1. (a)

Equation of line is $\frac{x}{a}+\frac{y}{a}=1$

$\Rightarrow x+y=a \Rightarrow y=-x+a$. Hence slope of the line is - 1.

2. (a)

Let the equation be $\frac{x}{a}+\frac{y}{-a}=1 \Rightarrow x-y=a$

But it passes through (-3, 2), hence $a=-3-2=-5$. Hence the equation of straight line is $x-y+5=0$.

3. (d)

Let the equation of line is $\frac{x}{a}+\frac{y}{-1-a}=1$, which passes through (4, 3). Then $\frac{4}{a}+\frac{3}{-1-a}=1 \Rightarrow a=\pm 2$

Hence equation is $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$.

4. (d)

The equation of a line passing through (2, 2) and perpendicular to $3x+y=3$ is

$y-2=\frac{1}{3}(x-2)$ or $x-3y+4=0$. Putting $x=0$ in this equation, we obtain $y=\frac{4}{3}$.

So y-intercept = $\frac{4}{3}$.

5. (a)

$p=\left|\frac{x_1-y_1-5}{\sqrt{1^2+1^2}}\right|=\left|\frac{-2-3-5}{\sqrt{1^2+1^2}}\right|=\left|\frac{-10}{\sqrt{2}}\right|=5\sqrt{2}$

6. (c)

Given lines $4x+3y=11$ and $8x+6y=15$, distance from the origin to both the lines are $\left|\frac{-11}{\sqrt{25}}\right|$ and $\left|\frac{-15}{\sqrt{100}}\right| \Rightarrow \frac{11}{5}, \frac{15}{10}$

Clearly both lines are on the same side of the origin.

Hence, distance between both the lines are, $\frac{11}{5}-\frac{15}{10}=\frac{7}{10}$.

7. (d)

Equation of line is $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow bx+ay-ab=0$

Perpendicular distance from origin to given line is

$p=\left|\frac{-ab}{\sqrt{a^2+b^2}}\right| \Rightarrow \frac{\sqrt{a^2+b^2}}{ab}=\frac{1}{p} \Rightarrow \frac{a^2+b^2}{a^2b^2}=\frac{1}{p^2}$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$

8. (a)

Let the point be $(h,0)$ then $a=\pm\frac{bh+0-ab}{\sqrt{a^2+b^2}}$

$\Rightarrow bh=\pm a\sqrt{a^2+b^2}+ab \Rightarrow h=\frac{a}{b}(b\pm\sqrt{a^2+b^2})$

Hence the point is $\left[\frac{a}{b}(b\pm\sqrt{a^2+b^2}),0\right]$

9. (b)

The set of lines is $4ax+3by+c=0$, where $a+b+c=0$

Eliminating c, we get $4ax+3by-(a+b)=0$

$\Rightarrow a(4x-1)+b(3y-1)=0$

They pass through the intersection of the lines $4x-1=0$ and $3y-1=0$ *i.e.,* $x=\frac{1}{4}, y=\frac{1}{3}$ *i.e.,* $\left(\frac{1}{4},\frac{1}{3}\right)$

10. (c)

The given equation can be written as

a(x + y -2) + b(x-y) = 0

or (x+y-2) + b/a (x-y) = 0.

This is a family of lines concurrent at x-y = 0 and x+y -2 = 0.

On solving these two equations we get (1, 1)

11. (d)

Slope of the line in the new position is b/a, since it is perpendicular to the line ax + by + c = 0 and it cuts the x-axis. Hence the required line passes through (2, 0) and its slope is b/a. The required equation is y - 0 = b/a (x-2)

Or, ay = bx - 2b or, ay - bx + 2b = 0

12. (a)

The four sides of the rhombus are

ax + by + c = 0, ax + by -c = 0, ax -by + c = 0, ax - by - c = 0 .

On solving these equations, we get the vertices as A (c/a, 0), B (0, c/b), C(-c/a, 0) and D(0, -c/b).

The length of the diagonal AC is 2c/a and that of the diagonal BD is 2c/b.

Theref ore the area of the rhombus is

$\frac{1}{2}\left(\frac{2c}{a}\right)\left(\frac{2c}{b}\right)=\frac{2c^2}{ab}$

13. (d)

Let the equation of line parallel to

3x - y -1 = 0

3x - y + k = 0 ...(1)

it passes through (1, 2)

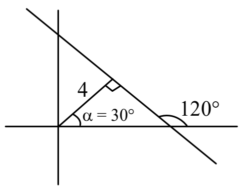
$\therefore$ 3 -2 + k = 0 $\Rightarrow$ k = -1

equation 3x - y -1 = 0

14. (c)

Let the equation

x cos α + y sin α = p



x cos 30^0 + y sin 30^0 = 4

$\frac{x\sqrt{3}}{2}+\frac{y.1}{2}=4$

$\sqrt{3}$ x + y = 8

15. (c)

Required line should be, $(3x-y+2)+\lambda(5x-2y+7)=0$ .....(i)

$\Rightarrow \quad (3+5\lambda)x-(2\lambda+1)y+(2+7\lambda)=0$

$\Rightarrow \quad y=\left(\frac{3+5\lambda}{2\lambda+1}\right)x+\frac{2+7\lambda}{2\lambda+1}$ ......(ii)

As the equation (ii) has infinite slope, $2\lambda+1=0 \Rightarrow \lambda=\frac{-1}{2}$

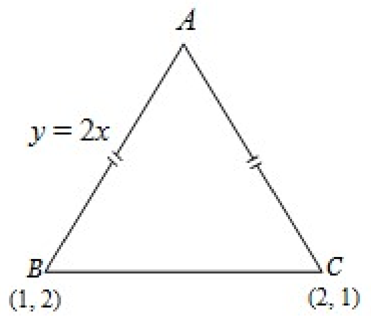
Putting $\lambda=\frac{-1}{2}$ in equation (i), We have

$(3x-y+2)+\left(\frac{-1}{2}\right)(5x-2y+7)=0 \Rightarrow \quad x=3$

16. (a)

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$; $\frac{2}{3}=\frac{2}{3}\neq\frac{7}{5}$. Hence, lines are parallel to each other.

