1. (c)

$$\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

Applying $C_2 \to C_1 + C_2$

$$\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_4 + {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_6 + {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_8 + {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

$$\Rightarrow \Delta = \begin{vmatrix} {}^{10}C_4 & {}^{11}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{12}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{13}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

Clearly $m = 5$ satisfies the above result

[$\because C_2, C_3$ will be identical]

2. (d)

$\because a_1, a_2, a_3, ..., a_n, ...$ are in G.P.

$\therefore a_{n+1}^2 = a_n.a_{n+2} \Rightarrow 2\log a_{n+1} = \log a_n + \log a_{n+2}$

$a_{n+4}^2 = a_{n+3}.a_{n+5} \Rightarrow 2\log a_{n+4} = \log a_{n+3} + \log a_{n+5}$

$a_{n+7}^2 = a_{n+6}.a_{n+8} \Rightarrow 2\log a_{n+7} = \log a_{n+6} + \log a_{n+8}$

Putting these values in the second column of the given determinant, we get

$$\Delta = \frac{1}{2}\begin{vmatrix} \log a_n & \log a_n + \log a_{n+2} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+3} + \log a_{n+5} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+6} + \log a_{n+8} & \log a_{n+8} \end{vmatrix} = \frac{1}{2}(0) = 0$$

a [$\because c_2$ is the sum of the elements, first identical with c_1 and second with c_3]

3. (a)

Applying $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2.2^x.2.2^{-x} & 2.3^x.2.3^{-x} & 2.5^x.2.5^{-x} \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix} = 0$$

[$\because R_1$ and R_2 are identical]

Trick : Putting $x = 0$, we get option (a) is correct

4. (c)

$\because x51 = 100x + 50 + 1$,

$y41 = 100y + 40 + 1$

$z31 = 100z + 30 + 1$

$$\therefore \Delta = \begin{vmatrix} 5 & 4 & 3 \\ 100x + 50 + 1 & 100y + 40 + 1 & 100z + 30 + 1 \\ x & y & z \end{vmatrix}$$

Applying $R_2 \to R_2 - 100R_3 - 10R_1$

$$\Delta = \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = x - 2y + z$$

$\because$ x, y, z are in A.P., $\therefore x - 2y + z = 0$, $\therefore \Delta = 0$

5. (a)

Expanding determinant, we get,

$\Delta = -(x-a)[-(x+b)(x-c)] + (x+b)[(x+a)(x+c)] = 0$

$\Rightarrow 2x^3 - (2\Sigma ab)x = 0$

$\Rightarrow$ Either $x = 0$ or $x^2 = \Sigma ab$.

Since $x = 0$ satisfies the given given equation.

Trick : On putting $x = 0$, we observe that the determinant becomes

$$\Delta_{x=0} = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

$\therefore x = 0$ is a root of the given equation.

6. (c)

$$(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Applying, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\therefore \tan x = -2, 1$ But $\tan x \neq -2$ in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. Hence

$\tan x = 1 \Rightarrow x = \frac{\pi}{4}$

7. (b)

Since it is an identity in λ so satisfied by every value of λ.

Now put $\lambda = 0$ in the given equation, we have

$$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -4 \\ -3 & 4 & 0 \end{vmatrix} = -12 + 30 = 18$$

8. (d)

$$\because D_1 = \begin{vmatrix} 1 & 15 & 8 \\ 1 & 35 & 9 \\ 1 & 25 & 10 \end{vmatrix}, D_2 = \begin{vmatrix} 2 & 15 & 8 \\ 4 & 35 & 9 \\ 8 & 25 & 10 \end{vmatrix}, D_3 = \begin{vmatrix} 3 & 15 & 8 \\ 9 & 35 & 9 \\ 27 & 25 & 10 \end{vmatrix}$$

$$, D_4 = \begin{vmatrix} 4 & 15 & 8 \\ 16 & 35 & 9 \\ 64 & 25 & 10 \end{vmatrix}, D_5 = \begin{vmatrix} 5 & 15 & 8 \\ 25 & 35 & 9 \\ 125 & 25 & 10 \end{vmatrix}$$

$$\Rightarrow D_1 + D_2 + D_3 + D_4 + D_5 = \begin{vmatrix} 15 & 75 & 40 \\ 55 & 175 & 45 \\ 225 & 125 & 50 \end{vmatrix}$$

