiv \left(x - y\frac{dx}{dy}, 0\right)$ and M $\equiv$ (x, 0)

$\frac{1}{2}\left(x - y\frac{dx}{dy} + x\right) = 0$

$\Rightarrow 2x - y\frac{dx}{dy} = 0$ or $2\int\frac{dy}{y} = \int\frac{dx}{x}$

$2\ell ny = \ell ncx$

$y^2 = cx$

curve passes through (1, 1)

$\therefore c = 1, y^2 = x$

60. (b)

Using LMVT

$f'(c) = \frac{f(x) - f(0)}{x - 0} \le \frac{1}{2}$

$\Rightarrow f(x) \le \frac{x}{2} \quad \forall x \in (0, 2)$

61. (d)

Put $\frac{dx}{dy} = 0$

62. (b)

Clearly x = 8 satisfies the given equation. Assume that $f(x) = e^{x-8} + 2x - 17 = 0$ has a real root α other than x = 8. We may suppose that $\alpha > 8$ (the case for $\alpha < 8$ is exactly similar). Applying Rolle's theorem on $[8, \alpha]$, we get $\beta \in (8, \alpha)$, such that $f'(\beta) = 0$. But $f'(\beta) = e^{\beta - 8} + 2$, So that $e^{\beta - 8} = -2$ which is not possible. Hence there is no real root other than 8.

63. (a)

Let r, l and h denotes respectively the radius, slant height and height of the cone at any time t. Then,

$l^2 = r^2 + h^2$

$\Rightarrow 2l\frac{dl}{dt} = 2r\frac{dr}{dt} + 2h\frac{dh}{dt}$

$\Rightarrow l\frac{dl}{dt} = r\frac{dr}{dt} + h\frac{dh}{dt}$

$\Rightarrow l\frac{dl}{dt} = 7 \times 3 + 24 \times (-4) \quad \left[\because \frac{dh}{dt} = -4 \text{ and } \frac{dr}{dt} = 3\right]$

$\Rightarrow l\frac{dl}{dt} = -75$

Where r = 7 and h = 24, we have

$l^2 = 7^2 + 24^2$

$[\because t^2 = r^2 + h^2]$

$\Rightarrow l = 25$

$l\frac{dl}{dt} = -75 \Rightarrow \frac{dl}{dt} = -3$

Let S denote the lateral surface area. Then,

$\Rightarrow \frac{dS}{dt} = \pi\left\{\frac{drl}{dt}\right\} = \pi\left\{\frac{dr}{dt}l + r\frac{dl}{dt}\right\} = \pi\{3 \times 25 + 7 \times (-3)\}$

$= 54\pi$ cm^2/min.

Hence (A) is the correct answer.

64. (d)

Let α, β be two different roots of $f(x) = 0$ in [0, 1] where $f(x) = x^3 - 3x + a$. Therefore, by Rolle's theorem $f'(x) = 0$ has a root between α and β, i.e. in (0, 1). But, $f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow x = \pm 1$

$\therefore f'(x) = 0$ has no roots in (0, 1)

Hence the given equation has no root lying between 0 and 1 for any value of a.

65. (c)

Consider the function $\phi(x) = f(x) - g(x)$. On the interval $[x_0, x]$. Then, $\phi(x)$ satisfies all the conditions of Lagrange's mean value theorem on $[x_0, x]$.

$\therefore$ There exists at least one $c \in (x_0, x)$ such that

$\phi(x) - \phi(x_0) = \phi'(c)(x - x_0)$

$\Rightarrow \phi(x) = \phi'(c)(x - x_0)$ ($\because \phi(x_0) = 0$) ... (1)

Also, $\phi'(x) = f'(x) - g'(x) \Rightarrow \phi'(c) = f'(c) - g'(c) > 0$

($\because f'(x) > g'(x)$ for $x > x_0$)

$\therefore$ from (1), $\phi(x) > 0$, for $x > x_0$

$\Rightarrow f(x) - g(x) > 0$ for $x > x_0$

or $f(x) > g(x)$ for $x > x_0$.

66. (c)

The intersection of the two curves is given by

$9x^2 + 6bx - 16 = 0$ (i)

Differentiating $y^2 = 6x$, we have $\frac{dy}{dx} = \frac{3}{y}$.

Differentiating $9x^2 + by^2 = 16$, we have $\frac{dy}{dx} = -\frac{9x}{by}$.

For curves to intersect at right angles, we must have at the points of intersection $\frac{3}{y}\left(\frac{-9x}{by}\right) = -1 \Rightarrow 27x = by^2$. Thus we must have

$9x^2 + by^2 = 16 \Rightarrow 9x^2 + 27x - 16 = 0$ (ii)

(i) and (ii) must be identical so $27 = 6b \Rightarrow b = 9/2$.