17. (b)



Slope of $BC=\frac{1-2}{2-1}=-1$

$\because \quad AB=AC$, $\therefore \quad \angle ABC=\angle ACB$

$\Rightarrow \left|\frac{2+1}{1+2(-1)}\right|=\frac{m+1}{1+m(-1)} \Rightarrow \frac{m+1}{1-m}=|-3| \Rightarrow \frac{m+1}{1-m}=\pm 3$

$\Rightarrow m=2,\frac{1}{2}$.

But slope of AB is 2; $\therefore \quad m=\frac{1}{2}$

(Here m is the gradient of the line AC)

Equation of the line AC is $y-1=\frac{1}{2}(x-2)$

$\Rightarrow \quad x-2y=0$ or $y=\frac{x}{2}$.

18. (c)

Let the line by $lx + my + 1 = 0$. . . . (1)

According to the given condition

$\frac{2l+1+2m+1+l+m+1}{\sqrt{l^2+m^2}}=0$

$\Rightarrow l + m + 1 = 0 \Rightarrow l(1) + m(1) + 1 = 0.$

Comparing it with (1), we find that line (1) is passing through (1, 1).

Hence (C) is the correct answer.

Alternate:

The three given points are collinear and (1, 1) is the mid-point of (2, 0) and (0, 2). Hence all the lines with given property will pass through (1, 1)

19. (b)

P and Q lie on the same side if sinθ and cosθ have opposite signs which is true for the 2nd and 4th quadrant

20. (a)

P(2, -1) goes 2 units along x + y = 1 upto A and 5 units along x - 2y = 4 upto B.

Slope of PA = -1 = tan135°.

Slope of PB = 1/2 = tanθ

$\Rightarrow \sin\theta=\frac{1}{\sqrt{5}}, \ \cos\theta=\frac{2}{\sqrt{5}}$.

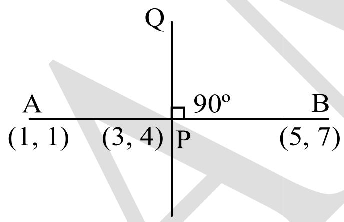
The coordinates of B

i.e. $(x_1 + r\cos\theta, \ y_1 + r\sin\theta)$ are $(2\sqrt{5} + 2, \sqrt{5} - 1)$.

The coordinates of A i.e. $(x_1 + r\cos135^0, \ y_1 + r\sin135^0)$ are $(2-\sqrt{2}, \sqrt{2}-1)$

21. (a)

$(x_1, y_1) = (3, 0)$



m = 3

equation

$y - y_1 = m(x - x_1)$

$y - 0 = 3(x - 3)$

$\Rightarrow 3x - y - 9 = 0$

22. (b)

$3x + 4y = 9$ .. (1)

and $6x + 8y = 15$

$\Rightarrow 3x + 4y = \frac{15}{2}$ ..(2)

Distance between (1) and (2)

$=\left|\frac{9-\frac{15}{2}}{\sqrt{9+16}}\right|=\frac{3}{10}$

23. (d)

Lines are concurrent

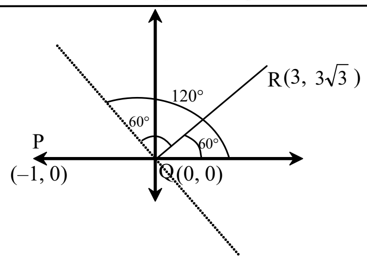
$\therefore \begin{vmatrix} 3 & -1 & -2 \\ 5 & a & -3 \\ 2 & 1 & -3 \end{vmatrix}=0$

$\Rightarrow 3(-3a+3) + 1(-15+6) - 2(5-2a) = 0$

$\Rightarrow a = -2$

24. (b)

slope of QR $=\frac{3\sqrt{3}-0}{3-0}=\sqrt{3}$



$\therefore$ equation of line

$y - 0 = \frac{-1}{\sqrt{3}}(x - 0)$

$x + \sqrt{3}\,y = 0$

25. (d)
If line is equally inclined from coordinate axes then slope will be equal to ± 1 and find y intercept if it is equidistant from both points.

26. (b)
The perpendicular distance of (1, 3) from the line $3x + 4y = 5$ is 2 units while,
$\sec^2\theta + 2\,\mathrm{cosec}^2\theta \geq 3 \quad \{\text{as } \sec^2\theta, \mathrm{cosec}^2\,\theta \geq 1\}$
Evidently, these will be two such points on the line.

27. (a)
We have, $3x + 5y = 2007$

$\Rightarrow x + \frac{5y}{3} = 669$

Clearly, 3 must divide 5y and so $y = 3k$ for some $k \in N$.
Thus, $x + 5k = 669$
$\Rightarrow 5k \leq 688$

$\Rightarrow k \leq \frac{688}{5} \quad \Rightarrow \quad k \leq 133.$

Thus, the odered pairs (x, y) can be given by
$(669 - 5k, 3k),\ 1 \leq k \leq 133.$

28. (a)
Given, $a^2 + b^2 - c^2 - 2ab = 0$
$\Rightarrow \quad (a - b)^2 - c^2 = 0$
$\Rightarrow \quad (a - b - c)(a - b + c) = 0$
$\Rightarrow \quad -a + b + c = 0 \quad \text{or} \quad a - b + c = 0$
On comparing with $ax + by + c = 0$.
The points of concurrency are (-1, 1) or (1, - 1)

29. (b)
Let $f \equiv 6x^2 + xy - 40y^2 - 35x - 83y + 1$

Solve $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

30. (a)
Consider two positions P1 and P2 of P. By geometry the points of trisection of MP1 and MP2 should lie on lines parallel to the base P1P2 and this is irrespective of the points with which we start to define the fixed point M.

