1. (c)

$F(x)\,G(N)]^{-1}\,[F(x)]^{-1}$

$$[F(x)]^{-1} = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(-x)$$

Similarly, $[G(n)]^{-1}$ -G(-y) F(-x)

2. (d)

$$AA^{-1} = \frac{1}{9}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 & 0 & x+4+2y \\ 0 & 9 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+4+y^2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow x = -2,\ y = -1$

$\Rightarrow x + y = -3$

3. (b)

$|A_r| = r^2 - (r-1)^2 = 2r - 1$

$\Rightarrow |A_1| + |A_2| + \ldots\ldots + |A_{2006}|$

$$= \sum_{r=1}^{2006}(2r-1) = (2006)^2$$

4. (a)

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow A^n = nA - (n-1)I$$

$\Rightarrow A = nA - (n-1)A = A$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$A^{m+1} = A^m.\,A = (mA - (m-1)A)\,A$

$= mA^2 - mA + A$

$$= m\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - mA + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} m+1 & 0 \\ m+1 & m+1 \end{bmatrix} - m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= (m+1)A - mA = A$

So (A) is true

5. (b)

$A = A^T \Rightarrow$ symmetric matrix

6. (c)

$A^T = -A$

$(A^n)^T = (A\,A\,\ldots.\,A)^T = (A^T\,A^T\,\ldots\,A^T) = (A^T)^n$

$= (-A)^n$

$= -A$

Hence skew symmetric matrix.

7. (c)

$$A^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$AA^T = \frac{1}{81}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix} = I$$

$\Rightarrow$ A is orthogonal matrix

Hence (C) is correct.

8. (c)

For n = 2 $\Rightarrow$(A-1 BA) (A-1BA) = A-1B2A

(A-1 BA)3 = (A-1BA)2 (A-1BA) = (A-1B2A) (A-1BA)

= A-1B3A and so on thus (A-1BA)n = A-1BnA

9. (d)

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 2c+4d \end{bmatrix}$$

$$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

if AB = BA, then a + 2c = a + 3b

$\Rightarrow 2c = 3b \quad \Rightarrow \quad b \neq 0$

b + 2d = 2a + 4b

$\Rightarrow \quad 2a - 2d = -3b$

$$\frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b - \frac{3}{2}b} = -1$$

10. (b)

$A^3 = O$

$(I + A + A^2)(I - A) = I - A^3 = I$

$\therefore I + A + A^2 = (I - A)^{-1}$

11. (d)

$x = (\alpha\gamma\ ABC^2D)^3$

$|x| = |\ \alpha\ \gamma\ ABC^2D\ |^3$

order of ABC^2D is 2×2

$|x| = (\alpha^2\,\gamma^2)^3\ |ABC^2D|^3$

$\therefore\ k = \alpha^6\,\gamma^6$

12. (b)

A' = A

$$\therefore \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix} = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$$

$\therefore x + 2 = 2x - 3 \Rightarrow x = 5$

13. (d)

U = [2 -3 4]

$$V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix},\ X = [0\ \ 2\ \ 3],\ Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$$\text{Then } U.V + X.\,Y = [2\ \ {-3}\ \ 4].\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + [0\ \ 2\ \ 3]\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

= [6 - 6 + 4] + [0 + 4 + 12]

= [4] + [16] = [20]

14. (d)

$$\text{Adj } A = \begin{bmatrix} +(-3) & -6 & +(-6) \\ -(-6) & +3 & -6 \\ +6 & -6 & +3 \end{bmatrix}$$

$$= 3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}'$$

$= 3.\ A^T$

15. (c)

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix} \Rightarrow A = \frac{1}{3}B \text{ (Let)}$$

and AA' = I

$$\Rightarrow \left(\frac{1}{3}.B\right).\ \left(\frac{1}{3}.B\right)' = I$$

= B.B' = 9 I

$$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}.\begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow x + 4 + 2y = 0,\ 2x + 2 - 2y = 0$

$\Rightarrow x = -2,\ y = -1$

$\therefore\ x + y = (-2) + (-1)$

$= -3$

16. (d)

