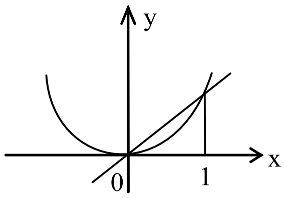
1. **(d)**

Required area = 4A

$$A = \int_0^{\pi} (x + \sin x)\,dx - \int_0^{\pi} x\,dx$$

$$\frac{\pi^2}{2} - \cos\pi + \cos 0 - \frac{\pi^2}{2} = 2 \text{ sq. units}$$

2. **(d)**



$$A_n = \int_0^1 (x - x^n)\,dx = \left[\frac{x^2}{2} - \frac{x^{n+1}}{n+1}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{n+1} = \frac{n-1}{2(n+1)}$$

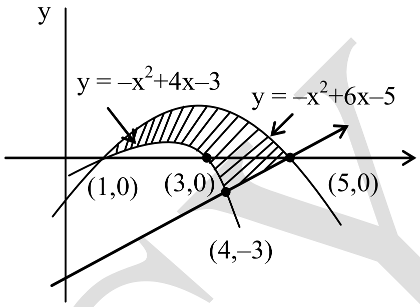
Thus $A_2 . A_3 . A_4 ..... A_n = \frac{1}{2^{n-1}}\left(\frac{1}{3}.\frac{2}{4}.\frac{3}{5}\cdots\frac{n-1}{n+1}\right)$

$$= \frac{1}{2^{n-2}.n(n+1)}$$

3. **(a)**

$$\text{Area} = \sum_{n=0}^{\infty} \frac{1}{2^{n-1}}\left(\frac{1}{2^{n-1}} - \frac{1}{2^n}\right) = \sum_{n=0}^{\infty} \frac{1}{2^{2n-1}} = \frac{8}{3}$$

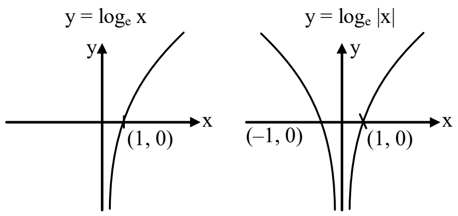
4. **(c)**

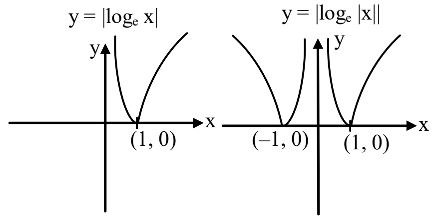


$$A = \int_1^4 \{(-x^2 + 6x - 5) - (-x^2 + 4x - 3)\}\,dx \; +$$

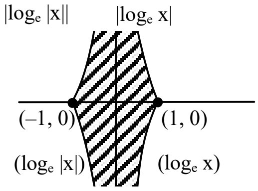
$$\int_4^5 \{(-x^2 + 6x - 5) - (-3x - 15)\}\,dx \quad A = \frac{73}{6}$$

5. **(c)**





So area bounded by four curve



Apply L'Hospital rule

$$\lim_{x\to 0} (x \log x - x)$$

$$\lim_{x\to 0} \left(\frac{\log x - 1}{\frac{1}{x}}\right) \quad (\infty/\infty)$$

$$\lim_{x\to 0} \frac{1/x}{-1/x^2} = \lim_{x\to 0} (-x) \quad = 0$$

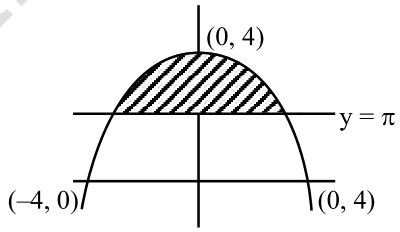
So area $4\left|\int_0^1 (\log_e x)dx\right| = 4\left|[x\log x - x]_0^1\right| = 4$

$$\left|1 - \lim_{x\to 0}(x\log x - x)\right|$$

= 4 |1 - 0| = 4 sq. units.

6. **(c)**

$0 \le \sin^2 x \le 1 \Rightarrow -1 \le -\sin^2 x \le 0$ ∴ϕ0 [– σιν² ξ] = 0 ορ – 1



let $\sec^{-1}$ is not defined hence $y = \sec^{-1}[-\sin^2 x] = \sec^{-1}(-1) =$

π Now $\pi = \frac{16 - x^2}{4} \Rightarrow x^2 = 16 - 4\pi = 4(4 - \pi) \Rightarrow x = \pm 2$

