1. (c)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & \lambda & 3 \end{vmatrix} = 5 - \lambda$$

For unique solution, $\lambda \neq 5$,

Now, $\Delta_1 = -\mu + 9$, $\Delta_2 = -\mu + 9$, $\Delta_3 = 2(\mu - 9)$

$\therefore$ If $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0 \Rightarrow$ infinite solution

i.e. $\lambda = 5, \mu = 9$

2. (c)

When a = b or b = c or c = a, the determinant reduces to zero. It is not necessary that a = b = c for determinant to be zero. Therefore triangle is isosceles

3. (b)

$$\frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2$$

4. (d)

Put a = i, b = i, c = 0

$$f(x) = \begin{vmatrix} 1-x & 0 & x \\ 0 & 1-x & x \\ 0 & 0 & 1 \end{vmatrix} = (1-x)^2$$

degree = 2

5. (a)

$$\Delta = -\frac{1}{2}\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix}$$

By $C_1 \to C_1 + C_2 + C_3$

$$= -\frac{1}{2}\begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

6. (b)

$3(bc)(ca)(ab) - [(ab)^3 + (bc)^3 + (ca)^3] = 0$

$\Rightarrow (ab + bc + ca)[(ab)^2 + (bc)^2 + (ca)^2 - ab^2c - bc^2a - ca^2b] = 0$

$\Rightarrow ba + cb + ca = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

7. (d)

given that $C = 2 - \cos\theta$

$$\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix} = C(C^2 - 1) - 1(C - 6)$$

$\Delta = 2\cos\theta(4\cos^2\theta - 1) - (2\cos\theta - 6)$

$\Delta = 4\cos^3\theta - 4\cos\theta + 6$

8. (a)

Apply operation

$C_2 = C_2 + C_3$

$$= \begin{vmatrix} 1 & x+y+z & y+z \\ 1 & x+y+z & z+x \\ 1 & x+y+z & x+y \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & y+z \\ 1 & 1 & z+x \\ 1 & 1 & x+y \end{vmatrix} = 0$$

(Because two columns are identical)

9. (d)

Apply $C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} (2x-1) & (2x-3) & (x-2)^2 \\ (2x-3) & (2x-5) & (x-3)^2 \\ (2x-5) & (2x-7) & (x-4)^2 \end{vmatrix}$$ $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 2 & 2 & (2x-5) \\ 2 & 2 & (2x-7) \\ (2x-5) & (2x-7) & (x-4)^2 \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= \begin{vmatrix} 0 & 0 & 2 \\ 2 & 2 & (2x-7) \\ (2x-5) & (2x-7) & (x-4)^2 \end{vmatrix} = -8$$

$\therefore$ Value of determinant is independent of x.

$\therefore$ a = b = c = 0 and d = - 8.

10. (c)

$C_1 \to C_1 + C_2 + C_3$

$$\alpha = \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix} \Rightarrow \alpha = 0$$

11. (b)

Det. is skew symmetric so value = 0

12. (d)

All the rows are in A.P. so

$\Delta = 0$

13. (b)

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_2$

$$= \begin{vmatrix} a+b & b & b \\ a+2b & b & b \\ a+4b & b & b \end{vmatrix} = 0$$

14. (b)

$R_2 \to R_2 - R_1$

$$\begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 D & \sin C + \sin^2 C \end{vmatrix}$$ $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 B & \sin^2 B & \sin^2 C \end{vmatrix}$$

= (sin A - sin B) × (sin B - sin C) × (sin C - sin A)

$\Rightarrow$ Either A = B or B = C or C = A So Δ is isosceles

15. (b)

$$\begin{vmatrix} 5 & 4 & 3 \\ 100x+51 & 100y+41 & 100y+31 \\ x & y & z \end{vmatrix}$$

$R_1 \to R_2 - (100R_3)$

$$\Delta = \begin{vmatrix} 5 & 4 & 3 \\ 51 & 41 & 31 \\ x & y & z \end{vmatrix} \Rightarrow \Delta = 0$$

16. (a)

$$\Delta = \begin{vmatrix} f & 2d & e \\ 2z & 4x & 2y \\ e & 2a & b \end{vmatrix}$$

$$= -\begin{vmatrix} 2d & f & e \\ 4x & 2z & 2y \\ 2a & e & b \end{vmatrix}$$

$$= \begin{vmatrix} 2d & e & f \\ 4x & 2y & 2z \\ 2a & b & e \end{vmatrix} = \begin{vmatrix} 2a & b & e \\ 4x & 2y & 2z \\ 2d & e & f \end{vmatrix}$$

$$= \begin{vmatrix} 2a & b & e \\ 2d & e & f \\ 4x & 2y & 2z \end{vmatrix} = \Delta_2$$

$$\therefore \frac{\Delta_1}{\Delta_2} = 1$$

17. (a)

We have,

$$\sum_{n=1}^{k} U_n = \begin{vmatrix} \sum_{n=1}^{k} 1 & k & k \\ 2\sum_{n=1}^{k} n & k^2+k+1 & k^2+k \\ 2\sum_{n=1}^{k} n - \sum_{n=1}^{k} 1 & k^2 & k^2+k+1 \end{vmatrix}$$

$$= \begin{vmatrix} k & k & k \\ k(k+1) & k^2+k+1 & k^2+k \\ k^2 & k^2 & k^2+k+1 \end{vmatrix}$$

$$= \begin{vmatrix} k & 0 & k \\ k^2+k & 1 & k^2+k \\ k^2 & 0 & k^2+k+1 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$]

$= k(k^2 + k + 1) - k^3 = k(k + 1) = 72$ (given)

$\Rightarrow k = 8$

Hence (A) is correct answer.