67. (c)

$\frac{dy}{dx} = 3x^2 - 2ax + 1 \geq 0;$

$\because A = 3 > 0 \Rightarrow D \leq 0 \Rightarrow 4a^2 - 12 \leq 0 \Rightarrow -\sqrt{3} \leq a \leq \sqrt{3}$

68. (b)

$f(x) = \log \sin x \to$ continuous & differentiable

$f(\pi/6) = \log \sin(\pi/6) = \log(1/2)$

$f(5\pi/6) = \log \sin(5\pi/6) = \log(1/2)$

$f'(x) = \cot x$

now

$f'(x) = \frac{f(5\pi/6) - f(\pi/6)}{5\pi/6 - \pi/6}$

$\Rightarrow \cot x = 0$

$\Rightarrow x = \pi/2 \in (\pi/6, 5\pi/6)$

69. (c)

$\frac{dr}{dt} = 6$ cm/sec

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi \times 10 \times 6$

$= 120\pi$ cm^2/sec

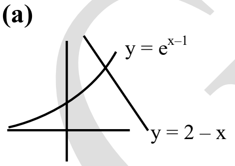
70. (c)

$ax + by + c = 0$ is

Normal to $xy = 1$

compare the slopes of Normal

71. (a)



$e^{x-1} = 2 - x$

72. (b)

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3}$

at $x = 2 \Rightarrow 2 = t^2 + 3t - 8 \Rightarrow t^2 + 3t - 10 = 0$

$= -5, 2$

at $y = -1 \Rightarrow -1 = 2t^2 - 2t - 5 \Rightarrow 2t^2 - 2t - 4 = 0$

$\Rightarrow t = 2, -1$

$\therefore$ put $t = 2$

$\therefore \frac{dy}{dx} = \frac{6}{7}$

73. (b)

According to Rolle's theorem between two roots of $f(x) = 0$ atleast one root of $f'(x) = 0$ lies.

74. (d)

at $x = \pi/2 \Rightarrow y = \pi/2$ $\therefore$ φ0 ποιντ $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

Now $\frac{dy}{dx} = 1 + \cos^2 x - \sin^2 x$

$\left(\frac{dy}{dx}\right)_{\left(\frac{\pi}{2}, \frac{\pi}{2}\right)} = 1 + 0 - 1 = 0$

$\Rightarrow y - \frac{\pi}{2} = -\frac{1}{0}\left(x - \frac{\pi}{2}\right) \Rightarrow 2x = \pi$

75. (a)

Let $x_1, x_2 \in R$ such that $x_1 < x_2$. Then, $x_1 < x_2$

$\Rightarrow f(x_1) < f(x_2)$ [$\because$ f is an increasing function]

$\Rightarrow g(f(x_1)) < g(f(x_2))$ [$\because$ g is an increasing function]

$\Rightarrow gof(x_1) < gof(x_2)$

Hence gof is an increasing function.

76. (b)

$f(x) = \log(1 + x) - x$; $f'(x) = \frac{1}{1+x} - 1 = \frac{-x}{1+x}$

If $x > 0 \Rightarrow f'(x) < 0 \Rightarrow f(x)$ is $\downarrow$

If $-1 < x \leq 0 \Rightarrow f'(x) \geq 0 \Rightarrow f\uparrow$

$\because x \leq 0 \Rightarrow f(x) \leq f(0)$ $(f(0) = 0)$

$\Rightarrow f(x) \leq 0 \Rightarrow \log(1 + x) \leq x$

77. (a)

$f'(x) = 3x^2 + 12x + p < 0$

$\because$ f(x) is decreasing in (- 3, - 1)

$\Rightarrow f'(x) \leq 0$ in (- 3, - 1)

$\Rightarrow 3 \times (-3)^2 + 12 \times (-3) + p \leq 0$ and

$3 \times (-1)^2 + 12 \times (-1) + p \leq 0 \Rightarrow p \leq 9 \Rightarrow p = 9$

78. (d)

$\because$ f(x) is decreasing function

$\therefore f'(x) \leq 0$

$\Rightarrow 3(a + 2)x^2 - 6ax + 9a \leq 0$

$\Rightarrow (a + 2)x^2 - 2ax + 3a \leq 0$

$\Rightarrow a + 2 < 0$ and $D \leq 0$

$\Rightarrow a < -2$ and $4a^2 - 12a(a + 2) \leq 0$

$a < -2$ and $-8a^2 - 24a \leq 0$

$\Rightarrow a < -2$ and $a(a + 3) \geq 0$

$a < -2$ and $a \leq -3$ or $a \geq 0$

$\Rightarrow a \leq -3 \Rightarrow -\infty < a \leq -3$

79. (b)

