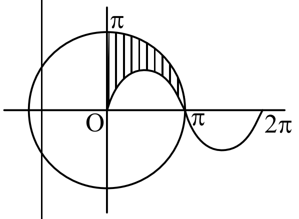
1. (a)

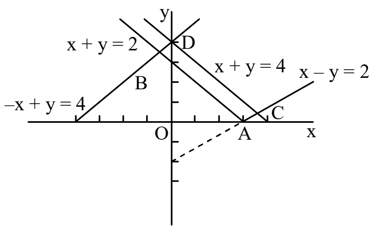


Area = $\int_0^{\pi}(y_2 - y_1)dx$

$= \int_0^{\pi}\{\sqrt{\pi^2 - x^2} - \sin x\}dx$

$= \int_0^{\pi}\sqrt{\pi^2 - x^2}\ dx - 2$

2. (d)



$y = |x - 2| = \begin{cases} x - 2 & \text{if } x \geq 2 \\ 2 - x & \text{if } x \leq 2 \end{cases}$ and

$\Rightarrow \begin{matrix} x - y = 2 & \text{if } x \geq 2 \\ x + y = 2 & \text{if } x \leq 2 \end{matrix}$

$y = 4 - |x| = \begin{cases} 4 - x & \text{if } x \geq 0 \\ 4 + x & \text{if } x < 0 \end{cases}$ and

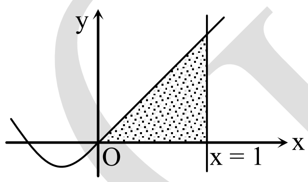
$\Rightarrow \begin{matrix} x + y = 4 & \text{if } x \geq 0 \\ -x + y = 4 & \text{if } x < 0 \end{matrix}$

A (2, 0), B (-1, 3), C(3, 1) D(0, 4)

ABCD is rectangle, AB = $\sqrt{18}$, AC = $\sqrt{2}$ Area of ABCD =

$\sqrt{18}\ .\ \sqrt{2} = \sqrt{36} = 6$

3. (b)



Since at point (x, y) of the curve, slope of the tangent to the curve is $\frac{dy}{dx}$, so as given

$\frac{dy}{dx} = 2x + 1$ Integrating, we get

$y = x^2 + x + c$ But the curve passes through the point (1, 2), so we have

$2 = 1 + 1 + c \Rightarrow c = 0$

$\therefore$ equation of the curve is $y = x^2 + x$ which is a parabola. Also the curve cuts x-axis at x = -1, and at x = 0. So the required area

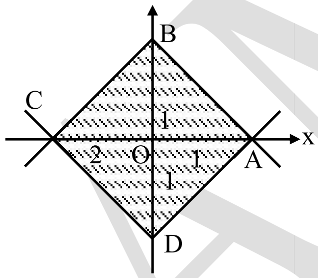
$= \int_0^1 (x^2 + x)\,dx = \left(\frac{x^3}{3} + \frac{x^2}{2}\right)_0^1 = \frac{1}{3} + \frac{1}{2}$

4. (b)

$y = |x| - 1 \Rightarrow \begin{cases} y = -x - 1, & x < 0 \\ y = x - 1, & x > 0 \end{cases}$

$y = -|x| + 1 \Rightarrow \begin{cases} y = x - 1, & x < 0 \\ y = -x + 1, & x > 0 \end{cases}$

$\therefore$ x + y = - 1, x - y = - 1, when x < 0 x - y = 1, x + y = `1, when x > 0



Obviously all these four lines form square ABCD whose side $= \sqrt{2}$. Hence required

area = area of the square = 2

5. (a)

In the given interval given curves meet at x = $\sqrt{3}/2$ So

required area = $\int_{1/2}^{\sqrt{3}/2}\{f(x) - g(x)\}dx$

$\int_{1/2}^{\sqrt{3}/2}[(1 - x) - (x - 1/2)^2]dx$

$= \left[x - \frac{x^2}{2} - \frac{(x - 1/2)}{3}\right]_{1/2}^{\sqrt{3}/2} = \frac{\sqrt{3}}{4} - \frac{1}{3}$

6. (c)

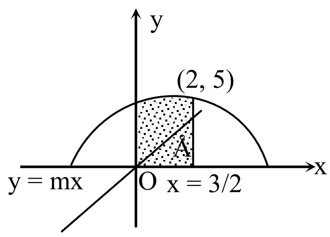
Area of one loop

$= \int_{\theta=0}^{\pi/3}(1/2)r^2 d\theta = \frac{1}{2}\int_0^{\pi/3} a^2 \sin^2 3\theta\, d\theta$

$= \frac{1}{2}\cdot\frac{1}{3}a^2\int_0^{\pi}\sin^2 t\,dt$ where $t = 3\theta$

$= \frac{1}{3}a^2\int_0^{\pi/2}\sin^2 t\,dt = \frac{1}{3}a^2 \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi a^2}{12}$

7. (c)



