1. (c)
According to formula, total number of functions = n^n
Here, n = 10. So, total number of functions = 10^{10}.

2. (b)
$f(-1)=\frac{-1-|-1|}{|-1|}=\frac{-1-1}{1}=-2$.

3. (c)
Given $f(y)=\log y \Rightarrow f(1/y)=\log(1/y)$, then $f(y)+f\left(\frac{1}{y}\right)$
$=\log y+\log(1/y)=\log 1=0$.

4. (c)
$f(x)=\log\left(\frac{1+x}{1-x}\right)$
$\therefore f\left(\frac{2x}{1+x^2}\right)=\log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right]=\log\left[\frac{x^2+1+2x}{x^2+1-2x}\right]$
$=\log\left[\frac{1+x}{1-x}\right]^2=2\log\left[\frac{1+x}{1-x}\right]=2f(x)$

5. (d)
$f(x)=\cos[\pi^2]x+\cos[-\pi^2]x$
$f(x)=\cos(9x)+\cos(-10x)=\cos(9x)+\cos(10x)$
$=2\cos\left(\frac{19x}{2}\right)\cos\left(\frac{x}{2}\right)$
$f\left(\frac{\pi}{2}\right)=2\cos\left(\frac{19\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$; $f\left(\frac{\pi}{2}\right)=2\times\frac{-1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}=-1$.

6. (d)
$f(x+y)=f(x)+f(y)$
put $x=1, y=0 \Rightarrow f(1)=f(1)+f(0)=7$
put $x=1, y=1 \Rightarrow f(2)=2.f(1)=2.7$; similarly $f(3)=3.7$ and so on
$\therefore \sum_{r=1}^{n} f(r)=7(1+2+3+.....+n)=\frac{7n(n+1)}{2}$.

7. (c)
$f(x)=\frac{1}{\sqrt{x+2\sqrt{2x-4}}}+\frac{1}{\sqrt{x-2\sqrt{2x-4}}}$
$f(11)=\frac{1}{\sqrt{11+2\sqrt{18}}}+\frac{1}{\sqrt{11-2\sqrt{18}}}$
$=\frac{1}{3+\sqrt{2}}+\frac{1}{3-\sqrt{2}}=\frac{3-\sqrt{2}}{7}+\frac{3+\sqrt{2}}{7}=\frac{6}{7}$.

8. (a)
For domain, $x^2-1>0 \Rightarrow (x-1)(x+1)>0$
$\Rightarrow x<-1$ or $x>1 \Rightarrow x\in(-\infty,-1)\cup(1,\infty)$.

9. (b)
For domain, $|x|-x>0 \Rightarrow |x|>x$.
This is possible, only when $x\in R^-$.

10. (c)
Here $f(x)=\frac{\log_2(x+3)}{x^2+3x+2}=\frac{\log_2(x+3)}{(x+1)(x+2)}$ exists if,
Numerator $x+3>0 \Rightarrow x>-3$ ...... (i)
and denominator $(x+1)(x+2)\neq 0 \Rightarrow x\neq -1,-2$ ...... (ii)
Thus, from (i) and (ii); we have domain of $f(x)$ is $(-3,\infty)-\{-1,-2\}$.

11. (b)
The quantity square root is positive, when
$-1-\sqrt{3}\le x\le -1+\sqrt{3}$. 8

12. (b)
$x^2-6x+7=(x-3)^2-2$ Obviously, minimum value is -2 and maximum ∞.

13. (d)
$f(x)=\sqrt{x-x^2}+\sqrt{4+x}+\sqrt{4-x}$
clearly $f(x)$ is defined if
$4+x\ge 0 \Rightarrow x\ge -4$
$4-x\ge 0 \Rightarrow x\le 4$
$x(1-x)\ge 0 \Rightarrow x\ge 0$ and $x\le 1$
$\therefore$ Domain of $f=(-\infty,4]\cap[-4,\infty)\cap[0,1]=[0,1]$.

14. (c)
The function $f(x)=\sqrt{\log(x^2-6x+6)}$ is defined when
$\log(x^2-6x+6)\ge 0$
$\Rightarrow x^2-6x+6\ge 1 \Rightarrow (x-5)(x-1)\ge 0$
This inequality hold if $x\le 1$ or $x\ge 5$. Hence, the domain of the function will be $(-\infty,1]\cup[5,\infty)$.