31. (b)

We have AB = 10, BC = 5. By bisector property $\frac{AD}{DC} = \frac{10}{5} = \frac{2}{1}$

$\Rightarrow$ co-ordinates of D are $\left(\frac{1}{3}, \frac{1}{3}\right)$ whence equation of BD is

$y - 1 = \frac{1/3 - 1}{1/3 - 5}(x - 5)$ or $x - 7y + 2 = 0$.

32. (b)
If lines (i) and (ii) are same then

$\frac{2\lambda + 1}{\mu + 3} = \frac{3\lambda + 1}{2\mu + 2} = \frac{5\lambda + 1}{6\mu + 4}$

Solve it value of $\lambda = -\frac{3}{7}$

Required line $x - 2y + 8 = 0$

33. (b)
Line at greatest distance from (3, 1) points will be perpendicular to line joining given two points (1, 2) and (3, 1).

34. (c)
Let the equation of the line L be
$y - 2 = m(x - 8),\ m < 0$

coordinates of P and Q are $P\left(8 - \frac{2}{m}, 0\right)$

and Q (0, 2 - 8m)

So $OP + OQ = 8 - \frac{2}{m} + 2 - 8m$

$= 10 + \frac{2}{-m} + 8(-m)$

$\geq 10 + 2\sqrt{\frac{2}{-m} \times 8(-m)} \geq 18$

absolute min. value of OP + OQ = 18.

35. (a)
Since origin and point $(a^2, a + 1)$ lie on the same side of both the lines , so
$3a^2 - (a + 1) + 1 > 0,\ a(3a - 1) > 0$ gives

$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

and $a^2 + 2(a + 1) - 5 < 0$
$a^2 + 2a - 3 < 0 \Rightarrow (a - 1)(a + 3) < 0 \Rightarrow a \in (-3, 1)$

By both the inequalities $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

36. (a)
The reflection of (α, β) in the line $y = 2x$ is

$(\alpha_1, \beta_1) \equiv \left(\frac{4\beta - 3\alpha}{5}, \frac{4\alpha + 3\beta}{5}\right)$

Since $\alpha_1\beta_1 = 1 \Rightarrow \left(\frac{4\beta - 3\alpha}{5}\right)\left(\frac{4\alpha + 3\beta}{5}\right) = 1$

$\Rightarrow 12\alpha^2 - 7\alpha\beta - 12\beta^2 + 25 = 0$

37. (d)

The slope of the line joining (2, 4) and (4, 7) is $\frac{3}{2}$ and the slope of the line perpendicular to this is - $\frac{2}{3}$ which is the slope of one of the given lines. The required line is 2x + 3y - 1 = 0.

38. (b)

Equation of bisectors $\frac{3x+4y-1}{5} = \pm \frac{11x+4y-1}{15}$

So bisectors have slope 8, - $\frac{1}{8}$

Equation of required lines y - 1 = 8(x - 1) and y - 1 = - $\frac{1}{8}$ (x - 1)

which intersect x-axis at $\left(\frac{7}{8}, 0\right)$ and (9, 0).

39. (d)

Family of lines passes through (0, 2)

pair of given lines

S: (x - 2y + 3) (x - y + 1) = 0

x2 + 2y2 + 4x - 3xy - 5y + 3 = 0

chord with middle point (h, k) is T = S1

xh + 2yk + 2 (x + h) - $\frac{3(xh+yk)}{2}$ - $\frac{5}{2}$ (y + k) + 3 = h2 + 2k2 + 4h - 3hk - 5k + 3

it passes through (0, 2)

4k + 2h - $\frac{3}{2}$ (2k) - $\frac{5}{2}$ (2 + k) + 3 = 1

8y + 4x - 10 - 5y + 6 = 0

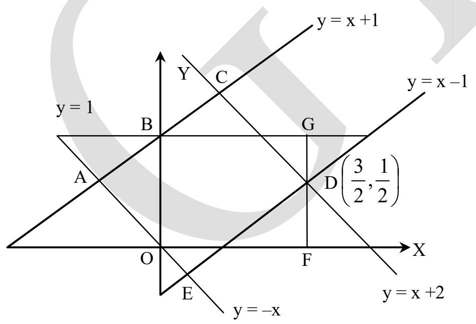
$\Rightarrow$ 4x + 3y = 4

40. (c)

Lines are y = 1, y = 0

y = -x, y = -x + 2

y = x + 1, y = x - 1



Area of OABCDE = area of OBGF (symmetry)

= $\frac{3}{2} \times 1 = \frac{3}{2}$

41. (a)

P $\left(\frac{2}{m}, 2\right)$, Q $\left(\frac{6}{m}, 6\right)$

$\sqrt{\left(\frac{6}{m} - \frac{2}{m}\right)^2 + 16} < 5$.

42. (b)

For a = 1, we have x = y = 1 for a > 1

a(4y + 1) = 1, 1 - 3y so y = $\frac{-(a-1)}{4a+3} < 0$

But a - 1 < 4a + 3, since a is positive, so y > - 1 (contradiction)

a = 1 only

43. (c)

BE ≡ 7x - 10y + 1 = 0

BN ≡ 3x - 2y + 5 = 0

So B ≡ (-3, -2), m_{AB} = $\frac{1}{5}$

ΔABN

$\tan\theta = \left|\frac{\frac{1}{5} - \frac{3}{2}}{1 + \frac{1}{5} \cdot \frac{3}{2}}\right| = 1$ ($\because \theta < \pi/2$)

ΔABN 1 = $\left|\frac{m - \frac{3}{2}}{1 + m - \frac{3}{2}}\right|$

$\Rightarrow 5m^2 + 24$ m -5 = 0 $\Rightarrow$ m = $\frac{1}{5}$ or - 5

BC ≡ y + 2 = -5 (x + 3) $\Rightarrow$ 5x + y + 17 = 0

44. (d)

$\because$ as altitude from A is fixed and orthocentre lies on altitude hence x + y = 3 is required locus.

