1. (a)

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 0 & c-a & a-b \\ 0 & c'-a' & a'-b' \\ 0 & c''-a'' & a''-b'' \end{vmatrix} = m \begin{vmatrix} a & b & c \\ a' & b' & c' \\ a'' & b' & c'' \end{vmatrix} \Rightarrow m = 0$$

2. (c)

Putting x = 0,

$$\begin{vmatrix} 0 & 3 & -2 \\ -2 & 0 & -1 \\ -3 & 2 & 0 \end{vmatrix} = f$$

or $f = 17$

3. (a)

$$D' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + pqr \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$$

(All other determinants vanish)

$D' = (1 + pqr) D$

4. (c)

$$[x] = -1, [y] = 0, [z] = 1 \therefore \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$$

5. (d)

$|AdjA|^2 = |A|^{(n-1)^2} = (11)^{2^2} = 11^4 = 14641$

6. (a)

$$\begin{vmatrix} x-\beta & \alpha-x & 0 \\ 0 & x-\gamma & 0 \\ \beta & \gamma & 1 \end{vmatrix} = 0$$

$\Rightarrow (x-\beta)(x-\gamma) = 0$

Hence roots are independent of α

7. (c)

$$\Delta(x) = \begin{vmatrix} 1-2\sin^2 x & \sin^2 x & 1-8\sin^2 x+8\sin^4 x \\ \sin^2 x & 1-2\sin^2 x & 1-\sin^2 x \\ 1-8\sin^2 x+8\sin^4 x & 1-\sin^2 x & 1-2\sin^2 x \end{vmatrix}$$

Put x = 0

$$\Delta(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$$

8. (d)

$$a^n b^n c^n \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$a^n b^n c^n (a - b)(b - c)(c - a)$

$\Rightarrow$ comparing with R.H.S

$n = 0$

9. (c)

If we put x = 0 then Three rows are same so

$(x - 0)^2$ is factor of Δ

Now, $C_1 \to C_1 + C_2 + C_3$ &

Take common $(x + \alpha + \beta + \gamma)$

So $\Delta = x^2 (x + \alpha + \beta + \gamma) \Rightarrow x = 0, -\alpha - \beta - \gamma$

10. (b)

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$$\Delta = \begin{vmatrix} x^2+4x+5 & x+2 & 5 \\ -2x & -1 & 0 \\ -18x & -9 & 0 \end{vmatrix}$$

$\Delta = 5(18x - 18x)$

$\Delta = 0$

So $y = \Delta = 0$

y = 0 is a line passing through origin.

11. (b)

$\Delta = -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

we know $\dfrac{a^2+b^2}{2} \geq ab$ (by AM $\geq$ GM)

$\dfrac{b^2+c^2}{2} \geq bc \Rightarrow \dfrac{a^2+c^2}{2} \geq ac \Rightarrow a^2 + b^2 + c^2 \geq ab + bc + ac$

$a^2 + b^2 + c^2 - ab - bc - ac \geq 0$

Now, $\Delta = -(+)(+)\ \Delta \leq 0$

12. (c)

$A^2 + A - I = 0 \Rightarrow A^{-1}(A^2 + A - I) = A^{-1}.0$

$\Rightarrow A + A^{-1}A - A^{-1}I = 0$

$\Rightarrow (A + I) - A^{-1} = 0 \qquad \Rightarrow A^{-1} = A + I$

13. (a)

If matrix A is singular. Then |A| =0

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$

$\Rightarrow \lambda = 3$

14. (d)

$\det(3A) = k\{\det(A)\}$

$\Rightarrow 3^3 \det(A) = k\{\det(A)\}$

$\therefore k = 27$

15. (b)

$x^3 - 1 = 0 \quad \therefore x = 1, \omega, \omega^2;$

Here $\alpha = \omega \quad \beta = \omega^2$

16. (a)

Let $\Delta(x) = A + Bx + Cx^2 + Dx^3 + \ldots\ldots\ldots$

$$\therefore \Delta(0) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0 \Rightarrow A = 0$$

$$\Delta'(0)=\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}+\begin{vmatrix}0&1&0\\0&1&1\\1&0&0\end{vmatrix}+\begin{vmatrix}0&1&0\\0&1&0\\1&0&0\end{vmatrix}=1$$

$\therefore$ B= 1

$\Rightarrow$ then $\Delta(x) = x + Cx^2 + Dx^3 + ...$

$\therefore$ Δx is divisible by x.