$\sqrt{4-\pi}$ The required area $= \int_{-2\sqrt{4-\pi}}^{2\sqrt{4-\pi}} \left(\frac{16 - x^2}{4} - \pi\right)dx$

$$= \frac{8}{3}(4 - \pi)^{3/2}$$

7. **(c)**

$r = |\cos^{-1}(\sin x)| - |\sin^{-1}(\cos x)|$

$= |\cos^{-1}\cos\left(\frac{\pi}{2}-x\right)| - |\sin^{-1}\sin\left(\frac{\pi}{2}+x\right)| = \left|\frac{5\pi}{2}-x\right| -$

$\left|x-\frac{3\pi}{2}\right| = 4\pi - 2x$ ∴ φ0 δεσιρεδ αρεα $= \int_{3\pi/2}^{2\pi}(4\pi-2x)dx =$

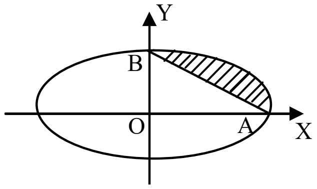
$\frac{\pi^2}{4}$

8. (c)

$A = \left|\int_{\pi/a}^{\pi/3a}\sin ax\ dx\right| = \left|-\frac{1}{a}(\cos ax)_{\pi/a}^{\pi/3a}\right|$

$= \left|-\frac{1}{a}\left(\cos\frac{\pi}{3}-\cos\pi\right)\right| = \left|-\frac{1}{a}\left(\frac{1}{2}+1\right)\right| = \frac{3}{2a}$

9. (d)



Area $= \frac{\pi}{4}$ ab - $\frac{1}{2}$ ab

10. (c)

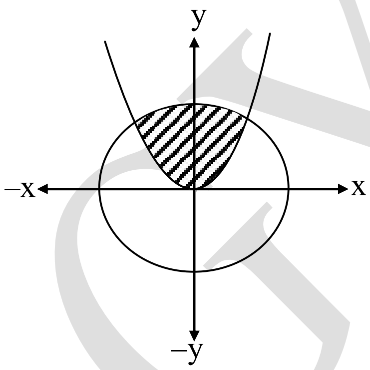
$\lim_{x\to 0}\left[1+\frac{f(x)}{x^3}\right]^{1/x}$ exists so $\lim_{x\to 0}\frac{f(x)}{x^3} = 0$ It means f(x) =

$a_4x^4 + a_5x^5 + \ldots\ldots + a_nx^n$, $a_n \neq 0$ $n \geq 4$

f(x) is of least degree $f(x) = a_4x^4$

$\lim_{x\to 0}\left[1+\frac{f(x)}{x^3}\right]^{1/x} = e$, $a_4 = 1$, $f(x) = x^4$ The graph of $y = x^4$

and $x^2 + y^2 = 2$



Area $= 2\int_0^1\left(\sqrt{2-x^2}-x^4\right)dx$

$= \frac{\pi}{2} - \frac{3}{5}$

11. (b)

If $1 \leq x < 2 \Rightarrow [x] = 1 \Rightarrow [y] = \pm 1$

$y \in [-1, 0) \cup [1, 2)$

If $2 \leq x < 3 \Rightarrow [x] = 2 \Rightarrow [y] = \pm 2$

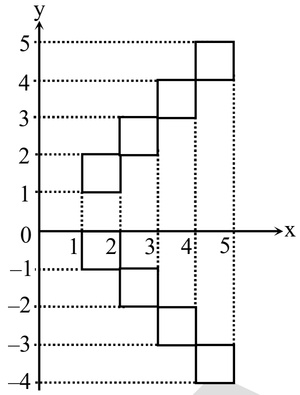
$y \in [-2, -1) \cup [2, 3)$

If $3 \leq x < 4 \Rightarrow [x] = 3 \Rightarrow [y] = \pm 3$

$y \in [-3, -2) \cup [3, 4)$

If $4 \leq x < 5 \Rightarrow [x] = 4 \Rightarrow [y] = \pm 4$

$y \in [-4, -3), \cup [4, 5)$



from figure required area consist of eight squares each of area unity Required area = 8 sq. unit

12. (c)

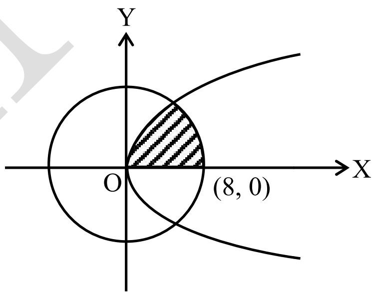
y = x + sin x its inverse is given by x = y + sin y ∴ required

area $= \int_{\pi/6}^{5\pi/6}(y+\sin y)\,dy$

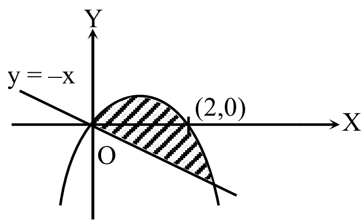
$= \left(\frac{y^2}{2}-\cos y\right)\Big|_{\pi/6}^{5\pi/6}$