18. (d)

$$\because \begin{vmatrix} 1 & \omega \\ \omega & \omega^2 \end{vmatrix} = 0, \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} = 0, \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix} = 0$$

Hence inverse does not exist.

19. (b)

We have $A' = -A$

Now $AA^{-1} = A^{-1}A = I_n$

$\Rightarrow (AA^{-1})' = (A^{-1}A)' = (I_n)'$

$\Rightarrow (A^{-1})'A' = A'(A^{-1})' = I_n$

$\Rightarrow (A^{-1})'(-A) = (-A)(A^{-1})' = I_n$ Thus, $(A^{-1})' = -(A^{-1})$ [inverse of a matrix is unique]

20. (a)

We have

$$|A| = \left(\frac{2}{x}\right)\begin{vmatrix} x & 2x^2 \\ 1/x & 2 \end{vmatrix}\begin{vmatrix} 1 & 2x^2 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & x \\ 1 & 1/x \end{vmatrix}$$

$$= \frac{2}{x}(0) + 2 - 2x^2 + \left(\frac{1}{x} - x\right) = \frac{2x(1-x^2) + 2(1-x^2)}{x}$$

$$= \frac{2(x+1)^2(1-x)}{x}$$

Now, $|A| = 0 \Rightarrow x = \pm 1$

21. (a)

$|adj\ (adj\ A)| = |A|^{(n-1)^2}$

$= (5)^{(3-1)^2} = 5^4 = 625$

22. (b)

A' = A & B' = B given

$\Rightarrow$ (AB - BA)' = (AB)' - (BA)'

= B'.A' - A'.B'

= B. A - A. B

= - (AB - BA)

$\therefore$ AB - BA is skew symmetric matrix

23. (c)

$$2A + 4B = \begin{bmatrix} 2 & 4 & 0 \\ 12 & -6 & 6 \\ -10 & 6 & 2 \end{bmatrix} \quad \ldots (1)$$

$$2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} \ldots (2)$$

On solving (1) and (2)

$$B = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix} \ \&\ A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

So Tr (A) - Tr (B) = 1 - (-1) = 2

24. (a)

$\because A^2 = I$ (given A is involutory)

So A. A = I $\Rightarrow \frac{A}{2}. 2A = I$

25. (c)

For $\theta = (2n+1)\frac{\pi}{2}$

$A=\begin{bmatrix} 0 & \mp 1 \\ \pm 1 & 0 \end{bmatrix}$ which is not symmetric for $\theta = n\pi$

$A = \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix}$ which is not skew symmetric

$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} \neq I$ so $A \neq A^{-1}$

26. (d)

$\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} = \frac{1}{2A^2+A-20}\begin{bmatrix} A & 5 \\ 4 & 2A+1 \end{bmatrix}$

So $\frac{1}{2A^2+A-20}\begin{bmatrix} A & 5 \\ 4 & 2A+1 \end{bmatrix}\begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix}$

$= \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}\frac{1}{2A^2+A-20}$

$\begin{bmatrix} A(A-5)+5(2A-2) & AB+5C \\ 4(A-5)+(2A+1)(2A-2) & 4B+C(2A+1) \end{bmatrix} = \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$

so $\frac{A^2+5A-10}{2A^2+A-20} = 14 \Rightarrow A = 3, \frac{-10}{3}$

27. (b)

(i) is false,

If $A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Thus, AB = 0 $\Rightarrow$ A = O or B = O

(iii) is false since matrix multiplication is not commutative.

(ii) is true as product AB is an identity matrix, if B is inverse of the matrix A.

28. (a)

We have

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = AA^{-1} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & x & 1 \end{bmatrix}$

$\begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & y \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & y+1 \\ 0 & 1 & 2(y+1) \\ 4(1-x) & 3(x-1) & 2+xy \end{bmatrix}$

1 - x = 0, x - 1 = 0, y + 1 = 0, y + 1 = 0, 2 + xy = 1

x = 1, y = -1.

29. (a)

Let $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ be the given skew symmetric matrix.

We have $|A| = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$

$[\because |A| = |A'|] = (-1)^3 \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = -|A|$

$\Rightarrow 2|A| = 0 \Rightarrow |A| = 0.$

30. (c)

$A(x)\,A(y) = (1-y)^{-1/2}\begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix}\begin{pmatrix} 1 & -y \\ -y & 1 \end{pmatrix}$

$= (1-x-y+xy)^{-1/2}\begin{bmatrix} 1+xy & -(x+y) \\ -(x+y) & 1+xy \end{bmatrix}$

$= \frac{1+xy}{\sqrt{1-(x+y)+(xy)}}\begin{bmatrix} 1 & \frac{-(x+y)}{1+xy} \\ \frac{-(x+y)}{1+xy} & 1 \end{bmatrix}$

$= \frac{\sqrt{1+xy}}{\sqrt{1-\frac{x+y}{1+xy}}}\begin{bmatrix} 1 & \frac{-(x+y)}{1+xy} \\ \frac{-(x+y)}{1+xy} & 1 \end{bmatrix}$

$= \sqrt{1+xy}\;A(z)$

$\Rightarrow k = \sqrt{1+xy}$.