45. (c)

Given equation can be written as

$P\left(\frac{y}{x}\right)^3 - 3\left(\frac{y}{x}\right)^2 + 3\left(\frac{y}{x}\right) + 3 = 0$

$\therefore Pm^3 - 3m^2 + 3m + 3 = 0$ … (1)

Let m_1, m_2, m_3 are roots

$\therefore m_1 m_2 m_3 = \frac{-3}{P}$

But two lines are perpendicular

$\therefore m_3 = \frac{3}{P}$

Put in (1)

P = -3

46. (a)

$(3x + 4y + 1)^2 + (x + y + 3)^2 = 0$

only when 3x + y + 1 = 0

and x + y + 3 = 0

which represents points of intersection of (1) & (2) i.e. (1, - 4)

47. (a)

Δ is right angled with right angle at (0, 0) so for other vertices

with $y = x$ $\left(\frac{1}{\ell+m}, \frac{1}{\ell+m}\right)$

with $y = -x$ $\left(\frac{1}{\ell-m}, -\frac{1}{\ell-m}\right)$

Now circumcentre (h, k)

$2h = \frac{1}{\ell+m} + \frac{1}{\ell-m}$ & $2k = \frac{1}{\ell+m} - \frac{1}{\ell-m}$

so $(h^2 - k^2)^2 = k^2 + h^2$

locus $(x^2 - y^2)^2 = x^2 + y^2$

48. (c)

Equation of line $\frac{x}{a} + \frac{y}{b} = 1$, through (2, 3)

$\Rightarrow \frac{2}{a} + \frac{3}{b} = 1 \Rightarrow 2b + 3a = ab$

area $= \frac{1}{2}|ab| = 12$

$|ab| = 24 \Rightarrow ab = \pm 24$

When ab = 24

$2b + 3a = 24$

$\Rightarrow 2ab + 3a^2 = 24a$

$\Rightarrow 3a^2 - 24a + 48 = 0$

$\Rightarrow a^2 - 8a + 16 = 0$

$\Rightarrow a = 4 \Rightarrow b = 6$

When ab = -24

$\Rightarrow 2b + 3a = -24$

$\Rightarrow a^2 + 8a - 16 = 0$

Two values of a

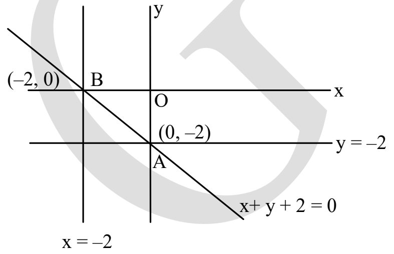
Hence 3 straight lines

49. (d)

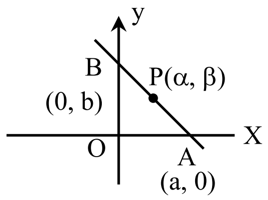
$x(y + 2) + 2(y + 2) = 0 \Rightarrow x = -2, y = -2$

Circumcentre of ΔOAB is mid point of AB

$\equiv (-1, -1)$



50. (b)



Area of $\Delta OAB = S = \frac{1}{2}ab$ ....(i)

equation of AB is $\frac{x}{a} + \frac{y}{b} = 1$

put (α, β)

$\frac{\alpha}{a} + \frac{\beta}{b} = 1$

$\Rightarrow \frac{\alpha}{a} + \frac{a\beta}{2S} = 1$ [using (i)]

$\Rightarrow a^2\beta - 2aS + 2\alpha S = 0$

$\therefore a \in R$

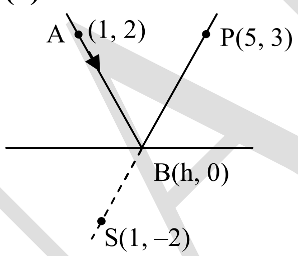
$\Rightarrow D \geq 0$

$4S^2 - 8\alpha\beta S \geq 0$

$S \geq 2\alpha\beta$

Least value of $S = 2\alpha\beta$

51. (d)



$\therefore$ Let B point is (h, 0)

$\therefore$ Reflection of A is (1, -2)

$\therefore$ Slope of SB = slope of PS

$\frac{2}{h-1} = \frac{5}{4} \Rightarrow \frac{8}{5} + 1 = h \Rightarrow h = 13/5$

Now find equation of AB

52. (b)

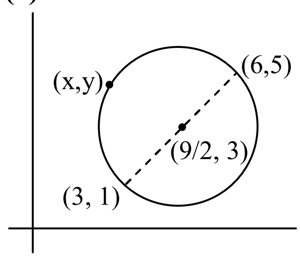
Let A (a, 0) and B(0, b) then area of $\Delta OAB = \frac{1}{2}ab = \frac{1}{2}\frac{a^2\beta}{a-\alpha}$

also $\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ \alpha & \beta & 1 \end{vmatrix} = 0$

53. (b)

use $h^2 = ab$ & $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

54. (a)



$\frac{(y-1)}{(x-3)} \times \left(\frac{y-5}{x-6}\right) = -1$

$y^2 - 6y + 5 = -[x^2 - 9x + 18]$

$x^2 + y^2 - 9x - 6y + 23 = 0$ ... (1)

radius of circle = 5/2 and area of $\Delta RPQ = 7$

$\therefore \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 6 & 5 & 1 \end{vmatrix} = \pm 7$

$\Rightarrow -4x + 3y + 9 = \pm 14$

$\Rightarrow -4x + 3y + 23 = 0$ … (2)

$\Rightarrow -4x + 3y - 5 = 0$ … (3)

We will have to solve equation (1) and equation (2) and also equation (1) and equation (3)

So, we can check whether straight lines (2) and (3) are cutting the circle (1). Distance of centre of circle from St. Line (1)

$= \frac{-4 \times \frac{9}{2} + 3 \times 3 + 23}{\sqrt{3^2 + 4^2}} = \frac{-18 + 9 + 23}{5} = \frac{14}{5}$

$\frac{14}{5}$ > radius of circle

Hence no. solution.