Time taken by the truck = $\frac{300}{x}$ hours

petrol consumed = $\left(2+\frac{x^2}{600}\right)\frac{300}{x}$ litre

$\therefore$ expenses on traveling

$E = 200 \times \frac{300}{x} + \left(2+\frac{x^2}{600}\right)\frac{3000}{x}$

$= \frac{60000}{x} + \frac{6000}{x} + 5x$

$= \frac{66000}{x} + 5x$

$\therefore \frac{dE}{dx} = -\frac{66000}{x^2} + 5 < 0$ for all $x \in [30, 60]$

$\therefore$ most economical speed is 60 kmph

80. (b)

Let $f(x) = \frac{x}{x^2+122}$ at $x = \sqrt{122}$ $f'(x) = 0$

$f(11) = \frac{11}{243}$, $f(12) = \frac{6}{133}$

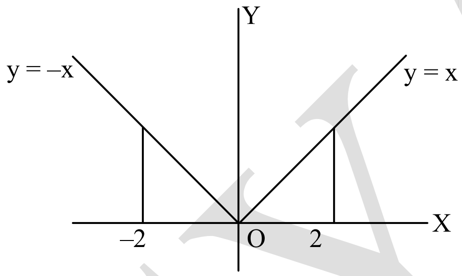
81. (d)

Let us redefine the function

x (+ ive) $\therefore$ $|x| = x$ for $0 < x < 2$

x (- ive) $\therefore$ $|x| = -x$ for $0 < -x < 2$

or $-2 < x < 0$



$\therefore f(x) = \begin{cases} -x, & -2 < x < 0 \\ 0, & x = 0 \\ x, & 0 < x < 2 \end{cases}$

$\frac{dy}{dx}$ does not exist at L.H.D. = -1

and R.H.D. = 1 at x = 0.

82. (a)

Let the increase in fixed charges be Rs. X per subscriber.

So, final income P = (500 - x) (300 + x)

$= 150000 + 200x - x^2$ …(1)

For maximum or minimum of P

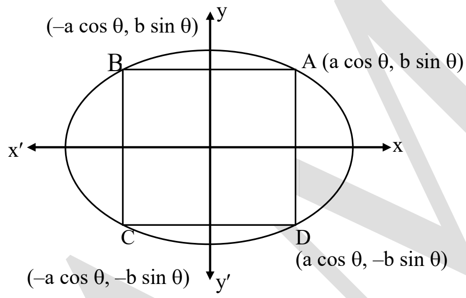
$\frac{dP}{dx} = (200 - 2x) = 0 \Rightarrow 200 - 2x = 0 \quad \Rightarrow x = 100.$

$\left.\frac{dP}{dx}\right|_{x=100}$ = negative $\therefore$ total income will be max if the increase in fixed charges in Rs. 100/-

83. (d)

Let the co-ordinates of the vertices of rectangle ABCD are A(a cos θ, b sin θ), B(-a cos θ, b sin θ), C (-a cos θ, - b sin θ) and D (a cos θ, - b sin θ), then length of rectangle, AB = 2 cos θ and breadth of rectangle, AD = 2b sin θ.

$\therefore$ Area of rectangle = AB × AD



$\Rightarrow$ Area of rectangle = 2a cos θ . 2b sin θ

$\Rightarrow$ Area of rectangle = 2 ab sin 2θ

$\frac{dA}{d\theta} = 2 \times 2\,ab \cos 2\theta$

Put $\frac{dA}{d\theta} = 0$, for maxima or minima

$\frac{dA}{d\theta} = 0$

$\Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

$\frac{d^2A}{d\theta^2} = -8ab \sin 2\theta$

Now, $\left(\frac{d^2A}{d\theta^2}\right) < 0 \Rightarrow \theta = \frac{\pi}{4}$

$\therefore$ Area is maximum at $\theta = \frac{\pi}{4}$

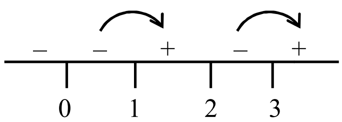
$\Rightarrow$ Maximum area of rectangle = 2ab.

[(from (i))].

84. (d)

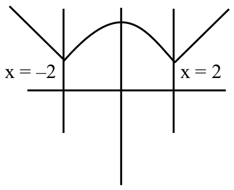
$f'(x) = x.(e^x - 1)(x - 1)(x - 2)^3 (x - 5)^5 = 0$

$\Rightarrow x = 0, 1, 2, 3,$



Min. at x = 1, 3

85. (a)



at $x = \pm 2$ $|6 - x^2| = |x|$

minima at $x = \pm 2$ $f(2) = 2$

86. (a)

$f(x) = x^3 + bx^2 + cx + d$

$\frac{dy}{dx} = 3x^2 + 2bx + c$

$D = 4b^2 - 4 \times 3 \times c = 4(b^2 - 3c)$, $b^2 < c = -$ ve

87. (c)