$A^3 - 5A^2 + 5A + k.I = 0$

$A^{-1}(A^3 - 5A^2 + 5A + kI) = A^{-1} \times (0)$

$A^2 - 5A + 5I + kA^{-1} = 0$

$$A^{-1} = -\frac{1}{k}(A^2 - 5A + 5I)$$

$\therefore\ A^{-1}$ exists if $k \neq 0$

17. (d)

$|B^{-1}.A.B| = |B^{-1}|.\ |A|\ .\ |B|$

$= (|B^{-1}|.\ |B|).\ |A|$

$= 1 \times |A|\ (\because\ |B^{-1}|.\ |B| = 1)$

$= |A|$

18. (c)

$(B^{-1}.\ A.\ B)^4$

$= (B^{-1}.\ A.B).(B^{-1}.A.B)\ (B^{-1}.A.B).(B^{-1}.A.B)$

$B^{-1}.\ A(I)\quad A\ .\ (I)\quad A\ .\ (I)\ A.B$

$= B^{-1}\ (A)\ (A)(A)(A).\ B$

$= B^{-1}.\ A^4.\ B$

19. (a)

$A\ .\ B = I \Rightarrow B = A^{-1}$

Also $B = A' \Rightarrow A^{-1} = A'$

20. (c)

A = Singular $\Rightarrow |A| = 0$

$$\begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$

$\Rightarrow 56\lambda - 128 - 36\lambda + 48 + 20 = 0$

$\Rightarrow 20\lambda = 60 \Rightarrow \lambda = 3$

21. (a)

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix},\ U_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix},\ U_3 = \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix}$$

$$A.\ U_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 \\ 2a_1 + b_1 \\ 3a_1 + 2b_1 + c_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a_1 = 1,\ b_1 = -2,\ c_1 = 1$

$$\Rightarrow U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\text{Similarly } U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix} \ \&\ U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \Rightarrow |U| = 3$$

22. (c)

$\because\ |A| = 68$

$\therefore\ A\ (\text{adj } A) = |A|.\ I$

23. (b)

Let order of B is $m \times n$

$\because\ A'B$ is define so m = 3

$\therefore BA'$ is define so n = 4

24. (a)

We have

$$E(\alpha)\ E(\beta) = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta\cos(\alpha-\beta) & \cos\alpha\sin\beta\cos(\alpha-\beta) \\ \sin\alpha\cos\beta\cos(\alpha-\beta) & \sin\alpha\sin\beta\cos(\alpha-\beta) \end{bmatrix}$$ As α and β differ by an odd multiple of $\pi/2$,

$\alpha - \beta = (2n + 1)\ \pi/2$ for some As α and β integer n. Thus,

$\cos\ [(2n + 1)\pi/2] = 0$

$\therefore\ E\ (\alpha)\ E\ (\beta) = O.$

25. (b)

We have $A' = -A$

Now, $AA^{-1} = A^{-1} A = I_n$

$\Rightarrow (AA^{-1})' = (A^{-1} A)' = (I_n)'$

$\Rightarrow (A^{-1})' A' = A'(A^{-1})' = I_n$

$\Rightarrow (A^{-1})' (-A) = (-A) (A^{-1})' = I_n$ Thus, $(A^{-1})' = -(A^{-1})$

[inverse of a matrix is unique].

26. (b)

If A is non-singular, A^{-1} exists.

Thus, $AB = AC \Rightarrow A^{-1}(AB) = A^{-1} (AC)$

$\Rightarrow (A^{-1}A) B = (A^{-1} A) C \Rightarrow IB = IC$

$\Rightarrow B = C.$

27. (b)

As $B = -A^{-1} BA$, we get $AB = -BA$ or $AB + BA = O$

Now, $(A + B)^2 = (A + B) (A + B)$

$= A^2 + BA + AB + B^2$

$= A^2 + O + B^2$

$= A^2 + B^2$

28. (a)

As A is a non-singular matrix $A^{-1} = \dfrac{1}{|A|}$ (adj A)

$\Rightarrow$ adj $A = |A|\, A^{-1} = B$(say).