Distance of centre of circle from St. Line (2)

$= \frac{-18 + 9 - 5}{5} = \frac{-14}{5} \left|\frac{-14}{5}\right|$ > radius of circle

Hence no solution in both case . So answer is (A) .

55. (c)

We have $b = \frac{a + c}{2}$

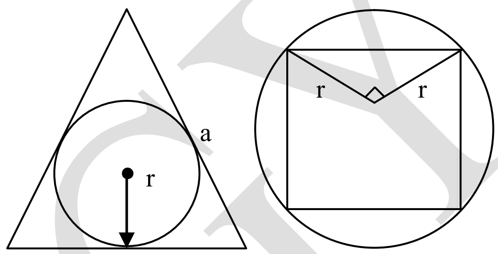
Then equation of the given line reduces to

$ax + \left(\frac{a+c}{2}\right) y + c = 0$

i.e. $a\left(x + \frac{y}{2}\right) + c\left(\frac{y}{2} + 1\right) = 0$

which passes through the fixed point (1, -2).

56. (c)



$r = \frac{1}{3} \times a \sin 60° = \frac{a}{2\sqrt{3}}$

Area of the square $= 4 \times \frac{1}{2} r^2 = \frac{a^2}{6}$

57. (d)

Let the given point on the line $\ell x + my + n = 0$ then $\ell\left(\frac{x^3}{x - 1}\right)$

$+ m\left(\frac{x^2 - 3}{x - 1}\right) + n = 0$

Where x = a, b, c are roots of equation.

$\ell x^3 + mx^2 + nx - 3m - n = 0$

$\Rightarrow a + b + c = -\frac{m}{\ell}$, $ab + bc + ca = \frac{n}{\ell}$ $abc = \frac{3m + n}{\ell}$

$\Rightarrow abc - (ab + bc + ca) + 3(a + b + c)$

$\Rightarrow \frac{3m + n}{\ell} - \frac{n}{\ell} - \frac{3m}{\ell} = 0$

$\Rightarrow \alpha = 1, \beta = 3, \gamma = -1$

Then $\alpha^2 + \beta^2 + \gamma^2 = 1 + 9 + 1 = 11$.

58. (a)

Since origin and the point $(a^2, a + 1)$ lie on the same side of both the lines, therefore we have

$3a^2 - (a + 1) + 1 > 0$

i.e. $a(3a - 1) > 0$

gives $a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

and $a^2 + 2(a + 1) - 5 < 0$

i.e. $a^2 + 2a - 3 < 0$

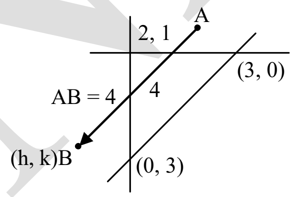
i.e. $(a - 1)(a + 3) < 0$

gives $a \in (-3, 1)$

Intersection of the above inequalities gives

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$.

59. (d)



Slope of AB = 1

i.e. $\frac{k - 1}{h - 2} = 1$

i.e. $h - k = 1$

and AB = 4

i.e. $(h - 2)^2 + (k - 1)^2 = 16$

i.e. $(k - 1)^2 + (k - 1)^2 = 16$

given $k = 1 - \sqrt{8}$

andh $= 2 - \sqrt{8}$.[$k = 1 + \sqrt{8}$ is not acceptable

$\because$ B lies in the third quadrant]

60. (c)

We have $3x + 4y = 9$

i.e $x = \frac{9 - 4y}{3} = 3 - \frac{4y}{3}$

Thus, points lying on the above line and having integral coordinates are given by

$P \equiv (3 - 4k, 3k)$ $k \in I$

If P also lies on $y + mx - 1 = 0$, then we have

$3k + m(3 - 4k) - 1 = 0$

gives $m = \frac{3k-1}{4k-3}, k \in I$

For m to be an integer, we have

$|4k - 3| \le |3k - 1|$

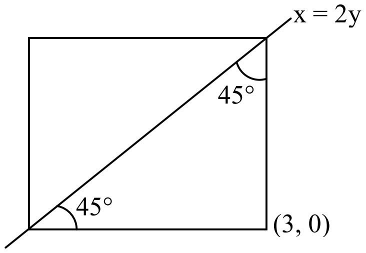
i.e. $-(3k - 1) \le (4k - 3) \le (3k - 1)$

i.e. $\frac{4}{7} \le k \le 2$

There are only two integral values of k lying in the above integral, viz. k = 1, 2, Hence, there are only two integral values of m.

61. (d)

The required equations are



$y = m(x - 3)$

where $m = \frac{\frac{1}{2} + \tan 45°}{1 - \frac{\tan 45°}{2}}, \frac{\frac{1}{2} - \tan 45°}{1 + \frac{\tan 45°}{2}}$

$m = 3, \frac{-1}{3}$

Hence, the equations are

$Y = 3(x - 3)$ and $y = \frac{-1}{3}(x - 3)$

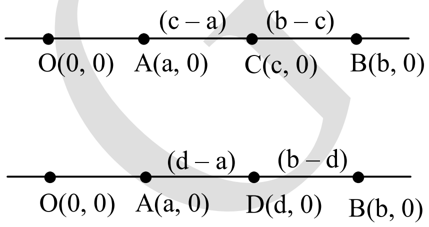
i.e. $3x - y - 9 = 0$ and $x + 3y - 3 = 0$.

62. (b)

We have

$\frac{CA}{CB} + \frac{DA}{DB} = 0$ i.e. $\left|\frac{c-a}{b-c}\right| + \left|\frac{d-a}{b-d}\right| = 0$

i.e. $\frac{c-a}{b-c} \pm \frac{d-a}{b-d} = 0$



Taking +ve sign, we have

$(a + b)(c + d) = 2(ab + cd)$ Taking -ve sign, we have $(a - b)(c - d) = 0$

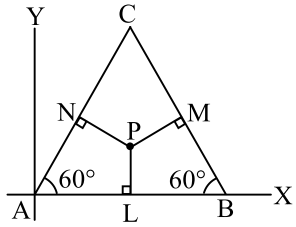
Which is not possible if the four points are distinct.

