1. (d)
Any relation from A to B will be a sub set of A × B. Now total elements in A × B is equal to mn. That implies that number of non-empty subsets of A × B is equal to $2^{nm} - 1$. Thus there are $2^{mn} - 1$ relations.

2. (b)
We must have
$2\{x\}^2 - 3\{x\} + 1 \geq 0 \Rightarrow \{x\} \geq 1$ or $\{x\} \leq 1/2$
Thus we have $0 < \{x\} \leq 1/2$
$\Rightarrow x \in \left[n, n+\frac{1}{2}\right]$, $n \in I$.

3. (c)
We have $1 \geq \cos x \geq -1$
$\Rightarrow 2 \geq -2\cos x \geq -2$
$\Rightarrow 3 \geq 1 - 2\cos x \geq -1$
$\Rightarrow \frac{1}{1-2\cos x} \in (-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

4. (d)
Clearly the domain of the function is [-1, 1]. We know that $\tan^{-1} x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ for $x \in [-1, 1]$. Thus range is $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

5. (c)
Period of $\sin^{-1}(\sin x)$ is 2π. Period of $e^{\tan x}$ is π. Thus period of $f(x) = \text{L.C.M.}(2\pi, \pi) = 2\pi$

6. (d)
Period of $\frac{2^x}{2^{[x]}}$ is 1. Period of $\sin^{-1}\{x\}$ is 1.
Period of $\sin^{-1}(\sqrt{\cos x})$ is 2π where as
$\text{Sin}^{-1}(\cos x^2)$ is non-periodic.

7. (d)
Periodic functions may not be bounded e.g., tanx, cotx.

8. (c)
$f(f(x)) = \frac{1}{1-\frac{1}{1-x}} = \frac{x-1}{x}$, $x \neq 0, 1$
$f(f(f(x))) = \frac{\frac{x-1}{x}-1}{\frac{x-1}{x}} = \frac{1}{1-x} = f(x)$, $x \neq 1, 0$

9. (b)
$\left[x^2+\frac{1}{2}\right] = \left[x^2-\frac{1}{2}+1\right] = 1+\left[x^2-\frac{1}{2}\right]$.
Thus for domain point of view.
$\left[x^2-\frac{1}{2}\right] = 0, 1 \Rightarrow \left[x^2+\frac{1}{2}\right] = 1, 0$
$\Rightarrow f(x) = \sin^{-1}(1) + \cos^{-1}(0)$
or $\sin^{-1}(0) + \cos^{-1}(-1)$
$\Rightarrow f(x) = \{\pi\}$

10. (d)
Odd extension of f(x) in $(0, \infty)$ can be obtained by replacing x by -x and multiplying throughout by -ve sign.

11. (c)
For domain point of view, $0 \leq x^2 + x + 1 \leq 1$. but $x^2 + x + 1 > \frac{3}{4} \Rightarrow \frac{\sqrt{3}}{4} \leq \sqrt{x^2+x+1} \leq 1$
$\Rightarrow \frac{\pi}{3} \leq \sin^{-1}(\sqrt{x^2+x+1}) \leq \frac{\pi}{2}$

12. (c)
$\log\left(\frac{x^4+x^2+1}{x^2+x+1}\right) = \log(x^2 - x + 1)$, which is neither odd nor even.

13. (b)
For f(x) to be odd, $\left[\frac{x^2}{|a|}\right]$ should not depend upon the value of x.
Since $x \in [-4, 4] \Rightarrow 0 \leq x^2 \leq 16$
$\Rightarrow \left[\frac{x^2}{|a|}\right] = 0$ if $|a| > 16$
$\Rightarrow a \in (-\infty, -16) \cup (16, \infty)$

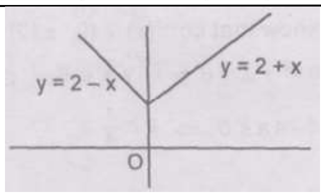
14. (c)
$f(x) + 2f(1 - x) = x^2 + 1$
Replacing $x \to (1 - x)$ we get; $f(1 - x) + 2f(x) = (1 - x)^2 + 1$
$\Rightarrow f(1 - x) + 2f(x) = 2 + x^2 - 2x$
From these equations we get $f(x) = \frac{1}{3}(x^2 - 4x + 3)$