Now,

adj (adj A) = adj (B) $= |B|B^{-1} = \|A|A^{-1}|\, (|A|\, A^{-1})^{-1}$

$= |A|^3\, |A^{-1}|\, |A|^{-1}\, (A^{-1})^{-1}$

[using scalar multiple property of determinants]

$= |A|^3\, \dfrac{1}{|A|} \cdot \dfrac{1}{|A|} A = |A|\, A.$

29. (c)

The system of equations will have a non-zero solution if and only if

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a+2 \\ a^3 & (a+1)^3 & (a+2)^3 \end{vmatrix} = 0$$

Using $C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$, we get

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & 3a^2+3a+1 & 3a^2+9a+7 \end{vmatrix} = 0$$

$\Rightarrow 6a + 6 = 0$ or $a + 1 = 0$

Or $a = -1$.

30. (c)

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 5 & -3 \\ 1 & 2 \end{bmatrix} = x\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \dfrac{5}{13} = 2x + y,\ \dfrac{-3}{13} = 3x$

$\Rightarrow \dfrac{1}{13} = -x,\ \dfrac{2}{13} = 5x + y$

$x = \dfrac{-1}{13},\ y = \dfrac{7}{13}.$

31. (a)

$A^2 = A \Rightarrow |A^2| = |A| \Rightarrow |A|^2 - |A| = 0$

$|A|\, [|A| - 1] = 0 \Rightarrow |A| = 0$ or $|A| = 1$

32. (c)

$\Rightarrow BC = CA\, CA^3\, C^{-1} \Rightarrow C^{-1}.B.C. = A = C.A.A.A.C^{-1}$

$\Rightarrow C(C^{-1}.B.C.)(C^{-1}.BC)(C^{-1}.BC).C^{-1} \Rightarrow (\pm)B.(I).(B).I(B).I$

$= B^3$

33. (b)

As $B = -A^{-1}BA$

$\Rightarrow AB = -BA \Rightarrow AB + BA = 0$

Now, $(A + B)^2 = A^2 + BA + AB + B^2$

$= A^2 + B^2$

34. (c)

$$A^2 = \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3/2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{64} = \begin{bmatrix} 1 & 32 \\ 0 & 1 \end{bmatrix}$$

35. (a)

$$\begin{bmatrix} x+2y+3z \\ 3x+y+2z \\ 2x+3y+z \end{bmatrix} = \begin{bmatrix} 8-2 \\ 0-6 \\ -2+2 \end{bmatrix}$$

$\Rightarrow x + 2y + 3z = 6$ …(1)

$\Rightarrow 3x + y + 2z = -6$ …(2)

$2x + 3y + z = 0$ …(3)

Solving equation (1), (2), (3) we get

$x = -4,\ y = 2,\ z = 2.$

36. (c)

$$A^TA = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ac+ab+bc \\ ab+bc+ca & b^2+c^2+a^2 & bc+ca+ab \\ ca+ab+bc & bc+ac+ab & c^2+a^2+b^2 \end{bmatrix}$$

$A^TA = I \Rightarrow a^2 + b^2 + c^2 = 1,\ ab + bc + ca = 0$

$\therefore (a + b + c)^2 = 1 + 2(0) = 1 \Rightarrow a + b + c = 1$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^2 = 3x\,1 + 1 = 4$

37. (b)

$\det.(M_r) = r^2 - (r-1)^2 = 2r - 1$

$$\sum_{r=1}^{2009}(2r-1) = 2 \times \left(\frac{2009 \times 2010}{2}\right) - 2009$$

$= (2009) \times (2009) = (2009)^2$

38. (b)

$|A|$ adj $(A^{-1}) = ?$

A^{-1} adj $(A^{-1}) = |A^{-1}|\, I_3$

$\Rightarrow A.\ A^{-1}$ adj $(A^{-1}) = |A^{-1}|\, AI_3$

$\Rightarrow$ adj $(A^{-1}) = |A^{-1}|$ A

$\Rightarrow$ |A| adj (A^{-1}) = A

Here |A| = 13But $|A| \neq 0$

39. (b)

Since A is a singular matrix

then $\begin{vmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$

$\lambda = 4$

40. (a)

tr (A) = 0