63. (c)

Let us choose A as the origin and AB as the X-axis. Then $B \equiv (a, 0)$ and the equations of lines AC and BC, are respectively given by

$y - \sqrt{3}x = 0$

$y + \sqrt{3}(x - a) = 0$



Let P (h, k) be the coordinates of the point whose locus is to be found. Now, according to the given condition, we have $PL^2 = PM \cdot PN$

i. e. $k^2 = \frac{|k - \sqrt{3}h|}{2} \cdot \frac{|k + \sqrt{3}(h-a)|}{2}$

i.e. $4k^2 = (k - \sqrt{3}h)[k + \sqrt{3}(h - a)]$ [∵ P lies below both the lines]

i.e. $3(h^2 + k^2) - 3ah + \sqrt{3}ak = 0$

i.e. $h^2 + k^2 - ah + \frac{a}{\sqrt{3}}k = 0$

Putting (x, y) in place of (h, k) gives the equation of the required locus as

$x^2 + y^2 - ax + \frac{a}{\sqrt{3}}y = 0$.

64. (d)

If possible equation of line is $y + 5 = m(x - 4)$

$\Rightarrow y - mx + 4m + 5 = 0$

then $\left|\frac{3 + 2m + 4m + 5}{\sqrt{1 + m^2}}\right| = 12$

$\Rightarrow (8 + 6m)^2 = 144(1 + m^2)$

$\Rightarrow 27m^2 - 24m + 20 = 0$

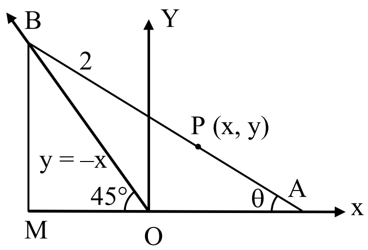
Here discriminate < 0

there for no such line possible for real value of m.

65. (a)

If $\angle BAO = \theta$ then $BM = 2\sin\theta$ and

$MO = BM = 2\sin\theta$, $MA = 2\cos\theta$



Hence $A = (2\cos\theta - 2\sin\theta, 0)$ and

$B = (-2\sin\theta, 2\sin\theta)$

Since P(x, y) is the mid point of AB,

$2x = (2\cos\theta) + (-4\sin\theta)$ or $\cos\theta - 2\sin\theta = x$

$2y = (2\sin\theta)$ or $\sin\theta = y$

Eliminating θ, we have $(x + 2y)^2 + y^2 = 1$ or ,

$x^2 + 5y^2 + 4xy - 1 = 0$.

66. (b)

Equation of pair of bisectors of angles between lines $ax^2 + 2hxy + by^2 = 0$ is

$\frac{x^2 - y^2}{xy} = \frac{a-b}{h} \Rightarrow h(x^2 - y^2) = (a - b)xy$ … (1)

But y = mx is one of these lines, then it will satisfy it.

Substituting y = mx in (1)

$h(x^2 - m^2x^2) = (a - b)x.\ mx$

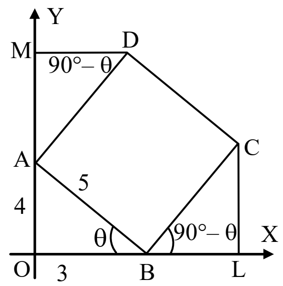
Dividing by x^2, $h(1 - m^2) = m(a - b)$.

67. (b)

The coordinates of A are (0, 4) and that of B are (3, 0).

Let CL and DM be perpendiculars on x-axis and y-axis respectively then if $\angle OBA = \theta$.

$\angle CBL = \angle ADM = 90° - \theta$ [See figure]



also, $BC = AB = \sqrt{3^2 + 4^2} = 5$

$\Rightarrow BL = BC \sin\theta$ and $CL = BC \cos\theta$

$\Rightarrow BL = 5 \times \frac{4}{5} = 4$ and $CL = 5 \times \frac{3}{5} = 3$

Similarly, MD = 4 and AM = 3.

So the co-ordinates of C are (OB + BL, CL) = (7, 3) and of D are (MD, OA + AM) = (4, 7)

The co-ordinates of the vertex farthest from the origin are therefore (4, 7).

68. (c)

$\because m = \pm 1$

So, equation of line is $y - (-2) = \pm(x - 1)$

Taking positive sign

$x - y - 3 = 0$

Taking negative sign

$x + y + 1 = 0$

69. (a)

Point (-1, - 2) satisfies the given equation of straight line.

70. (a)

Let the point be (x_1, y_1)

then $x_1 + y_1 = 4$ ….(i)

& $\left|\frac{4x_1 + 3y_1 - 10}{5}\right| = 1$

$\Rightarrow \frac{4x_1 + 3y_1 - 10}{5} = \pm 1$

Taking positive sign

$\Rightarrow 4x_1 + 3y_1 = 15$

…… (ii)

Taking negative sign

$4x_1 + 3y_1 = 5$ …..(iii)

solve (i) & (ii) $\Rightarrow$ point is (3, 1)

& solve (i) & (iii) $\Rightarrow$ point is (- 7, 11)

71. (c)

$y = (2 + \sqrt{3})\ x + 4$ ….(i)

$y = kx + 6$ ….(ii)

By (i) $\Rightarrow m_1 = (2 + \sqrt{3})$

By (ii) $\Rightarrow m_2 = k$

$\therefore \tan 60^0 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow k = -1$

72. (d)

$\frac{x}{a} + \frac{y}{b} = 1$

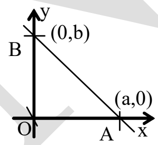
$P(2, -3) \Rightarrow \frac{2}{a} - \frac{3}{b} = 1$ ....(i)

$Q(4, -5) \Rightarrow \frac{4}{a} - \frac{5}{b} = 1$ ….(ii)

Solve (i) & (ii)

a = - 1 , b = - 1

73. (a)



Equation of line AB, x/a + y/b = 1 .....(i)

Given, Area of $\Delta OAB = 1/2ab = |6|$

$\Rightarrow ab = \pm 12$ ......(ii)

& $AB = |5| \Rightarrow a^2 + b^2 = 25$.....(iii)

Solve (ii) & (iii) $\Rightarrow$ {a= ?, b = ?