A point on the x-axis, (x, 0, 0)

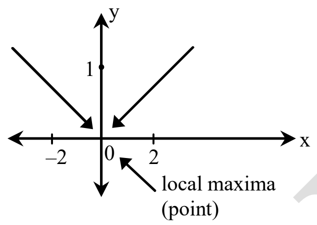
distance $d = \sqrt{(x-a)^2 + b^2 + c^2}$

$\Rightarrow d^2 = (x - a)^2 + b^2 + c^2$

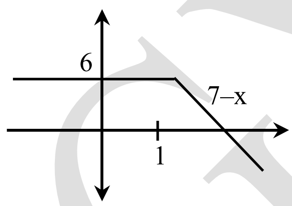
$\frac{d(d^2)}{dx} = 2(x - a) = 0 \Rightarrow x = a$

so $d_{min} = \sqrt{b^2 + c^2}$

88. (a)



89. (c)



From the graph, x = 1 is neither max. nor minimum.

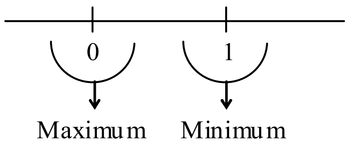
90. (a)

$f'(x) = x(e^x - 1)(x - 1)^9 (x + 2)^3 (x + 4)^4 \log(x + 1) .1$

↓ ↓ ↓ ↓ ↓

0 0 1 -2 0

($\because$ x = -2 is not possible)



91. (a)

$f(x) = x^x$

$f'(x) = x^x(1 + \ell nx) = 0 \Rightarrow x = \frac{1}{e}$

$\therefore a = \frac{1}{e}$ & $b = e^{-1/e}$

92. (b)

$f(x) - (x)^{a^2-b^2} \times (x)^{b^2-c^2} \times (x)^{c^2-a^2}$

$f(x) = (x)^{a^2-b^2+b^2-c^2+c^2-a^2} = (x)^0 = 1$

$f'(x) = 0$

93. (b)

If $y = \tan\left[\frac{1}{2} \times \tan^{-1} U + \frac{1}{2} \times 2\tan^{-1} U\right]$

$= \tan(2\tan^{-1} U) = \tan\left[\tan^{-1}\frac{2U}{1-U^2}\right]$

So $y = \frac{2U}{1-U^2} = x$ than $\frac{dy}{dx} = 1$

94. (c)

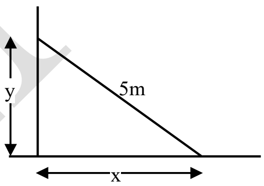
$\frac{dy}{dx} = 2e^{2x}$; $\left(\frac{dy}{dx}\right)_{0,1} = 2$

equation of tangent y-1= 2x at x-axis

y = 0; x = -1/2

95. (b)

At time t



$x^2 + y^2 = 5^2$

$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

Given $\frac{dx}{dt}$, x, y, we have to find $\frac{dy}{dt}$

96. (a)

x + 1 > x - 1

If f is increasing

f(x + 1) > f(x - 1)

g is decreasing g(f(x + 1)) < g(f(x - 1))

x + 1 > x - 1

g is decreasing g(x + 1) < g(x - 1)

f is increasing f(g (x + 1)) < f(g(x - 1))

97. (b)

Here, $f(x) = \sin^4 x + \cos^4 x$

$f'(x) = 4\sin^3 x \cdot \cos x + 4\cos^3 x(-\sin x)$

$f'(x) = 4\sin x \cos x(\sin^2 x - \cos^2 x)$

$f'(x) = 2(\sin 2x)(-\cos 2x)$

$f'(x) = -\sin 4x$

Now, $f'(x) \geq 0$ if $\sin 4x \leq 0$

$\Rightarrow \pi \leq 4x \leq 2\pi$

$\Rightarrow \pi/4 \leq x \leq \pi/2$

Here (b) is only subset of $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Therefore, (B) is the solution.

98. (d)

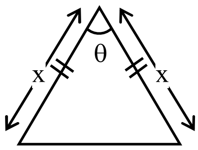
$3 \leq \frac{3f^2 f^1}{f^6 + 1}$ integrating from 0 to a

$\Rightarrow 3a \leq \tan^{-1}\sqrt{3} - \tan^{-1}1 = \frac{\pi}{12} \Rightarrow a \leq \frac{\pi}{36}$

99. (b)

Observing options (A), (C), (D) are rejected only (B) is accepted.

100.(c)



Area = $\frac{1}{2} \times x \times x \sin\theta$

Area = $\frac{x^2}{2} \sin\theta$

$\rightarrow (\text{Area})_{max}$ when $(\sin\theta)_{max} = 1$

$\Rightarrow$ max. area = $\frac{x^2}{2} \times 1 = \frac{x^2}{2}$