15. (d)
$2^y=2-2^x$
y is real if $2-2^x\ge 0 \Rightarrow 2>2^x \Rightarrow 1>x$
$x\in(-\infty, 1)$

16. (a)
$f(x)=\sin^{-1}[\log_2(x/2)]$
Domain of $\sin^{-1}x$ is $x\in[-1,1]$
$\Rightarrow -1\le\log_2(x/2)\le 1 \Rightarrow \frac{1}{2}\le\frac{x}{2}\le 2 \Rightarrow 1\le x\le 4$
$\therefore x\in[1,4]$.

17. (c)
$$f(x)=\begin{cases}\frac{1}{2}(-x-1), & x<-1\\ \tan^{-1}x, & -1\le x\le 1\\ \frac{1}{2}(x+1), & x>1\end{cases} \Rightarrow f'(x)=\begin{cases}-\frac{1}{2}, & x<-1\\ \frac{1}{1+x^2}, & -1<x<1\\ \frac{1}{2}, & x>1\end{cases}$$
$f'(-1-0)=-\frac{1}{2}$; $f'(-1+0)=\frac{1}{1+(-1+0)^2}=\frac{1}{2}$
$f'(1-0)=\frac{1}{1+(1-0)^2}=\frac{1}{2}$; $f'(1+0)=\frac{1}{2}$
$\therefore f'(-1)$ does not exist.

$\therefore$ domain of $f'(x) = R - \{-1\}$.

18. (d)

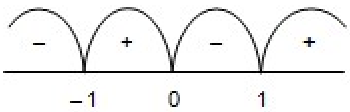
$f(x) = \dfrac{3}{4-x^2} + \log_{10}(x^3 - x)$

So, $4 - x^2 \neq 0 \Rightarrow x \neq \pm\sqrt{4} \Rightarrow x \neq \pm 2$

and $x^3 - x > 0 \Rightarrow x(x^2-1) > 0 \Rightarrow x > 0, |x| > 1$

$\therefore \quad D = (-1, 0) \cup (1, \infty) - \{2\}$

$D = (-1, 0) \cup (1, 2) \cup (2, \infty)$.



19. (c)

$f(x)$ is to be defined when $x^2 - 1 > 0$

$\Rightarrow x^2 > 1, \Rightarrow x < -1$ or $x > 1$ and $3 + x > 0$

$\therefore \quad x > -3$ and $x \neq -2$

$\therefore \quad D_r = (-3, -2) \cup (-2, -1) \cup (1, \infty)$.

20. (a)

$-\dfrac{\pi}{6} \le \sin^{-1}(2x) \le \dfrac{\pi}{2} \Rightarrow -\dfrac{1}{2} \le 2x \le 1 \Rightarrow x \in \left[-\dfrac{1}{4}, \dfrac{1}{2}\right]$.

21. (b)

$f(x) = \dfrac{x+2}{|x+2|} = \begin{cases} -1, & x < -2 \\ 1, & x > -2 \end{cases}$

$\therefore$ Range of $f(x)$ is $\{-1, 1\}$.

22. (a)

$f(x) = \sec\left(\dfrac{\pi}{4}\cos^2 x\right)$

We know that, $0 \le \cos^2 x \le 1$ at $\cos x = 0$, $f(x) = 1$

and at $\cos x = 1$, $f(x) = \sqrt{2}$

$\therefore \quad 1 \le x \le \sqrt{2} \Rightarrow x \in [1, \sqrt{2}]$.

23. (c)

$f(x) = 1 + \dfrac{1}{\left(x + \dfrac{1}{2}\right)^2 + \dfrac{3}{4}} \Rightarrow$ Range $= (1, 7/3]$.

24. (b)

f is one-one because $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow x_1 = x_2$

Further $f^{-1}(x) = \dfrac{x-3}{2} \notin N$ (domain) when x = 1, 2, 3 etc.

$\therefore$ f is into which shows that f is one-one into.

25. (b)

We have $f(x) = (x-1)(x-2)(x-3) \Rightarrow f(1) = f(2) = f(3) = 0$

$\Rightarrow f(x)$ is not one-one

For each $y \in R$, there exists $x \in R$ such that $f(x) = y$.

Therefore f is onto.

Hence, $f : R \to R$ is onto but not one-one.

26. (b)

Number of surjection from A to B $= \sum_{r=1}^{2} (-1)^{2-r}\,{}^2C_r (r)^4$

$= (-1)^{2-1}\,{}^2C_1(1)^4 + (-1)^{2-2}\,{}^2C_2(2)^4 = -2 + 16 = 14$

Therefore, number of surjection from A to B = 14.

Trick : Total number of functions from A to B is 2^4 of which two function $f(x) = a$ for all $x \in A$ and $g(x) = b$ for all $x \in A$ are not surjective. Thus, total number of surjection from A to B $= 2^4 - 2 = 14$.