$\Rightarrow$ put in (i)

74. (a)

Let the eq^n $3x - 2y + k = 0$ it passes through (-1, 1)

$\therefore -3 - 2 + k = 0 \Rightarrow k = 5$

eq^n $3x - 2y + 5 = 0$

75. (d)

$m = \tan 30^0 = \frac{1}{\sqrt{3}} \Rightarrow (x_1 y_1) = (0, -2)$

eq^n $y - y_1 = m(x - x_1) \Rightarrow y + 2 = \frac{1}{\sqrt{3}}(x - 0)$

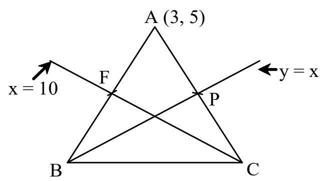
76. (c)

Clearly reflection of A on y = x and x = 10 should lie on the line BC

So, (5, 3) and (17, 5) should lies on BC

$\therefore$ Equation of BC is $\Rightarrow y - 3 = \frac{5-3}{17-5}(x - 5)$

$\Rightarrow$ 6y - x = 13



77. (c)

$$\frac{x}{3}+\frac{y}{4}=2$$

78. (c)

$L = x + 5y + \lambda = 0$

$$\frac{x}{-\lambda}+\frac{5y}{-\lambda}=1 \Rightarrow \frac{x}{-\lambda}+\frac{y}{\frac{-\lambda}{5}}=1$$

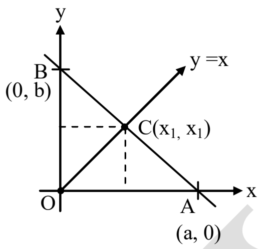
$$\Rightarrow \text{area of } \Delta = \frac{1}{2}(-\lambda)\left(\frac{-\lambda}{5}\right)$$

$$\Rightarrow\ 5=\frac{\lambda^2}{10} \Rightarrow\ \lambda=\pm\sqrt{50} \Rightarrow\ \lambda=\pm 5\sqrt{2}$$

79. (b)

$$\begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix}=0$$

80. (c)



Given $\Delta_{AOC} = 2\,(\Delta_{BOC})$

$$\Rightarrow \frac{1}{2}(OA)(x_1)=\frac{2\times 1}{2}(OB)(x_1)$$

$$\Rightarrow \boxed{a=2b}$$

Equation of AB $\Rightarrow \frac{x}{a}+\frac{y}{b}=1$ ……(i)

$$\frac{x}{2b}+\frac{y}{b}=1$$

…. (ii)

$\Rightarrow$ Since point C lies on the line (ii)

$$\Rightarrow \frac{x_1}{2b}+\frac{x_1}{b}=1$$

$$\Rightarrow x_1=\frac{2b}{3}=\frac{a}{3}$$

$$\Rightarrow C\left(\frac{2b}{3},\frac{2b}{3}\right)$$

81. (d)

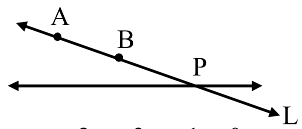
Angular bisector of angle B on the side containing origin is

$$\Rightarrow \frac{(x+y-1)}{\sqrt{2}}=+\frac{(7x-y-15)}{\sqrt{50}}$$

$\Rightarrow$ - x + 3y = -5

82. (a)

|PA - PB| is minimum



2x + 3y + 1 = 0

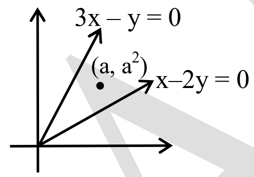
Equation of L is

x + y = 2

Now P is point of intersection of L and

2x + 3y + 1 = 0

83. (b)



$$\frac{3a-a^2}{-1}<0$$

a(a - 3) < 0 ....(i)

$$\frac{a-2a^2}{-2}>0$$

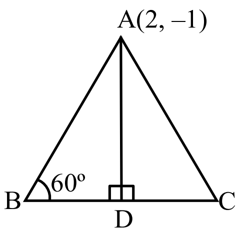
$2a^2$ - a > 0

$$2a\left(a-\frac{1}{2}\right)>0 \text{.......(ii)}$$

Solving (i) and (ii)

$$\frac{1}{2}<a<3$$

84. (b)



x + 2y = 1

$$AD=\frac{|2-2-1|}{\sqrt{5}}$$

$$=\frac{1}{\sqrt{5}}$$

AB = AD cosec 60^0

$$=\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{3}}$$

$$=\frac{2}{\sqrt{15}}$$

85. (a)

Let $k = 3x - 4y - 8$
then value of k at (3,4)
$= 3 \times 3 - 4 \times 4 - 8 = -15 < 0$
$\therefore$ For point P(x, y) we should have $k > 0$
$\Rightarrow 3x - 4y - 8 > 0$
$\Rightarrow 3x - 4(-3x) - 8 > 0 \{\because P(x,y)$ lie on $y = -3x\}$
$\Rightarrow 15x - 8 > 0 \Rightarrow x > 8/15$
and $-y - 4y - 8 > 0$
$y < -8/5$

86. [D]

We have $(x + 2) = \frac{r}{\sqrt{10}}$ & $(y - 1) = \frac{3r}{\sqrt{10}}$
Cartesian equation $(y - 1) = 3(x + 2)$
slope = 3

87. (a)

Point must be point of intersection of given line & perpendicular line passing through (1, 2)
i.e. $3x + y = 5$
$\Rightarrow (2, -1)$

88. (b)

The angle between the line joining the points (1, -2), (3, 2) and the line $x + 2y - 7 = 0$, is

89. (b)