15. (d)
We must have $-1 \leq \left[\log_2\left(\frac{x^2}{2}\right)\right] \leq 1$
$\Rightarrow -1 \leq \log_2\left(\frac{x^2}{2}\right) < 2$
$\Rightarrow \frac{1}{2} \leq \left(\frac{x^2}{2}\right) < 4 \Rightarrow 1 \leq x^2 < 8$
$\Rightarrow x \in (-\sqrt{8}, -1] \cup [1, \sqrt{8})$

16. (c)
$2^{f(x)} = 2 - 2^x \Rightarrow 2 - 2^x > 0 \Rightarrow 2^X < 2 \Rightarrow x < 1$

17. (b)
Putting x = 0, y = 0 in the functional equation we get $2f(0) = 2f^2(0) \Rightarrow f(0) = 0, 1$. But if f(0) is equal to zero then f(x) = 0 $\forall x \in R$. Thus f(0) = 1. Now putting x = 0 in the functional equation we get, $f(y) + f(-y) = 2f(0).\ f(y)$
$\Rightarrow f(y) = f(-y)$. Thus f(x) is even.

18. (a)
$f(f(x)) = \begin{cases} 2+f(x) & , f(x) \geq 0 \\ 2-f(x) & , f(x) < 0 \end{cases}$
Since $f(x) \geq 2\ \forall x \in R$
$\Rightarrow f(f(x)) = \begin{cases} 4+x & , x \geq 0 \\ 4-x & , x < 0 \end{cases}$



19. (b)

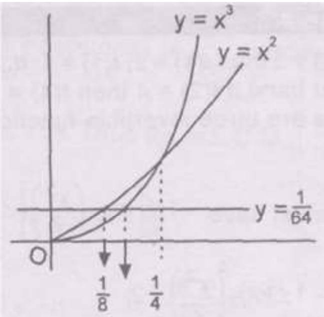
f(x) = [x] + [2x] + [3x] + ... + [nx] – (x + 2x + 3x + ...nx)

= -({x} + {2x} + {3x}+......+ {nx})

Period of f(x) = L.C.M. $\left(1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots, \frac{1}{n}\right) = 1$

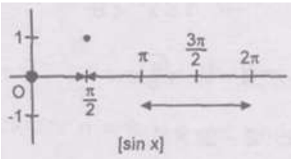
20. (c)

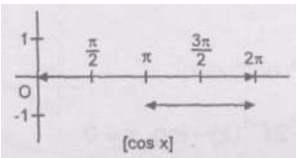
Clearly f(x) = $\begin{cases} \frac{1}{64}, & 0 \le x \le \frac{1}{8} \\ x^2, & \frac{1}{8} < x \le 1 \\ x^3, & x > 1 \end{cases}$

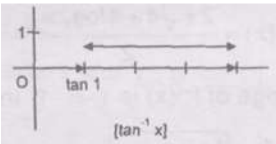


21. (c)

f(x) = [sinx] + [cosx] + [$\tan^{-1}$x]. We get







$$f(x) = \begin{cases} 1, & x = 0 \\ 0, & 0 < x < \tan 1 \\ 1, & \tan 1 \le x < \pi/2 \\ 2, & x = \pi/2 \\ 0, & \pi/2 < x < \pi \\ -1, & \pi \le x < 3\pi/2\pi \\ 0, & 3\pi/2 \le x < 2\pi \\ 2, & x = 2\pi \end{cases}$$

Thus range of f(x) = {-1, 0, 1, 2}.

22. (d)

$f(x) = \frac{n(n+1)}{2} + [\sin x] + \left[\sin \frac{x}{2}\right]$ +... $+ \left[\sin \frac{x}{n}\right]$

Since x ∈ [0, π] . Thus range is

$\left\{\frac{n(n+1)}{2}, \frac{n(n+1)}{2} + 1\right\}$

$\Rightarrow \left(\left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1\right) = 0$

Clearly $g(x) = \left(\frac{2}{5}\right)^x + \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$ is a decreasing function and $\lim_{x \to \infty} g(x) = -1$, $\lim_{x \to -\infty} g(x) = \infty$ also g(0) = 1. Thus f(x) = 0 has exactly one root.