$= 15(3125) - 75(-7375) + 40(-32500)$

$= 46875 + 553125 - 1300000 = -700000$

9. (a)

$$\sum_{n=1}^{N} U_n = \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left\{\frac{N(N+1)}{2}\right\}^2 & 3N^2 & 3N \end{vmatrix}$$

$$= \frac{N(N+1)}{12}\begin{vmatrix} 6 & 1 & 5 \\ 4N+2 & 2N+1 & 2N+1 \\ 3N(N+1) & 3N^2 & 3N \end{vmatrix}$$

Applying $C_3 \to C_3 + C_2$

$$= \frac{N(N+1)}{12}\begin{vmatrix} 6 & 1 & 6 \\ 4N+2 & 2N+1 & 4N+2 \\ 3N(N+1) & 3N^2 & 3N(N+1) \end{vmatrix} = 0$$

[$\because$ C_1 and C_3 are identical]

10. (b)

$$\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} = x^3 - 3abx \Rightarrow \frac{d}{dx}(\Delta_1) = 3(x^2 - ab) \text{ and}$$

$$\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab$$

$$\therefore \frac{d}{dx}(\Delta_1) = 3(x^2 - ab) = 3\Delta_2$$

11. (d)

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} = \begin{vmatrix} \sin mx & m\cos mx & -m^2 \sin mx \\ -m^3 \cos mx & m^4 \sin mx & m^5 \cos mx \\ -m^6 \sin mx & -m^7 \cos mx & m^8 \sin mx \end{vmatrix}$$

Taking $-m^6$ common from R_3, R_1 and R_3 becomes identical. Hence the value of determinant is zero.

12. (a)

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 + a(a)^2 = 0 \Rightarrow a^3 = -1 \Rightarrow a = -1$$

13. (c)

As the system of the equations has a non-trivial solution

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$$

$$\Rightarrow a(b-1)(c-1) - 1(1-a)(c-1) - 1(1-a)(b-1) = 0$$

$$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0 \Rightarrow \frac{1}{1-a} - 1 + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

14. (a)

For the equations to be inconsistent $D = 0$

$$\therefore D = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & k+3 \\ 2k+1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow k = -3 \text{ and}$$

$$D_1 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0$$, Hence system is inconsistent for $k = -3$

15. (c)

Each of the first three options contains $m = 3$. When $m = 3$, the last two equations become $x + 2y + 3z = 10$ and $x + 2y + 3z = n$.

Obviously, when $n = 10$ these equations become the same.

So we are left with only two independent equations to find the values of the three unknowns.

Consequently, there will be infinite solutions.

16. (c)

On operating [$R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$],

$$f(\theta) = \begin{vmatrix} 0 & 1-e^{i\theta} & -2 \\ 1 & e^{i\theta} & 1 \\ 0 & -1-e^{i\theta} & -1-e^{i\theta} \end{vmatrix} = (-1)\,[(1-e^{i\theta})$$

$(-1 - e^{-i\theta}) - 2(-1 - e^{i\theta}] = 2$ when $\theta = \frac{\pi}{2}$

17. (c)

When a = b or b = c or c = a the determinant reduces to zero. It is not necessary that a = b = c for determinant to be zero. Therefore triangle is isosceles

18. (d)

By putting x = 0 on both sides of the equation we have

$$= \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix} = 9$$

19. (a)

Putting x = 1 y = 1,

and z = 1 on both sides, we get k = 4

20. (b)

Since elements of the rows are in A.P. with common difference 7, given determinant is zero

21. (a)

Since $\Delta = \overline{\Delta} \Rightarrow \Delta$ is purely real.

22. (a)

$$\text{Here } \Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

= (a - b) (b - c) (c - a).

23. (c)

$$\Delta = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & 5\lambda \end{vmatrix} = 7(5\lambda + 5) = 0 \Rightarrow \lambda - 1.$$

24. (c)

$a_r = \frac{20}{r}$, r = 1, 2, 3, ... 9

$\Rightarrow$ the value of the determinant is $\frac{50}{21}$

25. (b)

$$\Delta' = \begin{vmatrix} e^x & \sin x & 1 \\ -\sin x & \ell n(1+x^2) & 1 \\ 1 & x^2 & 1 \end{vmatrix}$$

$$+ \begin{vmatrix} e^x & \cos x & 1 \\ \cos x & \frac{2x}{1+x^2} & 1 \\ x & 2x & 1 \end{vmatrix} + 0 = b + 2cx$$

Now use x = 0

$$b = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -1$$