$x + 2y - 7 = 0$ .... (1)
slope of line (1) $= m_1 = -\frac{1}{2}$
M_2 = slope of line PQ $= \frac{2-(-2)}{3-1} = 2$
When P(1, - 2) & Q (3, 2)
$\because m_1, m_2 = -1$
angle b/w line (1) & line PQ $= \frac{\pi}{2}$

90. (b)

Let equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$
Which meet the axes at A(a, 0) and B (0, b)
If (1, 2) are the coordinate of centroid of ΔOAB then
$\frac{0+a+0}{3} = 1$ & $\frac{0+0+b}{3} = 2$
$\Rightarrow a = 3$ & $b = 6$
$\therefore$ equation of line is $\frac{x}{3} + \frac{y}{6} = 1 \Rightarrow 2x + y = 6$

91. (b)

The sides of triangle are $x + y - 4 = 0$ …(i)
$x - 1 = 0$ … (ii),
$y - 2 = 0$ … (iii)
So, the triangle is right angled at (1, 2)
The hypotenuse is $x + y - 4 = 0$ whose ends are (1, 3) and (2, 2)

Circumcentre is $\left(\frac{1+2}{2}, \frac{3+2}{2}\right) \equiv \left(\frac{3}{2}, \frac{5}{2}\right)$ and
circum-radius is $\frac{1}{2}\sqrt{(1-2)^2 + (3-2)^2} = \frac{1}{\sqrt{2}}$
$\therefore$ equation of circle is $\left(x - \frac{3}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{1}{2}$
$\therefore x^2 + y^2 - 3x - 5y + 8 = 0$

92. (d)

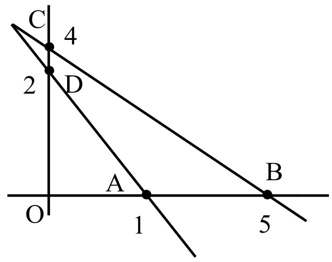
Let $(ax^2 + 2hxy + by^2) = (y - m_1x)(y - m_2x)$
$\therefore m_1 + m_2 = \frac{-2h}{b}$, $m_1m_2 = a/b$
New lines will be
$(x + m_1y)(x + m_2y) = 0$
$x^2 + (m_1 + m_2)xy + m_1m_2y^2 = 0$
Put values of $m_1 + m_2$ and m_1m_2

93. (c)

lines are concurrent
$$\begin{vmatrix} 1 & 2 & -1 \\ a & 1 & 3 \\ b & -1 & 2 \end{vmatrix} = 0$$
$\Rightarrow 7b - 3a + 5 = 0$
locus of (a, b) is $3x - 7y = 5$
least distance from (0, 0) = length of perpendicular from (0, 0) $= \frac{5}{\sqrt{58}}$

94. (a)

In given quadrilateral, only six points will be inside whose coordinates are +ve integers.



95. (a)

Let the two curves be S_1 and S_2. Then $(g' \times S_1) - (g \times S_2)$ gives
$(ag' - a'g)x^2 + 2(g'h - gh')xy + (bg' - b'g)y^2 = 0$
Which is a homogeneous second degree equation and thus represents a pair of straight lines passing through the origin.
The two lines will be at right angles if
coeff. of x^2 + coeff. of $y^2 = 0$
i.e. $(a + b)g' = (a' + b')g$.

96. (d)

We have

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix} = \frac{1}{2}(a^2+b^2+c^2-ab-bc-ca)$$

Now, area of other triangle is

$$\Delta' = \frac{1}{2}\begin{vmatrix} ac-b^2 & ab-c^2 & 1 \\ ba-c^2 & bc-a^2 & 1 \\ cb-a^2 & ca-b^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} (c-b)(a+b+c) & (a-c)(a+b+c) & 0 \\ (a-c)(a+b+c) & (b-a)(a+b+c) & 0 \\ bc-a^2 & ac-b^2 & 1 \end{vmatrix}$$

$[R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3]$

$$= \frac{1}{2}(a+b+c)^2\begin{vmatrix} c-b & a-c & 0 \\ a-c & b-a & 0 \\ bc-a^2 & ac-b^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}(a+b+c)^2(a^2+b^2+c^2-ab-bc-ca)$$

$= (a+b+c)^2\,\Delta.$

97. (a)

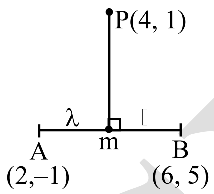
Let the eq^n. $\frac{x}{a}+\frac{y}{b}=1$ given $b = -a$

$\therefore \frac{x}{a}+\frac{y}{-a}=1$ it passes through (-3, 2)

$\therefore \frac{-3}{a}+\frac{2}{-a}=1 \Rightarrow a = -5$

$\therefore$ eq^n $\quad x - y + 5 = 0$

98. (b)



Let, $\frac{Am}{mB}=\frac{\lambda}{1}$

$\therefore m\left\{\frac{6\lambda+1(2)}{\lambda+1}, \frac{5\lambda+(-1)}{\lambda+1}\right\}$ $\quad \therefore pm \perp AB$

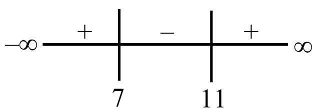
$\Rightarrow \begin{pmatrix}\text{slope of} \\ \text{PM}\end{pmatrix} \times \begin{pmatrix}\text{slope of} \\ \text{AB}\end{pmatrix} = -1 \Rightarrow \lambda = \frac{5}{8}$

99. (d)

$(3(1) - 5(2) + a)\,(3 \times 3 - 5 \times 4 + a) > 0$

$(3 - 10 + 9)\,(9 - 20 + a) > 0$

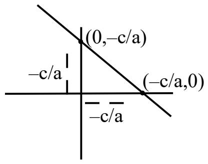
$(a - 11)\,(a - 7) > 0$



$a \in (-\infty, 7) \cup (11, \infty)$

or $a < 7$ or $a > 11$

100.(d)



For first quadrant $-\frac{c}{a} > 0$ & $-\frac{c}{b} > 0$

c & a have opp sing, C δ b have opp sigh $ac < 0$, $bc < 0$