23. (d)

The function in **(a)** is periodic with period 1 and the function in **(b)** is also periodic since $f(x+r) = f(x)$ for every rational r. The function in **(c)** is equal to $\frac{4}{|\sin x|}$ and thus has period π.

All are periodic. In 'b' there is no period.

24. (b)

The given function is f (x) = $\frac{x}{1+x}$ = 1 - $\frac{1}{1+x}$.

Hence its range is [0, 1) which is a subset of [0, ∞). Also the function is one-one

25. (c)

Here, $\frac{x^2+1}{x^2+2} = 1 - \frac{1}{x^2+2}$

Now, $2 \le x^2 + 2 < \infty$ for all x∈R

$\Rightarrow \frac{1}{2} \ge \frac{1}{x^2+2} > 0 \Rightarrow -\frac{1}{2} \le \frac{-1}{x^2+2} < 0$

$\Rightarrow \frac{1}{2} \le 1 - \frac{1}{x^2+2} < 1 \Rightarrow \frac{\pi}{6} \le \sin^{-1}\left(1 - \frac{1}{x^2+2}\right) < \frac{\pi}{2}$

26. (a)

Clearly period of |sin 2x| = π/2 and period of |cos 8x| = π/8.

Now L.C.M. (π/8, π/2) = π/2 ⇒ Period of the given function is π/2

27. (b)

f(x) = $\sqrt{3}$ sin x - cos x + 2 = 2 sin $\left(x - \frac{\pi}{6}\right)$ + 2.

Since f(x) is one-one and onto, f is invertible.

Now fof $^{-1}$(x) = x

$\Rightarrow 2 \sin\left(f^{-1}(x) - \frac{\pi}{6}\right) + 2 = x \Rightarrow \sin\left(f^{-1}(x) - \frac{\pi}{6}\right) = \frac{x}{2} - 1$

$\Rightarrow f^{-1}(x) = \sin^{-1}\left(\frac{x}{2} - 1\right) + \frac{\pi}{6}$, because $\left|\frac{x}{2} - 1\right| \le 1$ for all x ∈ [0, 4]

28. (b)

For f(x) to be defined $\log_{10}(1+x^3) > 0$

$\Rightarrow 1 + x^3 > 10^0 = 1$ or $x^3 > 0 \Rightarrow x \in (0, \infty)$

29. (c)

fog = f(g(x)) = |sin x| = $\sqrt{\sin^2 x}$.

Also gof = g(f(x)) = $\sin^2\sqrt{x}$.

Obviously, $\sqrt{\sin^2 x} = \sqrt{g(x)}$ and $\sin^2\sqrt{x} = \sin^2(f(x))$

i.e. g(x) = $\sin^2 x$ and f(x) = $\sqrt{x}$

30. (a)

Here $\left|\sin\frac{x}{2}\right|$ is a periodic function with period 2π and $|\cos x|$ is a periodic function with period π.

Therefore period of f(x) is 2π

31. (c)

Let $f_1 = \sqrt{\log_{1/4}\left(\frac{5x-x^2}{4}\right)}$ and $f_2 = {}^{10}C_x$.

Clearly f_1 is defined for $\log_{1/4}\left(\frac{5x-x^2}{4}\right) \geq 0$

$\Rightarrow 0 < \frac{5x-x^2}{4} \leq 1$

$\Rightarrow \frac{5x-x^2}{4} > 0$ and $\frac{5x-x^2}{4} \leq 1$

$\Rightarrow x(x-5) < 0$ and $x^2 - 5x + 4 \geq 0$

$\Rightarrow x \in (0, 5)$ and $x \in (-\infty, 1] \cup [4, \infty)$

$\Rightarrow f_1$ is defined for $x \in (0, 1] \cup [4, 5)$ and f_2 is defined for $x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \Rightarrow f(x)$ is defined for $x \in D_{f1} \cap D_{f2} = \{1, 4\}$

32. (c)

$\lim_{x\to3+} f(x) = 5-3 = 2$ and $\lim_{x\to3-} f(x) = \frac{2}{5-3} = 1$

33. (d)

L.H.L. $= \lim_{x\to1-0} f(x) = \lim_{h\to0} f(1-h) = \lim_{h\to0} 3(1-h)$

$= \lim_{h\to0}(3-3h) = 3-3.0 = 3$

R.H.L. $= \lim_{x\to1+0} f(x) = \lim_{h\to0} f(1+h) = \lim_{h\to0}[5-3(1+h)]$

$= \lim_{h\to0}(2-3h) = 2-3.0 = 2$

Hence $\lim_{x\to1} f(x)$ does not exists.