Equation of the given curve is $(x - 2)^2 = -(y - 5)$ which is a parabola given area = $\int_0^{3/2} y\,dx = \int_0^{3/2} (1+4x-x^2)\,dx$

$= \left[x+2x^2-\frac{x^3}{3}\right]_0^{3/2}$ = 3/2 + 9/2 -9/18 = 39/8 Also A′ =

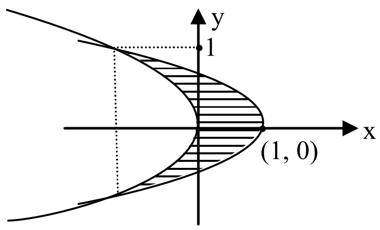
$\int_0^{3/2} mx\,dx = \frac{m}{2}\left[x^2\right]_0^{3/2} = \frac{9}{8}m$

As given $\frac{9}{8}m = \frac{1}{2}\left(\frac{39}{8}\right) \Rightarrow m = 13/6$

8. (c)

Area = $2\int_0^1 (1-3y^2)-(-2y^2)\,dy$

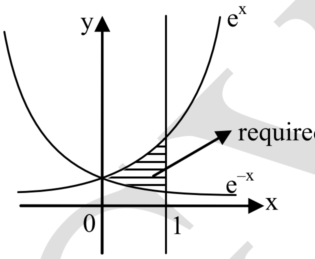
$= 2\int_0^1 (1-y^2)\,dy$



$= 2.\left(y-\frac{y^3}{3}\right)_0^1 = 2.\left(1-\frac{1}{3}\right) = \frac{4}{3}$

9. (d)

$A = \int_0^1 (e^x - e^{-x})\,dx$



$\Rightarrow A = (e^x + e^{-x})_0^1$

$A = (e + e^{-1}) - (1 + 1)$

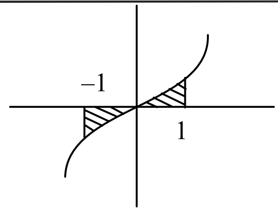
$A = e + \frac{1}{e} - 2$

10. (a)

Side of equilateral triangle $\sqrt{3}+2+\sqrt{3}$ (by trigonometry)

$A = (\text{side})^2\,\frac{\sqrt{3}}{4}$

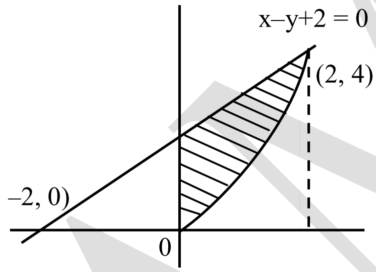
11. (c)



$y = x\,|x| \Rightarrow$ Area $\left|\int_{-1}^{0}(-x^2).dx\right| + \int_0^1 x^2.dx$

12. (c)

Area $\int_0^2 (x+2-x^2)dx = \frac{10}{3}$sq.units



13. (b)

$A_1 = 2\int_0^a \sqrt{4ax}\,dx = \frac{8a^2}{3}$

$A_2 = 2\left[\int_0^{2a}\sqrt{4ax} - \int_0^a \sqrt{4ax}\right] = \frac{16\sqrt{2}}{3}a^2 - \frac{8a^2}{3}$

$\Rightarrow \frac{A_1}{A_2} = \frac{1}{2\sqrt{2}-1} = \frac{2\sqrt{2}+1}{7}$

14. (d)

Area = $\int_0^2 2^x - (2x-x^2)\,dx = \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$

15. (a)

Area = $\int_0^{\pi/4} \tan x\,dx + \int_0^{\pi/4} \cot x\,dx$ =lo $\sqrt{2} + \log\sqrt{2} = \log 2$

16. (b)

Area = $\frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 3 \times 3 = 5$

17. (b)

$A_1 = \int_0^{\pi/3} \cos x dx = \frac{\sqrt{3}}{2}$

$A_2 = \int_0^{\pi/2} \cos 2x dx = \frac{\sqrt{3}}{4}$

$\therefore \frac{A_1}{A_2} = \frac{2}{1}$

18. (a)

Area bounded by the curve y = f(x), x-axis and x = 1 & x = b

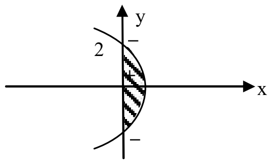
is $\int_1^b f(x)\,dx = (b-1)\sin(3b+4)$

On differentiating w.r.t b we get f(b) · 1 = 3(b - 1) cos (3b + 4) + sin (3b + 4)

$\Rightarrow$ f(x) = 3 (x - 1) cos (3x + 4) + sin (3x + 4)

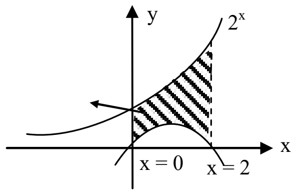
19. (b)

$x = 2y - y^2 \Rightarrow x = -y(y-2)$



Required area = $\int_0^2 (2y - y^2)\, dy$

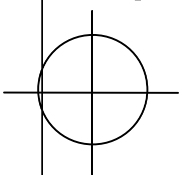
20. (a)



