1. The improper integral $\int_0^{\infty} e^{-x} dx$ is …… and the value is….

(a) Convergent, 1 (b) Divergent, 1
(c) Convergent, 0 (d) Divergent, 0

2. $\int_1^2 \frac{x+1}{\sqrt{x-1}} dx$ is

(a) Convergent and equal to $\frac{14}{3}$

(b) Divergent and equal to $\frac{3}{14}$

(c) Convergent and equal to ∞

(d) Divergent and equal to ∞

3. $\int_1^2 \frac{dx}{x^2-5x+4} dx$ is

(a) Convergent and equal to $\frac{1}{3}\log 2$

(b) Convergent and equal to 3/log2

(c) Divergent

(d) None of these

4. $\int_{-10}^{20} [\cot^{-1} x] dx$, where [.] denotes greatest integer function

(a) $30 + \cot 1 + \cot 3$

(b) $30 + \cot 1 + \cot 2 + \cot 3$

(c) 8 $30 + \cot 1 + \cot 2$

(d) None of these

5. $\int_0^2 [x^2 - x + 1] dx$, where [.] denotes greatest integer function

(a) $\frac{7-\sqrt{5}}{2}$ (b) $\frac{7+\sqrt{5}}{2}$

(c) $\frac{\sqrt{5}-3}{2}$ (d) None of these

6. If $f(x) = \int_{1/x}^{\sqrt{x}} \cos t^2 \, dt$ $(x > 0)$ then $\frac{df(x)}{dx}$ is

(a) $\frac{\sqrt{x}\cos x + 2\cos(x^{-2})}{2x\sqrt{x}}$

(b) $\frac{x\sqrt{x}\cos x + 2\cos(x^{-2})}{2x^2}$

(c) $2\sqrt{x}\cos x - \frac{2}{x}\cos\left(\frac{1}{x}\right)$

(d) None of these.

7. $\int_0^{\pi} x \, f(\sin x) \, dx$ is equal to

(a) $\pi \int_0^{\pi} f(\sin x) dx$ (b) $\pi \int_0^{\frac{\pi}{2}} f(\sin x) dx$

(c) $2\pi \int_0^{\frac{\pi}{2}} f(\sin x) dx$ (d) None of these

8. If $f(a + b - x) = f(x)$ then $\int_a^b x \, f(x) \, dx$ is equal to

(a) $\frac{a-b}{2} \int_a^b f(x) \, dx$

(b) $\left(\frac{a+b}{2}\right) \int_a^b f(x) \, dx$

(c) 0

(d) None of these

9. The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\tan^5 x}$ is

(a) 1 (b) $\pi/12$ (c) $\pi/6$ (d) None of these

10. If $I = \int_{-1}^{1} \left(\frac{x^2 + \sin x}{1+x^2}\right) dx$ then

(a) 0 (b) 2 (c) $\pi/2$ (d) $2 - \pi/2$

11. The value of $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{r}{n+r}\right)$ is

(a) ln 2 (b) 1 + ln 2 (c) 1 - ln 2 (d) 0

12. If f is an odd function, then $I = \int_{-a}^{a} \frac{f(\sin x)}{f(\cos x) + f(\sin^2 x)} dx$

(a) Can't be evaluated (b) I = 0

(c) $I = \frac{\pi}{2}$ (d) None of these

13. $\int_0^{\infty} [2\, e^{-x}] dx$, where [.] denotes the greatest integer function, is equal to

(a) 0 (b) ln 2 (c) e^2 (d) $2e^{-1}$

14. The value of $\int_0^{\pi/2} \operatorname{cosec}(x - \pi/3) \operatorname{cosec}(x - \pi/.6) dx$ is

(a) 2log3 (b) -2log 3 (c) log3 (d) None of these

15. If $I_n = \int_0^{\pi/4} \tan^n x \, dx, n \in N$, then $I_{n+2} + I_n$ equals

(a) $\frac{1}{n}$ (b) $\frac{1}{n-1}$

(c) $\frac{1}{n+1}$ (d) $\frac{1}{n+2}$

16. If the value of the integral $\int_1^2 e^{x^2} dx = \alpha$, then the value of $\int_e^{e^4} \sqrt{\ell n\, x} \, dx$ is

(a) $e^4 - e - \alpha$ (b) $2e^4 - e - \alpha$
(c) $2(e^4 - e) - \alpha$ (d) None

17. If $I_1 = \int_0^{\pi/2} \frac{x}{\sin x} dx$ and $I_2 = \int_0^1 \frac{\tan^{-1} x}{x} dx$, then $\frac{I_1}{I_2} =$

(a) 1 (b) 1/2 (c) 2 (d) $\pi/2$

18. $\int_0^{\pi/4} \left(\frac{x}{x \sin x + \cos x}\right)^2 dx =$

(a) $\frac{3+\pi}{4-\pi}$ (b) $\frac{4-\pi}{3+\pi}$

(c) $\frac{4-\pi}{4+\pi}$ (d) $\frac{4+\pi}{4-\pi}$

19. Which of the following is not correct ?

(a) $\int_{-a}^{a} x.(f(\cos x))^2.dx = 0$

(b) $\int_0^{n\pi} f(\cos^2 x)dx = n\int_0^{\pi} f(\cos^2 x)dx \quad n \in N$

(c) $\int_0^{b-c} f(x+c)dx = \int_b^c f(x)dx$

(d) $\int_a^{\pi-a} x.f(\sin x)dx = \frac{\pi}{2}\int_a^{\pi-a} f(\sin x)dx$

20. Let $I_1 = \int_0^1 \frac{e^x dx}{x+1}$ and $I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)}$, then $\frac{I_1}{I_2}$ is equal to

(a) $\frac{3}{e}$ (b) $\frac{e}{3}$

(c) 3e (d) $\frac{1}{3e}$