$= \sqrt{3} + \frac{\pi^2}{3}$

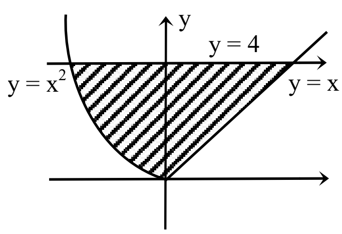
13. (b)



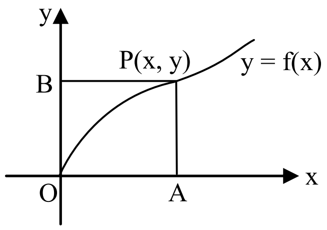
14. (a)



15. (d)



16. (b)



Area of OAPB= xy ; area OAPO = $\int_0^x f(x)dx$

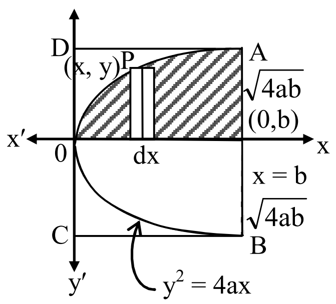
$\Rightarrow \int_0^x f(x)dx = \frac{2}{3} x\, f(x)$

Diff. w.r.t. 'x' to get f(x) = $\frac{2}{3}$ f(x) + $\frac{2}{3}$ x f ′ (x) or f(x) = 2f ′ (x)

$x \Rightarrow \frac{y}{x} = 2\frac{dy}{dx}$ or $\ell n\,|y| = 2\ell n|x| + c$

$\Rightarrow y = cx^2$

17. (c)



Let $y^2 = 4ax$ be a parabola and let x = b be a double ordinate. Then, A_1 = Area enclosed by the parabola $y^2 = 4ax$ and the double ordinate x = b.

$= 2\int_0^b y\,dx = 2\int_0^b \sqrt{4ax}\,dx = 4\sqrt{a}\int_0^b \sqrt{x}\,dx$

$= 4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^b$

$= 4\sqrt{a} \times \frac{2}{3}b^{3/2} = \frac{8}{3}a^{1/2}b^{3/2}$ And, A_2

= Area of the rectangle ABCD = AB × AD = $2\sqrt{4ab}$ × b = 4 $a^{1/2}\,b^{3/2}$

$\therefore A_1 : A_2 = 8/3\, a^{1/2}\, b^{3/2} : 4\, a^{1/2}\, b^{3/2} = 2/3 : 1 = 2 : 3$.

18. (b)

Required area

$= \int_{-2}^{3}|[x-3]|\,dx = \int_{-2}^{-1}|[x-3]|\,dx + \int_{-1}^{0}|[x-3]|\,dx + \int_{0}^{1}|[x-3]|\,dx + \int_{1}^{2}|[x-3]|\,dx + \int_{2}^{3}|[x-3]|\,dx = \int_{-2}^{-1}5\,dx + \int_{-1}^{0}4.\,dx + \int_0^1 3.dx + \int_1^2 2.dx + \int_2^3 1.dx = 5(1) + 4(1) + 3(1) + 2(1) + 1(1)$

= 15 sq. units.

19. (b)

$f'(x) = 4x^3 - 6x^2 + 2x = 2x(2x^2 - 3x + 1) = 2x(2x - 1)(x - 1)$.Since f is a differentiable function, so extremum points of $f(x)$, we must have $f'(x) = 0$ so x = 0, 1/2, 1. Now $f''(x) = 12x^2 - 12x + 2$, $f''(0) = 2$ $f''(1) = 2$ and $f''(1/2) = 3 - 6 + 2 = -1$. Thus the function has minimum at x = 0 and x = 1.

Therefore, the required area = $\int_0^1 (x^4 - 2x^3 + x^2 + 3)\,dx$

$= \left(\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} + 3x\right)\Big|_0^1 = \frac{1}{5} - \frac{1}{2} + \frac{1}{3} + 3 = \frac{91}{30}$.

20. (d)

Required area = $\int_{-5\pi/6}^{\pi}|\cos x|\,dx$

$= -\int_{-5\pi/6}^{-\pi/2}\cos x dx + \int_{-\pi/2}^{\pi/2}\cos x dx - \int_{\pi/2}^{\pi}\cos x dx = -\sin x\Big|_{-5\pi/6}^{-\pi/2} + \sin x\Big|_{-\pi/2}^{\pi/2} - \sin x\Big|_{-\pi/2}^{\pi} = 7/2.$