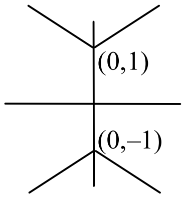
Required area = $\int_0^2 2^x - (2x - x^2)\, dx$

21. (b)

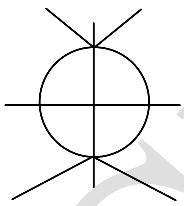
Graph of $x^2 + y^2 = 1$ is



and graph of $|y| - |x| = 1$ is

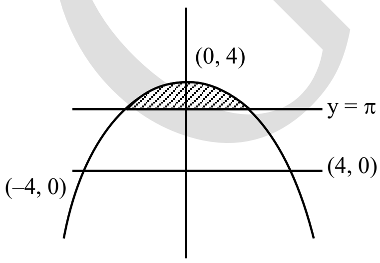


So area enclosed by two graph is zero.



22. (c)

$0 \le \sin^2 x \le 1$



$\Rightarrow -1 \le -\sin^2 x \le 0 \;\therefore\; [-\sin^2 x] = 0$ or -1 but $\sec^{-1}(0)$ is nto defined hence $y = \sec^{-1}[-\sin^2 x] = \sec^{-1}(-1) =$ now $\pi =$

$\frac{16 - x^2}{4} \Rightarrow x^2 = 16 - 4\pi = 4(4 - \pi) \Rightarrow x = \pm 2\sqrt{4-\pi}$ The

required area = $\int_{-2\sqrt{4-\pi}}^{2\sqrt{4-\pi}} \left(\frac{16 - x^2}{4} - \pi\right) dx \;=\; \frac{8}{3}(4 - \pi)^{3/2}$.

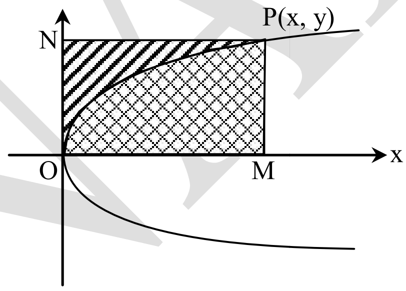
23. (b)

Let P(x, y) be the point on the curve passing through the origin O(0, 0), and let PN and PM be the lines parallel to the x-axes and y-axes, respectively. If the equation of the curve is y = y(x), the area POM equals

$\int_0^x y\, dx$ and the area PON equals $xy - \int_0^x y\, dx$ Assuming that 2

(POM) = PON, we therefore have $2\int_0^x y\, dx = xy - \int_0^x y\, dx$

$\Rightarrow 3\int_0^x y\, dx = xy.$



Differentiating both sides of this gives

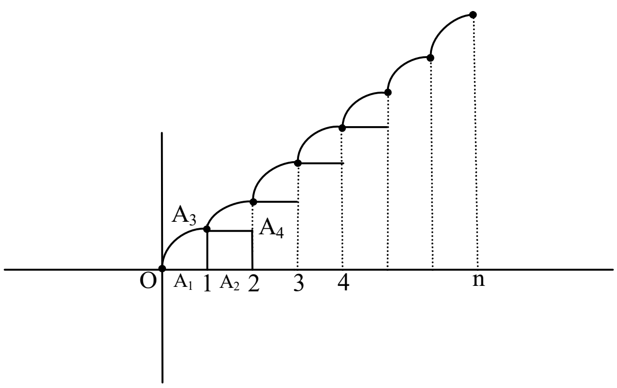
$3y = x\frac{dy}{dx} + y \Rightarrow 2y = x\frac{dy}{dx} \Rightarrow \frac{dy}{y} = 2\frac{dx}{x}$

$\Rightarrow \log|y| = 2\log|x| + C \Rightarrow y = Cx^2$, with C being a constant.

This solutions represents a parabola. We will get a similar result if we had started instead with 2(PON) = POM.

24. (c)

Curve



Area	$0 \le x < 1$	$\int_0^1 \sqrt{x}\, dx$
Area	$1 \le x < 2$	$\int_0^1 \sqrt{x}\, dx + 1 \times 1$ (Area of A_1

$A_2 A_3 A_4$) Area $2 \le x < 3$ $\int_0^1 \sqrt{x}\ dx + 2 \times 1$

Area $n - 1 \le x < n$ $\int_0^1 \sqrt{x}\ dx + (n - 1) \times 1$ So

total

area $= n\int_0^1 \sqrt{x}\ dx + [1 + 2 + 3 + \ldots\ \ldots + (n - 1)] =$

$\frac{2n}{3} + \frac{n(n-1)}{2}$

25. (c)

As given $\int_1^b f(x)dx = (b - 1) \sin (3b + 4)$ Now differentiating with respect to b (using [P-9] from definite integral)

$f(b).1 = \sin (3b + 4) + 3(b - 1) \cos (3b + 4)$

$\therefore\ f(x) = \sin(3x + 4) + 3(x - 1) \cos (3x + 4)$