17. (a)

Let $\begin{vmatrix}-2a & a+b & a+c\\ b+a & -2b & b+c\\ c+a & c+b & -2c\end{vmatrix} = k(b+c)(c+a)(a+b)$ Putting a = b = c = □

then $\begin{vmatrix}-2\lambda & 2\lambda & 2\lambda\\ 2\lambda & -2\lambda & 2\lambda\\ 2\lambda & 2\lambda & -2\lambda\end{vmatrix} = k(2\lambda)(2\lambda)(2\lambda)$

$\therefore\ \kappa = \begin{vmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{vmatrix} = -1(0)-1(-2)+1(2)=4$

18. (b)

$$f(x)=x(x+1)\begin{vmatrix}1&1&1\\2x&x-1&x\\3x(x-1)&(x-1)(x-2)&x(x-1)\end{vmatrix}$$

$$= x(x+1)(x-1)\begin{vmatrix}1&1&1\\2x&x-1&x\\3x&x-2&x\end{vmatrix}$$

Applying $\begin{matrix}C_2\to C_2-C_1\\ C_3\to C_3-C_1\end{matrix}$ $\quad\therefore\quad f(200)=0$

19. (b)

Let $\Delta=\begin{vmatrix}x+1&x+2&x+\lambda\\x+2&x+3&x+\mu\\x+3&x+4&x+\nu\end{vmatrix}$

Applying $R_2 \to R_2 - \frac{1}{2}(R_1+R_3)$

$$=\begin{vmatrix}x+1&x+2&x+\lambda\\0&0&\mu-\frac{\lambda+\nu}{2}\\x+3&x+4&x+\nu\end{vmatrix}$$

$$=\left(\mu-\frac{\lambda+\nu}{2}\right)(x+1)(x+4)-(x+2)(x+3)=0$$

$$=\left(\mu-\frac{\lambda+\nu}{2}\right)(-2)=0$$

$\Rightarrow 0=0\ \ (\because \lambda,\mu,\nu\ are\ in\ A.P.)$

Hence Δ is an identify in x.

20. (d)

$$\therefore\ f(-x)=\begin{vmatrix}-x&\cos x&e^{x^2}\\-\sin x&x^2&\sec x\\-\tan x&1&2\end{vmatrix}=-f(x)$$

$\therefore \int_{-\pi/2}^{\pi/2} f(x)\,dx = 0 \qquad [\because f(x)\ is\ odd\ function]$

21. (b)

For non-trivial solution

$$\begin{vmatrix}a-1&-1&-1\\1&1-b&1\\1&1&1-c\end{vmatrix}=0$$

Applying $C_1 \to C_1 - C_2$ *and* $C_2 \to C_2 - C_3$ then

$$\begin{vmatrix}a&0&-1\\b&-b&1\\0&c&1-c\end{vmatrix}=0$$

$\Rightarrow a(-b+bc-c)-0-1(bc+0)=0$

$\Rightarrow -ab+abc-ac-bc=0$

$\therefore ab+bc+ca=abc$

22. (d)

$\alpha+\beta+\gamma$ = -a, $\sum\alpha\beta = 0$

using $R_1 \to R_1+R_2+R_3$

$$\Delta=\begin{vmatrix}\alpha+\beta+\gamma&\alpha+\beta+\gamma&\alpha+\beta+\gamma\\\beta&\gamma&\alpha\\\gamma&\alpha&\beta\end{vmatrix}\ \Delta = -a\begin{vmatrix}1&1&1\\\beta&\gamma&\alpha\\\gamma&\alpha&\beta\end{vmatrix}$$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

or $\Delta = -a\begin{vmatrix}1&0&0\\\beta&\gamma-\beta&\alpha-\beta\\\gamma&\alpha-\gamma&\beta-\gamma\end{vmatrix}$

23. **(c)**

For non trivial solution $\Rightarrow \Delta = 0$

$\Delta=\begin{vmatrix}1&a&a\\b&1&b\\c&c&1\end{vmatrix}=\Delta=\begin{vmatrix}1-a&0&a\\b-1&1-b&b\\c&c-1&1\end{vmatrix}=0\Rightarrow$n(1-a)(1-b)(1-c)×

$\begin{vmatrix}1&0&\frac{a}{1-a}\\-1&1&\frac{b}{1-b}\\0&-1&\frac{1}{1-c}\end{vmatrix}$ = 0⇒ 1/1-a + b/1-b + c/1-c = - 1

24. **(d)**

$$\frac{1}{abc}\begin{vmatrix}ab^2c^2&abc&ab+ac\\bc^2a^2&abc&bc+ab\\a^2b^2c&abc&ac+bc\end{vmatrix}$$

$$= \frac{a^2b^2c^2}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ac & 1 & bc+ab \\ ab & 1 & ac+bc \end{vmatrix} = abc\begin{vmatrix} bc & 1 & ab+bc+ca \\ ac & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix} = 0$$

25. (c)

The system of linear equation has a non zero solution.

$$\Delta = \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

Solving $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow$ a, b, c are in H.P.