27. (d)

Total number of one-one onto functions = 3!

28. (d)

$f(-1) = f(1) = 1 \quad \therefore$ function is many-one function.

Obviously, f is not onto so f is neither one-one nor onto.

29. (b)

For any $x, y \in R$, we have

$f(x) = f(y) \Rightarrow \dfrac{x-m}{x-n} = \dfrac{y-m}{y-n} \Rightarrow x = y$ ∴ φ0 φ ισ ονε–ονε

Λετ $\alpha \in R$ such that $f(x) = \alpha \Rightarrow \dfrac{x-m}{x-n} = \alpha \Rightarrow x = \dfrac{m - n\alpha}{1-\alpha}$

Clearly $x \notin R$ for $\alpha = 1$. So, f is not onto.

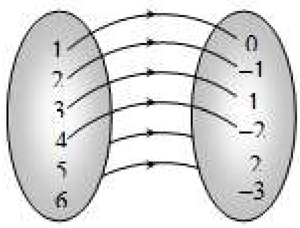
30. (c)

Function $f : R \to R$ is defined by $f(x) = e^x$. Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$ or $e^{x_1} = e^{x_2}$ or $x_1 = x_2$. Therefore f is one-one. Let $f(x) = e^x = y$. Taking log on both sides, we get $x = \log y$. We know that negative real numbers have no pre-image or the function is not onto and zero is not the image of any real number. Therefore function f is into.

31. (c)

$f : N \to I$

$f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2, f(5) = 2$ and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

32. (a)

We have : $f(x) = x\left(\dfrac{a^x - 1}{a^x + 1}\right)$ $f(-x) = -x\left(\dfrac{a^{-x} - 1}{a^{-x} + 1}\right) = -x\left(\dfrac{\dfrac{1}{a^x} - 1}{\dfrac{1}{a^x} + 1}\right)$

$= -x\left(\dfrac{1 - a^x}{1 + a^x}\right) = x\left(\dfrac{a^x - 1}{a^x + 1}\right) = f(x)$

So, $f(x)$ is an even function

33. (b)

$f(x)=\sqrt{x^4+15} \Rightarrow f(-x)=\sqrt{(-x)^4+15}=\sqrt{x^4+15}=f(x)$

$\Rightarrow f(-x)=f(x)\Rightarrow f(x)$ is an even function

$\Rightarrow f(x)$ is symmetric about y-axis.

34. (a)

$fog(x)=f\{g(x)\}=f(x^2+1)=(x^2+1+1)^2=(x^2+2)^2$

$\Rightarrow fog(-3)=(9+2)^2=121$.

35. (a)

$f(x)=\sin^2 x+\sin^2(x+\pi/3)+\cos x\cos(x+\pi/3)$

$=\frac{1-\cos 2x}{2}+\frac{1-\cos(2x+2\pi/3)}{2}+\frac{1}{2}\{2\cos x\cos(x+\pi/3)\}$

$=\frac{1}{2}[1-\cos 2x+1-\cos(2x+2\pi/3)+\cos(2x+\pi/3)+\cos\pi/3]$

$=\frac{1}{2}\left[\frac{5}{2}-\{\cos 2x+\cos\left(2x+\frac{2\pi}{3}\right)\}+\cos\left(2x+\frac{\pi}{3}\right)\right]$

$=\frac{1}{2}\left[\frac{5}{2}-2\cos\left(2x+\frac{\pi}{3}\right)\cos\frac{\pi}{3}+\cos\left(2x+\frac{\pi}{3}\right)\right]=5/4$ for all x.

$\therefore\ gof(x)=g(f(x))=g(5/4)=1$ [$\because$ g(5/4) =1 (given)]

Hence, $gof(x)=1$, for all x.

36. (a)

$g(x)=x^2+x-2 \Rightarrow (gof)(x)=g[f(x)]=[f(x)]^2+f(x)-2$ Given,

$\frac{1}{2}(gof)(x)=2x^2-5x+2$

$\therefore\ \frac{1}{2}[f(x)]^2+\frac{1}{2}f(x)-1=2x^2-5x+2$

$\Rightarrow [f(x)]^2+f(x)=4x^2-10x+6$

$\Rightarrow f(x)[f(x)+1]=(2x-3)[(2x-3)+1] \Rightarrow f(x)=2x-3$.