34. (d)

$f(x) = a\cos(bx+c)+d$ ….. (i)

For minimum $\cos(bx+c) = -1$

from (i), $f(x) = -a+d = (d-a)$,

for maximum $\cos(bx+c) = 1$

from (i), $f(x) = a+d = (d+a)$

$\therefore$ Range of $f(x) = [d-a, d+a]$ ..

35. (b)

$f(x) = \log(x+\sqrt{x^2+1})$ and $f(-x) = -\log(x+\sqrt{x^2+1}) = -f(x)$,

so $f(x)$ is an odd function.

36. (b)

In option (a), $f(-x) = \frac{a^{-x}+1}{a^{-x}-1} = \frac{1+a^x}{1-a^x} = -\frac{a^x+1}{a^x-1} = -f(x)$ So, It is an odd function.

In option (b),

$f(-x) = (-x)\frac{a^{-x}-1}{a^{-x}+1} = -x\frac{(1-a^x)}{1+a^x} = x\frac{(a^x-1)}{(a^x+1)} = f(x)$ So, It is an even function.

In option (c), $f(-x) = \frac{a^{-x}-a^x}{a^{-x}+a^x} = -f(x)$ So, It is an odd function.

In option (d), $f(-x) = \sin(-x) = -\sin x = -f(x)$ So, It is an odd function.

37. (b)

$f(x) = \sin\left(\log(x+\sqrt{1+x^2})\right)$

$\Rightarrow f(-x) = \sin[\log(-x+\sqrt{1+x^2})]$

$\Rightarrow f(-x) = \sin\log\left((\sqrt{1+x^2}-x)\frac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}\right)$

$\Rightarrow f(-x) = \sin\log\left[\frac{1}{(x+\sqrt{1+x^2})}\right]$

$\Rightarrow f(-x) = \sin\left[\log(x+\sqrt{1+x^2})^{-1}\right]$

$\Rightarrow f(-x) = \sin\left[-\log(x+\sqrt{1+x^2})\right]$

$\Rightarrow f(-x) = -\sin\left[\log(x+\sqrt{1+x^2})\right] \Rightarrow f(-x) = -f(x)$

$\therefore f(x)$ is odd function.

38. (a)

$f(x) = 2\cos\frac{1}{3}(x-\pi) = 2\cos\left(\frac{x}{3}-\frac{\pi}{3}\right)$

Now, since $\cos x$ has period $2\pi \Rightarrow \cos\left(\frac{x}{3}-\frac{\pi}{3}\right)$ has period $\frac{2\pi}{\frac{1}{3}} = 6\pi$

$\Rightarrow 2\cos\left(\frac{x}{3}-\frac{\pi}{3}\right)$ has period $= 6\pi$.

39. (b)

Here $|\sin 2x| = \sqrt{\sin^2 2x} = \sqrt{\frac{(1-\cos 4x)}{2}}$

Period of $\cos 4x$ is $\frac{\pi}{2}$. Hence, period of $|\sin 2x|$ will be $\frac{\pi}{2}$

Trick : $\because$ $\sin x$ has period $= 2\pi \Rightarrow \sin 2x$ has period $= \frac{2\pi}{2} = \pi$

Now, if $f(x)$ has period p then $|f(x)|$ has period $\frac{p}{2}$

$\Rightarrow |\sin 2x|$ has period $= \frac{\pi}{2}$.

40. (a)

Given, $f(x)$ is an odd periodic function. We can take $\sin x$, which is odd and periodic.

Now since, $\sin x$ has period = 2 and $f(x)$ has period = 2.

So, $f(x) = \sin(\pi x) \Rightarrow f(4) = \sin(4\pi) = 0$.