37. (c)

$$f[g(y)]=\frac{y/\sqrt{1+y^2}}{\sqrt{1-\left(\frac{y}{\sqrt{1+y^2}}\right)^2}}=\frac{y}{\sqrt{1+y^2}}\times\frac{\sqrt{1+y^2}}{\sqrt{1+y^2-y^2}}=y$$

38. (b)

Here $g(x)=1+n-n=1, x=n\in Z$

$1+n+k-n=1+k,\ x=n+k$ (where $n\in Z,\ 0<k<1$)

Now $f(g(x))=\begin{cases}-1, & g(x)<0\\ 0, & g(x)=0\\ 1, & g(x)>0\end{cases}$

Clearly, $g(x)>0$ for all x. So, $f(g(x))=1$ for all x.

39. (d)

Here $f(2)=\frac{5}{4}$

Hence $(fof)(2)=f(f(2))=f\left(\frac{5}{4}\right)=\frac{2\times\frac{5}{4}+1}{3\times\frac{5}{4}-2}=2$

40. (d)

$g(f(x))\le f(g(x)) \Rightarrow g(|x|)\le f[x] \Rightarrow [|x|]\le|[x]|$.

This is true for $x\in R$.

41. (b)

Clearly, $f:R\to R$ is a one-one onto function. So, it is invertible.

Let $f(x)=y$. then, $3x-5=y\Rightarrow x=\frac{y+5}{3}\Rightarrow f^{-1}(y)=\frac{y+5}{3}$.

Hence, $f^{-1}(x)=\frac{x+5}{3}$.

42. (c)

$f(x)=3x-4=y \Rightarrow y=3x-4 \Rightarrow x=\frac{y+4}{3}$

$\Rightarrow f^{-1}(y)=\frac{y+4}{3} \Rightarrow f^{-1}(x)=\frac{x+4}{3}$.

43. (c)

Let h (x) = |x| then g(x) |f(x)| = h)(f(x))

Since composition of two continuous function is continuous, g is continuous if f is continuous.

44. (d)

It is given that $2^x+2^y=2\ \forall x, y\in R$

But $2^x.\ 2^y>0\ \forall x, y\in R$

Therefore, $2^x=2-2^y<2$

$\Rightarrow 0<2^x<2$

Taking log for both side with base 2 $\Rightarrow \log_2 0<\log_2 2^x<\log_2 2$. Hence domain is $-\infty<x<1$.

45. (b)

g(x) = 1 + x – [x]; f(x) $\begin{cases}-1, x<0\\ 0,\ x=0\\ 1,\ x>0\end{cases}$

For integral values of x; g(x) = 1 > 0

For x < 0; x - [x] > 0 $\Rightarrow$ g(x) > 0

(but not integral value)

For x > 0; x –[x] > 0 $\Rightarrow$ g(x) > 0

$\therefore$ g(x) > 0, $\forall$ x

$\therefore$ f(g(x)) = 1, $\forall$ x

$\therefore$ '’(b) is the correct alternative.

46. (a)

f: $[1,\infty)\to[2,\infty)$

F(x) $x+\frac{1}{x}=y\Rightarrow x^2-yx+1=0\Rightarrow x\ \frac{y\pm\sqrt{y^2-4}}{2}$

But $\begin{cases}x>1\\ y>2\end{cases}\ \therefore\phi 0\ \ x=\frac{y+\sqrt{y^2-4}}{2}$

$\therefore\ f^{-1}(x)=\frac{x+\sqrt{x^2-4}}{2}$

$\therefore$ 'A' is the correct alternative

47. (d)

For domain of f(x) = $\frac{log_2 9x+3}{x^2+3x+2}$

$x^2+3x+2\neq 0$ and $x+3>0$

$\Rightarrow x \neq -1, -2$ and $x > -3$

$\therefore D_f = (-3, \infty) - (-1, -2)$

$\therefore$ (d) is the correct alternative.

48. (a)

From E to F we can define, in all $2 \times 2 \times 2 \times 2 = 16$ functions (2 options for each element of E) out of which 2 are into, when all the elements of E either map to 1 or to 2.

$\therefore$ No. of onto functions = 16 - 2 = 14

$\therefore$ (a) is the correct alternative.

49. (a)

Any element of set A, say x_i, can be connected with the elements of set B in n ways. Hence these are exactly n^m functions.

50. (a)

For function to be one-one, each element of set A must have a different image in set B. We first of all choose any 'm' elements in set B. This can be done in ${}^nC_m$ ways. Now one-one correspondence of elements of set A with these selected elements can be done in m! ways. Thus total number of one-one functions will be equal to ${}^nC_m.m!$ i.e., ${}^np_m$.